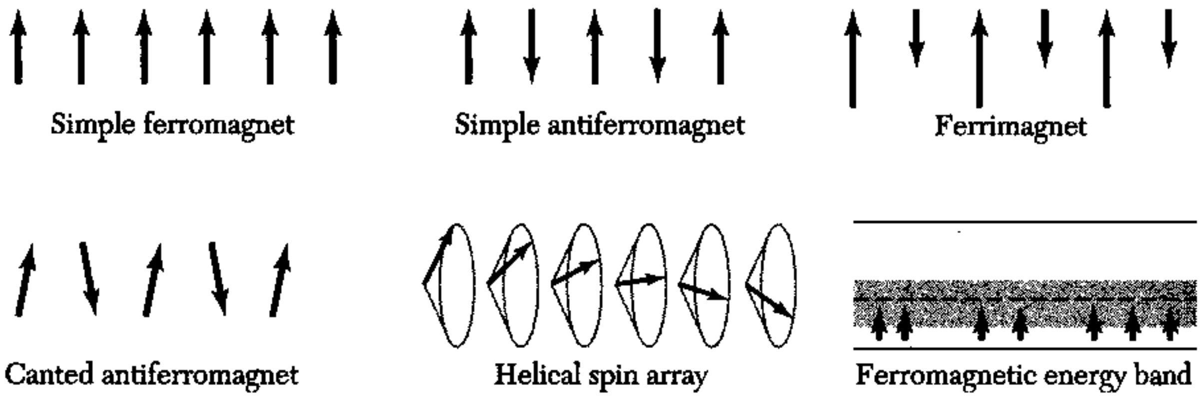
Dept of Phys

M.C. Chang

Ferromagnetism and antiferromagnetism

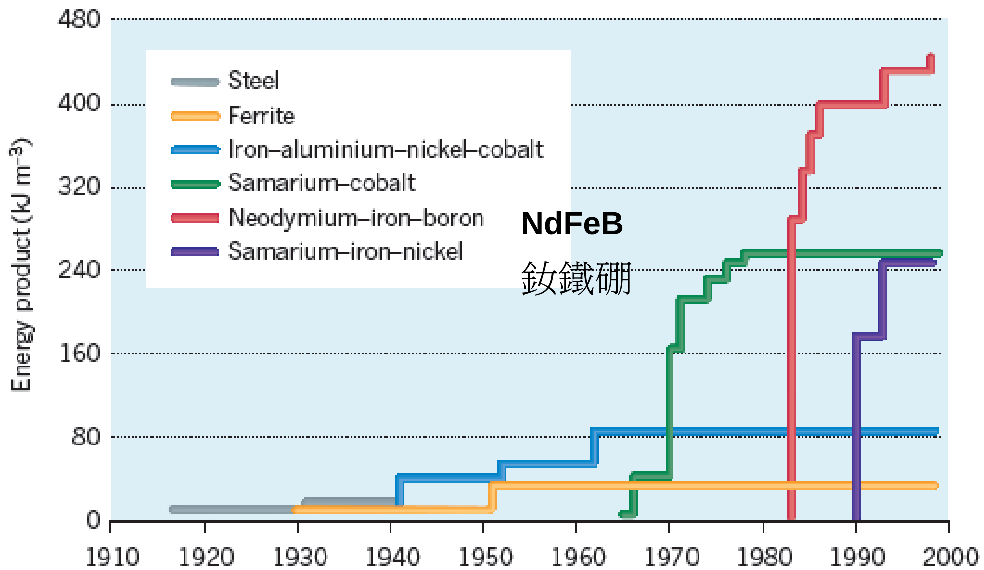
- ferromagnetism (FM)
    - exchange interaction, Heisenberg model
    - spin wave, magnon
- antiferromagnetism (AFM)
- ferromagnetic domains
- nanomagnetic particles

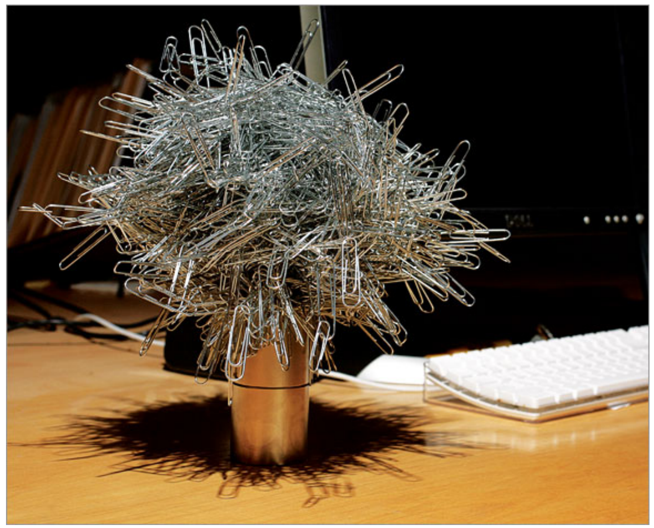
Magnetic order:



