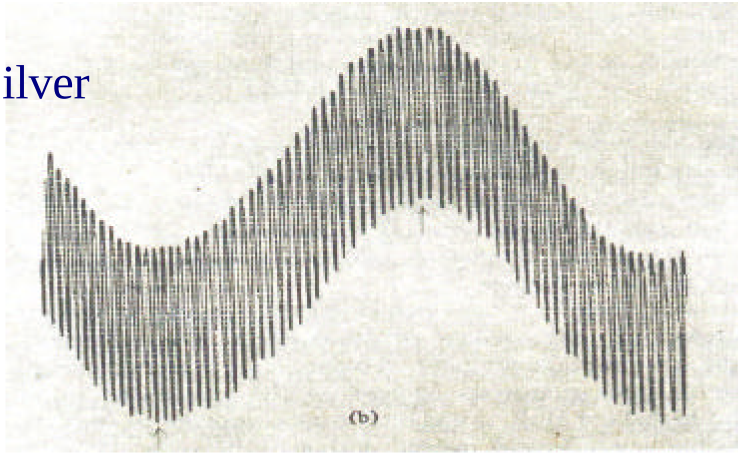
Measuring FS using the de Haas-van Alphen effect

In a high magnetic field, the magnetization M of a crystal oscillates as the magnetic field increases (dHvA effect, 1930)

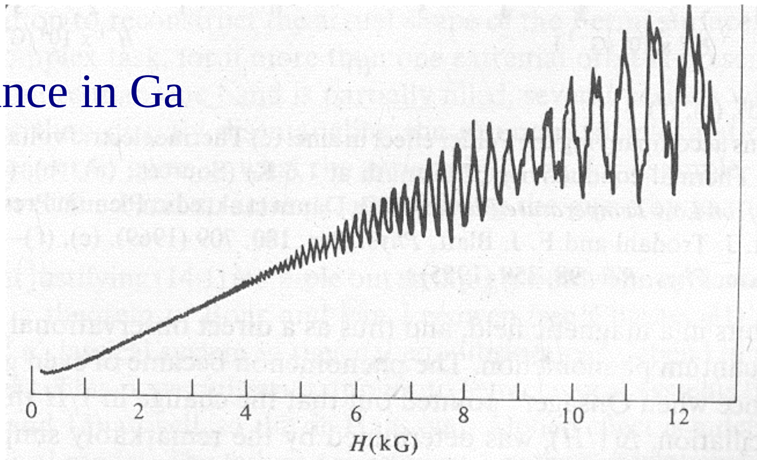
Silver



Similar oscillatons are observed in other physical quantities

Eg., magnetoresistivity (Shubnikov-de Haas effect, 1930), specific heat, sound attenuation… etc

Resistance in Ga



Basically, they are all due to the quantization of electron energy levels in a magnetic field (Landau levels, also 1930)

- Quantization of the cyclotron orbits

In Chap 12, the radius of the cyclotron orbit can be varied continuously; but because of their wave nature, the electron orbits are quantized.

- Bohr-Sommerfeld quantization rule

$$\boxed{\oint d\vec{r}\cdot\vec{p} = \left(n+\frac{1}{2}\right)h}$$

$$\text{where } \vec{p} = \vec{p}_{kin} + \vec{p}_{field} = \hbar\vec{k} + \frac{q}{c}\vec{A},\ q = -e$$

$$\begin{cases} \oint d\vec{r}\cdot\hbar\vec{k} = -\dfrac{e}{c}\oint d\vec{r}\cdot\vec{r}\times\vec{H} = 2\dfrac{e}{c}\Phi \\ \dfrac{e}{c}\oint d\vec{r}\cdot\vec{A} = \dfrac{e}{c}\Phi \end{cases}$$

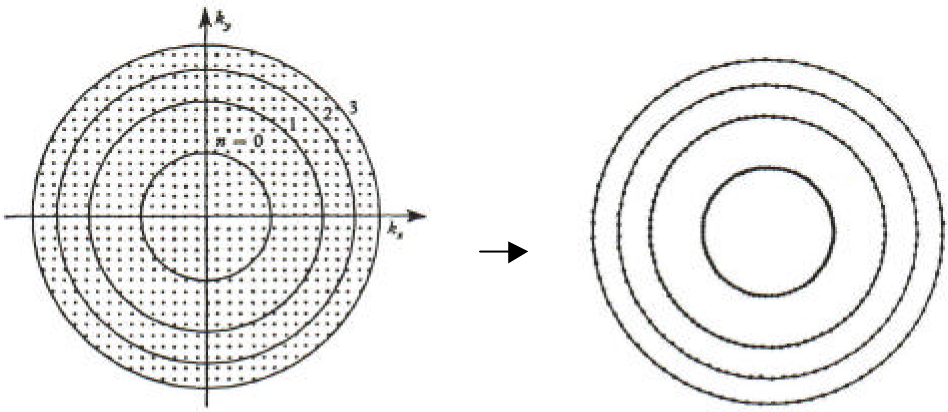
→ $\Phi_n=(n+1/2)\ hc/e$, the orbit is quantized

