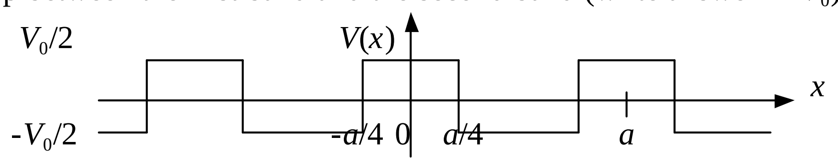
Solid State Physics – Final Exam 9:10 AM -- 11:00 AM (1/20/2005)

1. [40%] Consider a free electron gas in 2-dimension.
   a) Find out the connection between electron density n and Fermi wave vector k_F.
   b) If you are given an unknown sample, briefly explain how do you determine its n value experimentally? Also, how can you estimate the value of t by experiment?
   c) The electron density n for the two-dimensional electron gas (2DEG) is 6.4×10^{11} cm^{-2}. Find out its Fermi energy (in eV).
   d) We known that the electron specific heat C_e is proportional to T at low temperature for three-dimensional free electrons. Is this still true for 2DEG? If it is, explain why? If not, give the correct T-dependence. Also explain what do you mean by "low" temperature?

2. [40%] Consider a 1-dim crystal with potential $V(x)$ and lattice constant a. We would like to discuss the behavior of ε(k) near Brillouin zone boundary k=π/a.
   (a) Assume the wave function is simply $y = b_0 e^{ikx} + b_1 e^{i(kx-2px/a)}$, and $V(x)$ is decomposed as $V(x) = \sum_{n=-\infty}^{\infty} C_n e^{i2pnx/a}$, ($C_0$=0). Write down the matrix equation for the unknowns b_0 and b_1. (C_n are regarded as known.)
   (b) Follow the approximation used in (a). For the potential below, find the energy gap between the first band and the second band (write answer in V_0).



   (c) Follow (b), find the effective mass for the electron on the top of the first band.

3. [20%] In many semiconductors, the dependence of the electron energy, E, on the wave vector, k, for the lowest minimum of the conduction band differs from a simple parabolic relation. A more accurate equation linking E and k is given by

$$E(1+aE) = \hbar^2 k^2 / 2m,$$

where k is the wave vector, and a is a constant (called a non-parabolicity constant). In this case, the electron effective mass is a function of energy. Find out the dependence of the effective mass m^* on energy.

h=6.6×10^{-34} Js; m_e=9.1×1^{-31} kg; e= 1.6×10^{-19} C; 1 eV = 1.6×10^{-19} J