15 elements are magnetically ordered in the solid state

H																	He
Li	Be											B	C	N	**O**	F	Ne
Na	Mg											Al	Si	P	S	Cl	Ar
K	Ca	Sc	Ti	V	**Cr**	**Mn**	**Fe**	**Co**	**Ni**	Cu	Zn	Ga	Ge	As	Se	Br	Kr
Rb	Sr	Y	Zr	Nb	Mo	Tc	Ru	Rh	Pd	Ag	Cd	In	Sn	Sb	Te	I	Xe
Cs	Ba	La	Hf	Ta	W	Re	Os	Ir	Pt	Au	Hg	Tl	Pb	Bi	Po	At	Rn
Fr	Ra	Ac															
				Ce	Pr	**Nd**	Pm	**Sm**	**Eu**	**Gd**	**Tb**	**Dy**	**Ho**	**Er**	**Tm**	Yb	Lu
				Th	Pa	U	Np	Pu	Am	Cm	Bk	Cf	Es	Fm	Md	No	Lw





Nature, April 2011

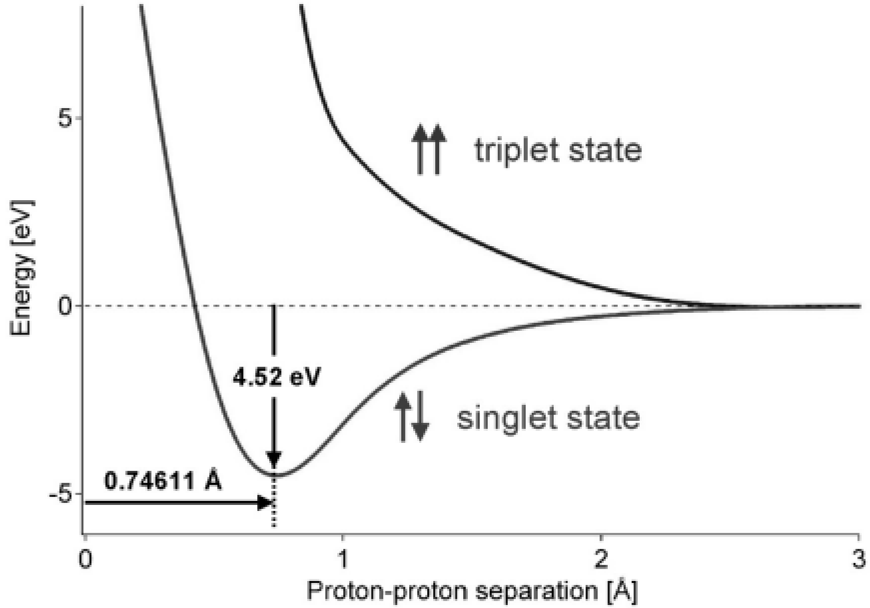
Ferromagnetic insulator (no itinerant electron)

• FM is not from magnetic dipole-dipole interaction, nor the SO interaction. It is a result of electrostatic interaction!

• Estimate of order:

Dipole-dipole
$$U = \frac{1}{r^3}\left[\vec{m}_1 \cdot \vec{m}_2 - 3\left(\vec{m}_1 \cdot \hat{r}\right)\cdot\left(\vec{m}_2 \cdot \hat{r}\right)\right]$$
$$\cong \frac{\left(g\mu_B\right)^2}{r^3} \cong 10^{-4} \text{ eV } (\sim 1 \text{ K}) \text{ for } r \cong 2\text{A}$$

• Because of the electrostatic interaction, some prefers ↑ ↑ , some prefers ↑ ↓ (for example, H2).



• Effective interaction between a pair of spinful ions

$$\vec{S}_1 \cdot \vec{S}_2 = \begin{cases} -3/4 & \text{for singlet} \\ 1/4 & \text{for triplet} \end{cases}$$

$\therefore$ Heisenberg wrote

$$U = -(E_s - E_t)\vec{S}_1 \cdot \vec{S}_2 + \frac{1}{4}(E_s + 3E_t) \rightarrow \begin{cases} E_s \\ E_t \end{cases}$$

$$= -J\vec{S}_1 \cdot \vec{S}_2 + \text{constant} \quad \text{(Heisenberg model)}$$

• J is called the exchange coupling const.