($hc/e \equiv \Phi_0 = 4.14\cdot10^{-7}$ gauss·cm^2 is the flux quantum)

- Since a k-orbit (circling an area S) is closely related to a r-orbit (circling an area A), the orbits in k-space are also quantized

$$S_n = A_n/\lambda_B^4$$

$$= (n+1/2)\,(2\pi e/\hbar c)\,H,$$ Onsager, 1952



- The number of points collected by each orbit

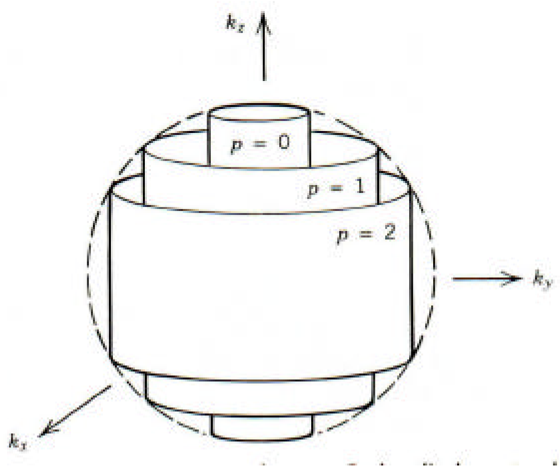
$$D = (2\pi eH/\hbar c)/(2\pi/L)^2 = HL^2/(hc/e) = \Phi_{sample}/\Phi_0$$

- Energy of the orbit (for spherical FS)

$$E_n = (\hbar k_n)^2/2m = (n+1/2)\hbar\omega_c \leftarrow \text{Landau levels}$$

- The kz direction is not quantized

$$E_{n,k_z} = \left(n+\frac{1}{2}\right)\hbar w_c + \frac{\hbar^2 k_z^2}{2m^*}$$



Note:

1. In the presence of H, the Fermi sphere becomes a stack of cylinders.

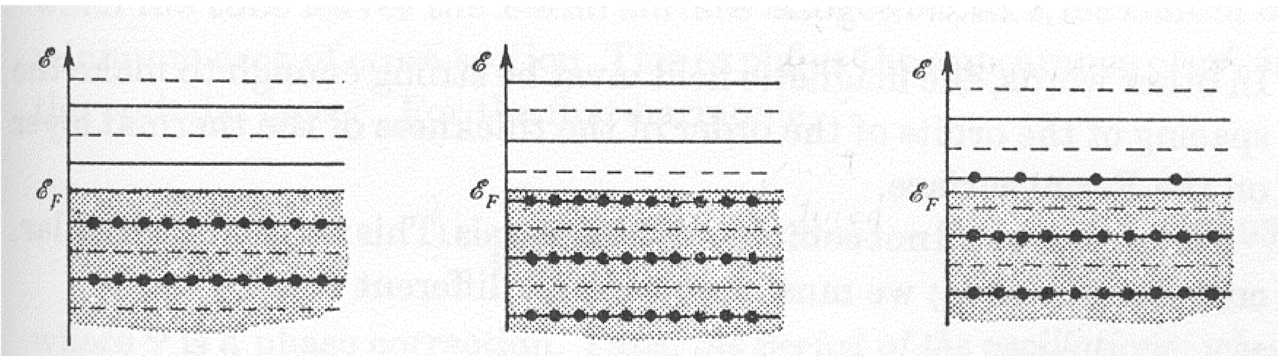
2. Fermi energy ≈ 1 eV,

   cyclotron energy ≈ 0.1 meV (for H = 1 Tesla)

   ∴ the number of cylinders usually ≈ 10000!