(for 2-e system, the GND state must be a singlet)

• FM has J>0, AFM has J<0

• The tendency for an ion to align the spins of nearby ions is called an exchange field H_E (or molecular field, usually much stronger than applied field.)

• Weiss mean field $H_E = \lambda M$ for FM

$$\vec{M} = \chi_p(\vec{H} + \vec{H}_E), \text{ where } \chi_p = C/T \text{ is PM susceptibility}$$

$$\chi_p = n\left(g_J \mu_B\right)^2 \frac{J(J+1)}{3kT} \equiv \frac{C}{T}$$

$$\Rightarrow \quad \chi = \frac{M}{H} = \frac{C}{T - C\lambda} \equiv \frac{C}{T - T_C} \quad \text{(Curie-Weiss law, for } T > T_c \text{ only)}$$

$$\lambda = \frac{T_c}{C} = \frac{3k_B T_c}{ng^2 \mu_B^2 S(S+1)}$$

For iron, $T_c \sim 1000$ K, $g \sim 2$, $S \sim 1$

$\therefore \lambda \sim 5000$ (no unit in cgs)

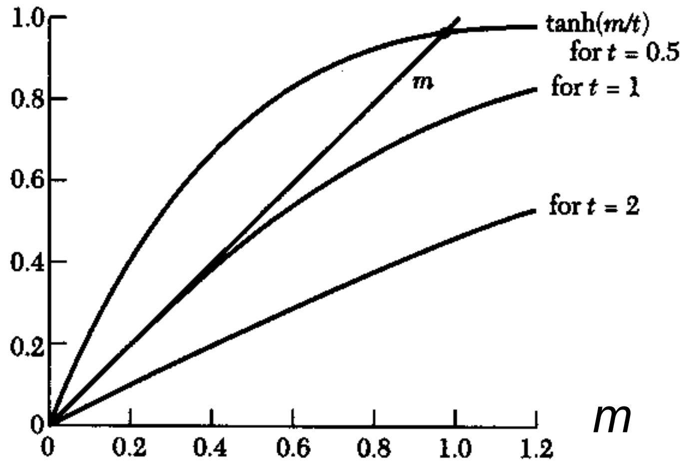
$M_s \sim 1700$ G, $H_E \sim 10^3$ T.

Temperature dependence of magnetization

$$M = \frac{N}{V} g_J \mu_B J B_J\left(\frac{g_J \mu_B J H}{kT}\right)$$

$$\text{where } B_J(x) \equiv \frac{2J+1}{2J}\coth\left(\frac{2J+1}{2J}x\right) - \frac{1}{2J}\coth\left(\frac{x}{2J}\right)$$

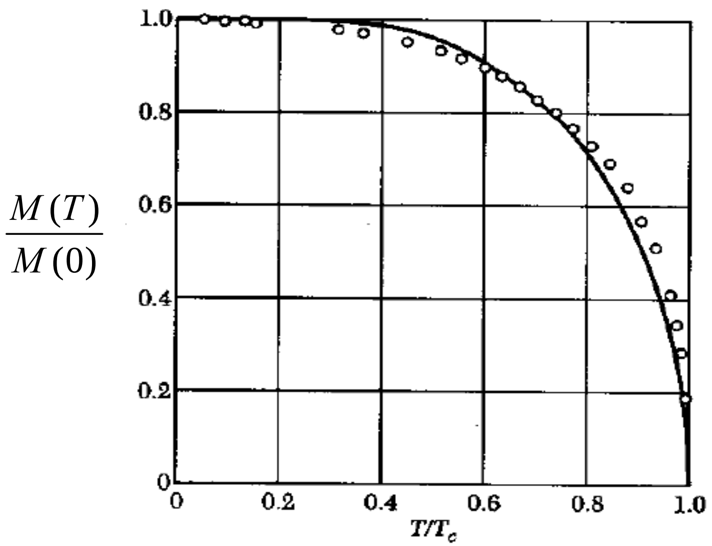
$$= \tanh\ x \quad (\text{for } J = 1/2)$$



$$H \approx \lambda M \quad (H_{ext} \text{ neglected})$$

$$\therefore \quad M = n\mu \tanh \frac{\mu\lambda M}{kT} \qquad (\mu \equiv g_J \mu_B J)$$

$$\text{or } \ m \equiv \frac{M}{n\mu} = \tanh \frac{m}{t} \qquad \left(t \equiv \frac{kT}{n\mu^2\lambda}\right)$$



At low T,

$$\Delta M \equiv M(0) - M(T)$$

$$\approx 2n\mu e^{-2\lambda n\mu^2/kT}$$

Does not agree with experiment, which is ~ $T^{3/2}$. Explained later using spin wave excitation.

Spin wave in 1-dim FM

Heisenberg model $H = -2J\sum_{p=1}^{N} \vec{S}_p \cdot \vec{S}_{p+1} \quad (J>0)$

- Ground state energy $E_0 = -2NJS^2$
- Excited state:

Flip 1 spin costs $8JS^2$. But there is a cheaper way to create excited state.

Classical approach,

$$H = -J\sum_{p=1}^{N} \vec{S}_p \cdot \left(\vec{S}_{p-1} + \vec{S}_{p+1}\right) = -\sum_{p=1}^{N} \vec{\mu}_p \cdot \vec{B}_p$$

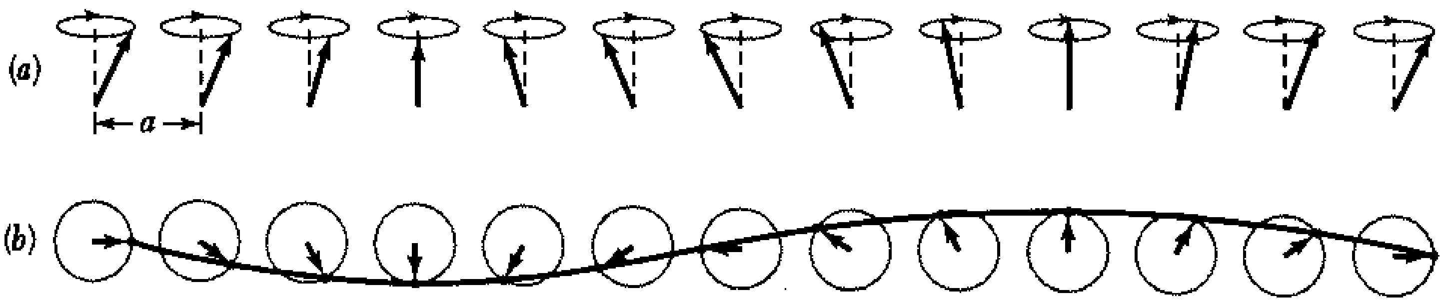
where $\vec{\mu}_p = -g\mu_B \vec{S}_p \qquad \left(\mu_B = e\hbar/2mc\right)$

$\hbar S$ is the classical angular momentum

$$\vec{B}_p \equiv -\frac{J}{g\mu_B}\left(\vec{S}_{p-1} + \vec{S}_{p+1}\right)$$

effective B field (exchange field)

$$\hbar\frac{d\vec{S}_p}{dt} = \vec{\mu}_p \times \vec{B}_p = J\left(\vec{S}_p \times \vec{S}_{p-1} + \vec{S}_p \times \vec{S}_{p+1}\right)$$



Dispersion of spin wave

assume $S_p^z \approx S;\ S_p^x, S_p^y << S_p^z,$

neglect nonlinear terms in S_p^x, S_p^y

$$\Rightarrow \begin{aligned} \hbar \frac{dS_p^x}{dt} &= JS\left(2S_p^y - S_{p-1}^y - S_{p+1}^y\right), \\ \hbar \frac{dS_p^y}{dt} &= -JS\left(2S_p^x - S_{p-1}^x - S_{p+1}^x\right). \end{aligned}$$

let $S_p^x = ue^{i(pka-\omega t)};\ S_p^y = ve^{i(pka-\omega t)},$

then $\begin{pmatrix} i\hbar\omega & 2JS(1-\cos ka) \\ -2JS(1-\cos ka) & i\hbar\omega \end{pmatrix}\begin{pmatrix} u \\ v \end{pmatrix} = 0$

$\Rightarrow \hbar\omega = 2JS(1-\cos ka) \quad \propto k^2$ at long wave length

- Quantized spin wave is called magnon ($\in$ boson)
- magnon energy $\varepsilon_k = \left(n_k + 1/2\right)\hbar\omega_k$
- magnons, like phonons, can interact with each other if nonlinear spin interaction is included