   need low T and high H to observe the fine structure

3 Radius of cylinders ∝ H, so they expand as we increase H. The orbits are pushed out of the FS one by one.



$\longrightarrow$ increasing H

● Successive H's that produce orbits with the same area:

$$S_n = (n+1/2)\ 2pe/\hbar c\ H$$

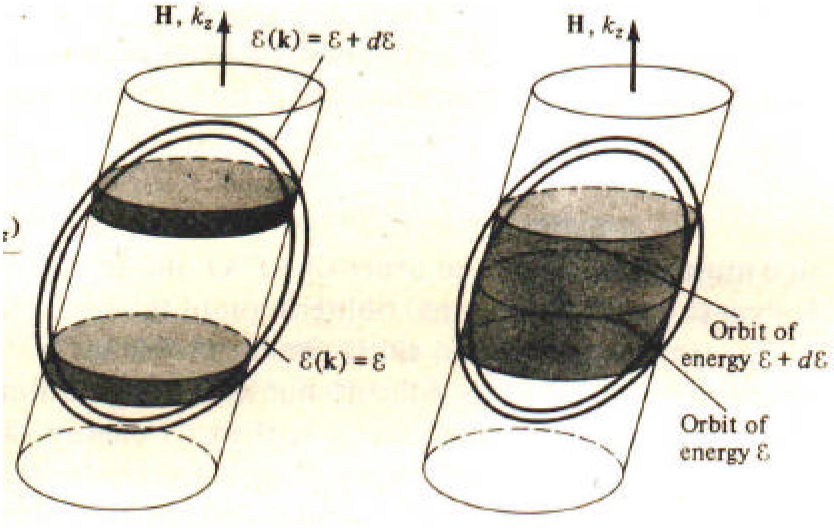
$$S_n' = (n-1/2)\ 2pe/\hbar c\ H' \quad (H' > H)$$

$$\rightarrow \quad \boxed{S\left(\frac{1}{H} - \frac{1}{H'}\right) = \frac{2pe}{\hbar c}}$$

→ equal increment of 1/H reproduces similar orbits

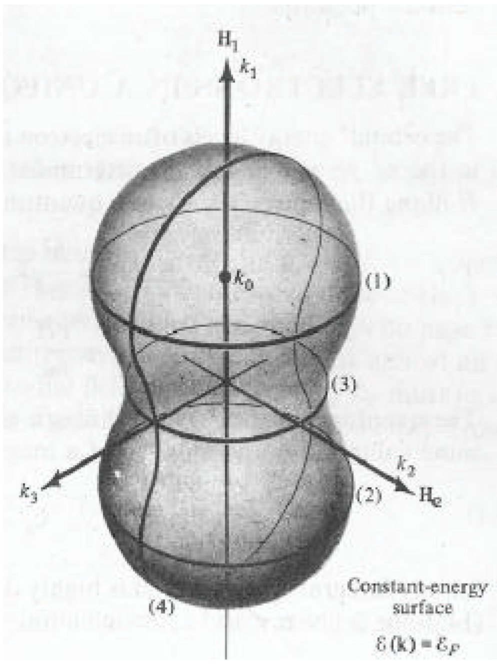
Oscillation of the DOS at the Fermi energy

w/o extremal orbit   w/ extremal orbit



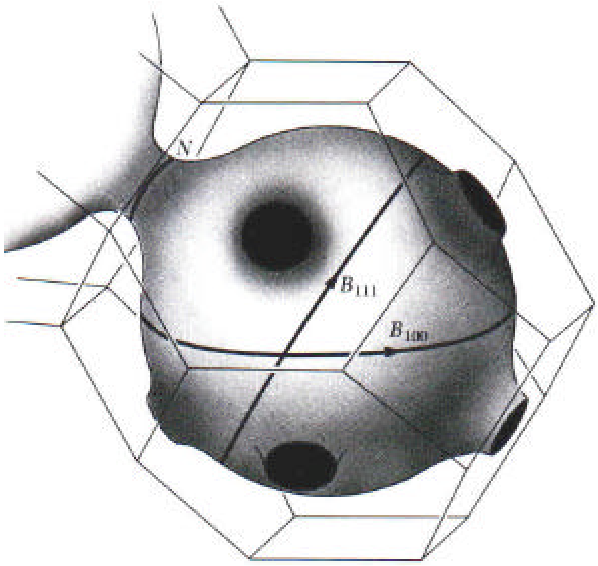
- The number of states at E_F are highly enhanced when there are extremal orbits on the FS
- There are extremal orbits at regular interval of 1/B
- This oscillation in 1/B can be detected in any physical quantity that depends on the DOS