Thermal excitations of magnons

$$M = NS - \sum_k \langle n_k \rangle$$

Number of magnons being excited,

$$\sum_k \langle n_k \rangle = \int d\omega D(\omega) \langle n_k \rangle$$

$$\langle n_k \rangle = \frac{1}{e^{\hbar\omega_k / kT} - 1}$$

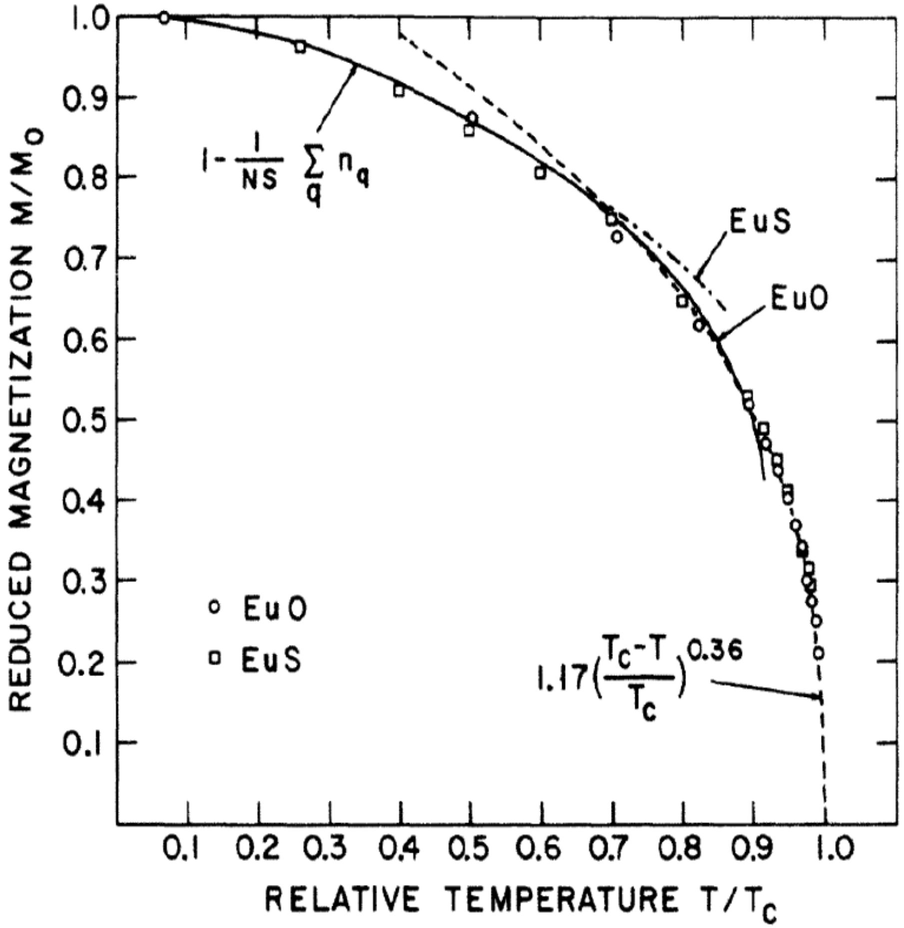
DOS in 3-dimension,

$$D(\omega) = \frac{1}{4\pi^2} \left(\frac{\hbar}{JSa^2} \right)^{3/2} \omega^{1/2}$$

$$\therefore \quad \sum_k \langle n_k \rangle = \frac{1}{4\pi^2} \left(\frac{kT}{JSa^2} \right)^{3/2} \int_0^\infty dx \frac{x^{1/2}}{e^x - 1}$$

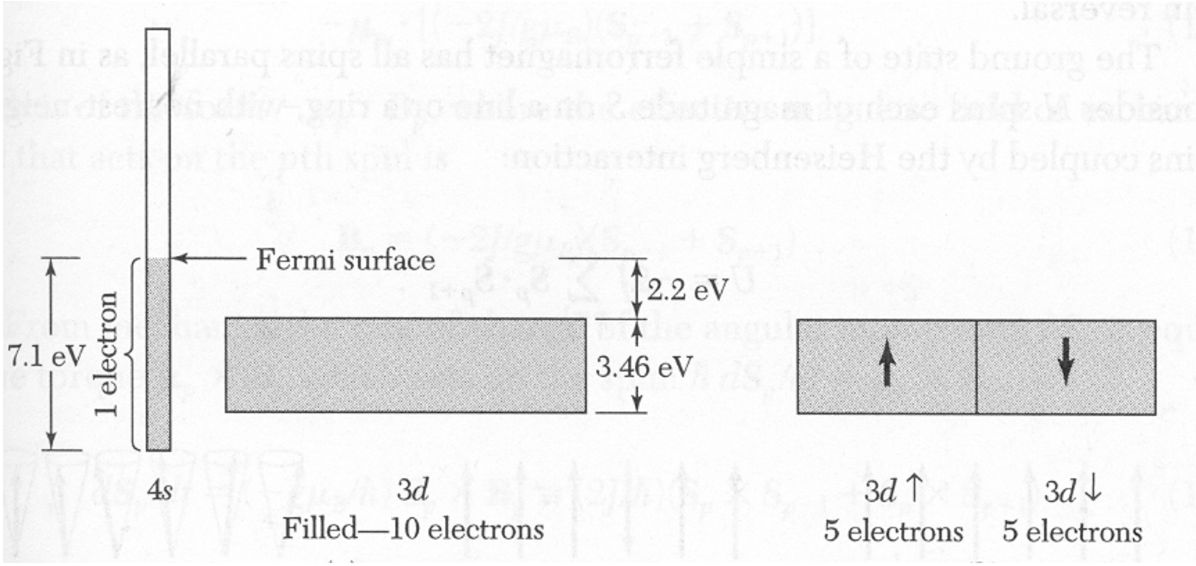
$$0.0587(4\pi^2)$$

$$\frac{\Delta M}{M(0)} = \frac{\sum_k \langle n_k \rangle}{NS} \propto T^{3/2} \quad \text{(Bloch } T^{3/2} \text{ law)}$$

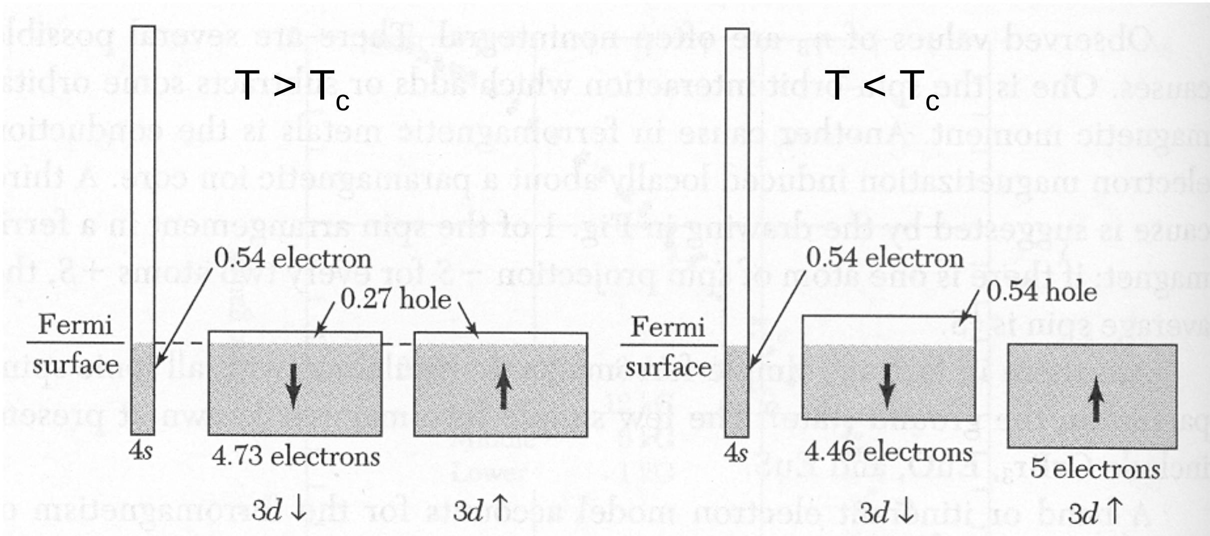


FM in Fe, Co, Ni (Itinerant electrons)