Two extremal orbits



Determination of FS

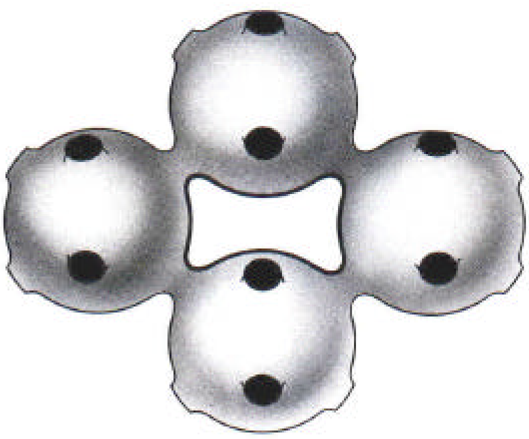
In the dHvA experiment of silver, the two different periods of oscillation are due two different extremal orbits



Recall that $$S\left(\frac{1}{H}-\frac{1}{H'}\right)=\frac{2pe}{\hbar c}$$

Therefore, from the two periods we can determine the ratio between the sizes of the "neck" and the "belly"

The extremal orbit at the [110] direction (no double oscillation)



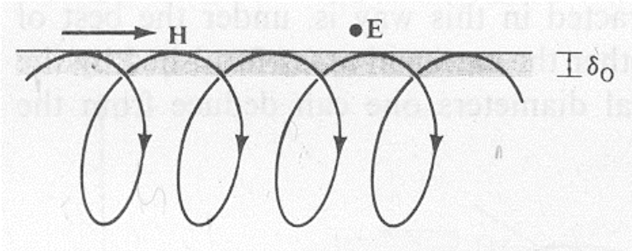
Other Fermi surface probes:

- Azbel-Kaner cyclotron resonance (1956)

  = a steady magnetic field to make cyclotron motion

  + an oscillating electric field to induce resonance

  set-up



typical cyclotron frequency

$$\omega_c = eH/m_c{*}c \approx 10^{11} \text{ (rad) at 1 Tesla}$$

radius of the orbit

$$H \times \pi r_c^{\,2} \approx 10^4\, \Phi_0 = 10^4\, hc/e$$

$\rightarrow$ $r_c \approx 10^2\, (hc/eH)^{1/2} = 10^2\, \lambda_B = 2.56\times10^4$ A at 1 Tesla

penetration depth δ_0 of the oscillating E field at microwave frequency $= c/(2\pi\mu\omega\sigma)^{1/2} \approx 0.1\ \mu$m (for copper)

$\therefore$ electron is accelarated by E field only near the surface

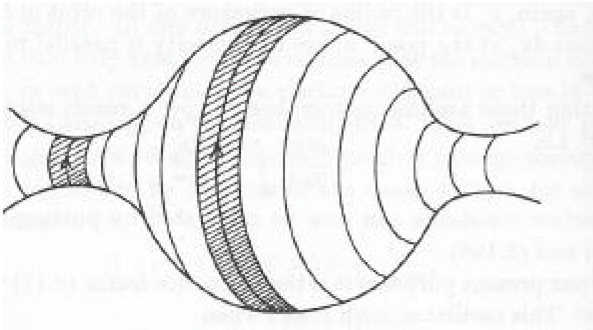
If $\omega_E = n\omega_c$, then electron will absorb energy from the field.

$\rightarrow$ determine $m_c{*}$

(usually we fix E and vary H to get the resonance)

One problem:

Given a H, there can be many cyclotron orbits, with different m_c^* (if the FS is more complicated than an ellipsoid)



It can be shown that the absorption is “likely” to be dominated by the extremal orbits, as in the dHvA effect

Cyclotron resonance near Cu(100) surface

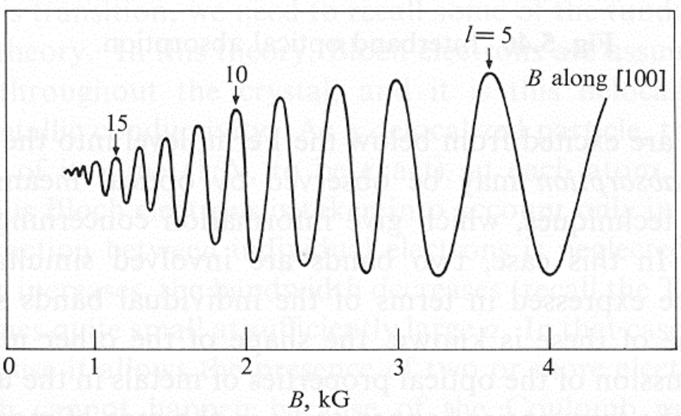


Fig. 5.45 AKCR spectrum in Cu at $T = 4.2°K$. The crystal surface (upper surface) is cut along the (100) plane. The ordinate of the curve represents the derivative of the surface resistivity with respect to the field. [After Haüssler and Wells, *Phys. Rev.*, **152,** 675, 1966]

(periodic in 1/H)