Cu, nonmagnetic



Ni, magnetic

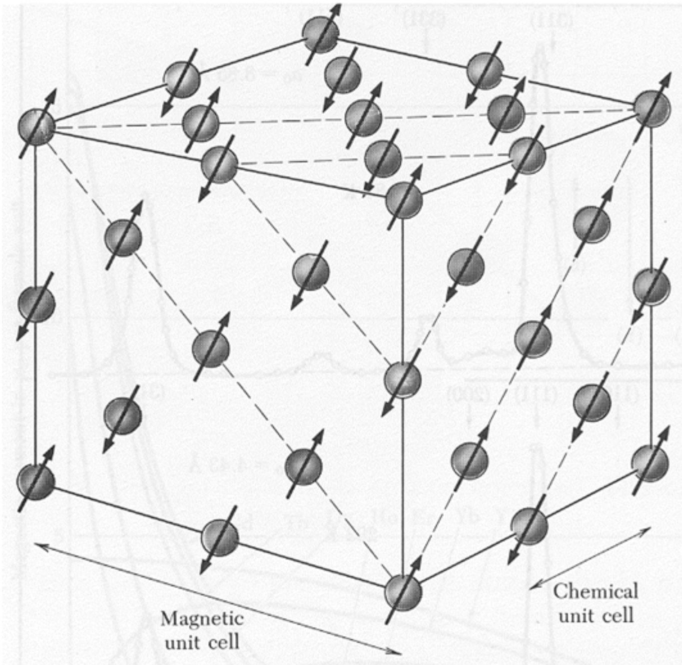


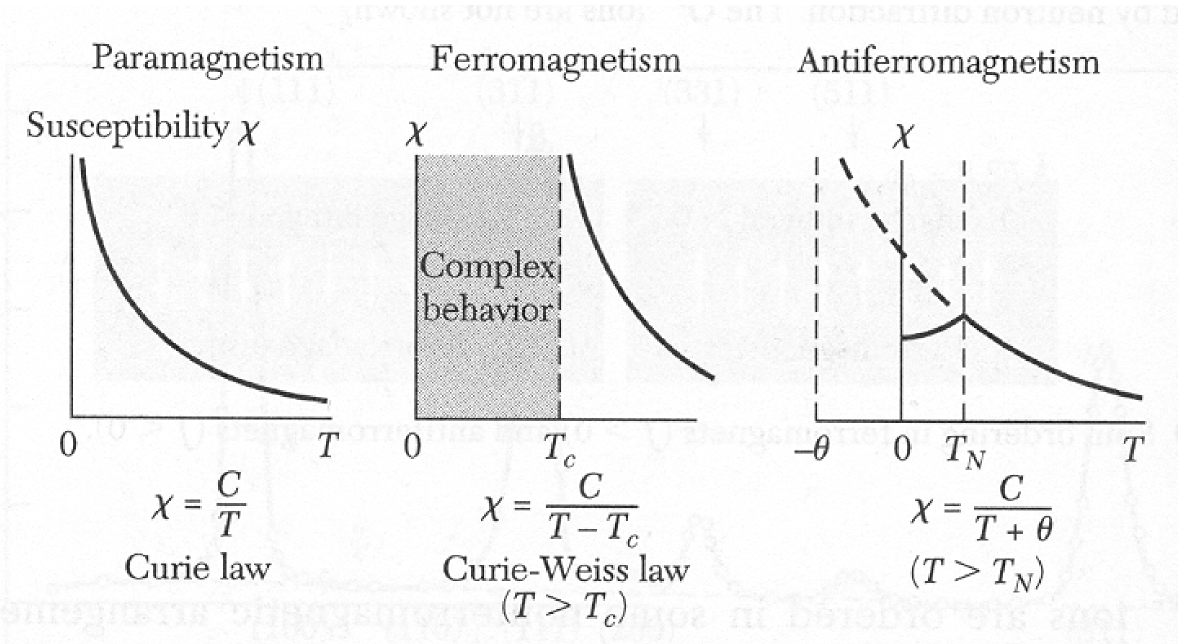
- ferromagnetism (FM)
- antiferromagnetism (AFM)
  - susceptibilities
  - ferrimagnetism
- ferromagnetic domains
- nanomagnetic particles

Antiferromagnetism (predicted by Neel, 1936)

- many AFM are transition metal oxides
- net magnetization is zero, not easy to show that it's a AFM. First confirmed by Shull at 1949 using neutron scattering.

MnO, transition temperature=610 K





T-dependence of susceptibility for $T > T_N$

Consider a AFM consists of 2 FM sublattices A, B.

$$\vec{H}_A = -\lambda \vec{M}_B; \quad \vec{H}_B = -\lambda \vec{M}_A$$

Use separate Curie consts C_A, C_B for sublattices A, B

$$M_A T = C_A (H - \lambda M_B);$$

$$M_B T = C_B (H - \lambda M_A).$$

$$\rightarrow \begin{pmatrix} T & \lambda C_A \\ \lambda C_B & T \end{pmatrix} \begin{pmatrix} M_A \\ M_B \end{pmatrix} = \begin{pmatrix} C_A \\ C_B \end{pmatrix} H$$

There is non-zero solution when $H = 0$

only if det $= 0$

$$\Rightarrow \quad T_N = \lambda (C_A C_B)^{1/2}$$

At $T > T_N$,

$$\chi = \frac{M_A + M_B}{H} = \frac{(C_A + C_B)T - 2\lambda (C_A C_B)}{T^2 - T_N^2}$$

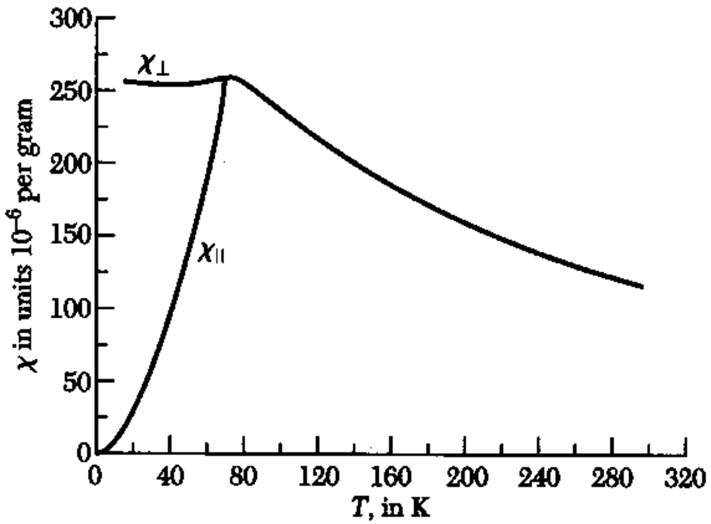
For identical sublattices,

$$\chi = \frac{2CT - 2\lambda C^2}{T^2 - (\lambda C)^2} = \frac{2C}{T + T_N}, \quad T_N = \lambda C$$

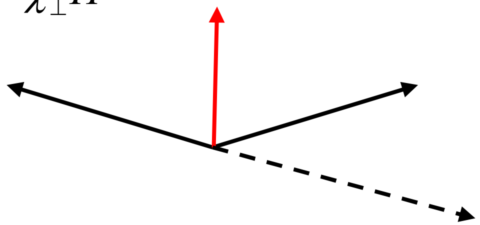
Experiment: $\chi = \dfrac{2C}{T + \theta}$

Substance	Paramagnetic ion lattice	Transition temperature, T_N, in K	Curie-Weiss θ, in K	$\frac{\theta}{T_N}$	$\frac{\chi(0)}{\chi(T_N)}$
MnO	fcc	116	610	5.3	$\frac{2}{3}$
MnS	fcc	160	528	3.3	0.82
MnTe	hex. layer	307	690	2.25	
MnF_2	bc tetr.	67	82	1.24	0.76
FeF_2	bc tetr.	79	117	1.48	0.72
$FeCl_2$	hex. layer	24	48	2.0	<0.2
FeO	fcc	198	570	2.9	0.8
$CoCl_2$	hex. layer	25	38.1	1.53	
CoO	fcc	291	330	1.14	
$NiCl_2$	hex. layer	50	68.2	1.37	
NiO	fcc	525	~2000	~4	
Cr	bcc	308			

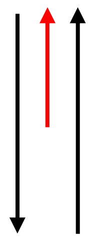
• Susceptibility for $T < T_N$ (Kittel, p343)



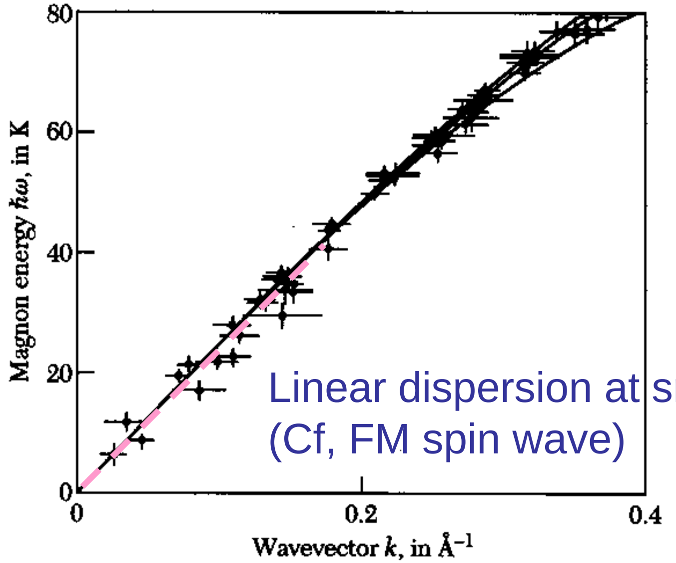
$$M = \chi_{\perp} H$$



$$M = \chi_{//} H$$



• Dispersion relation for AFM spin wave (Kittel, p344)



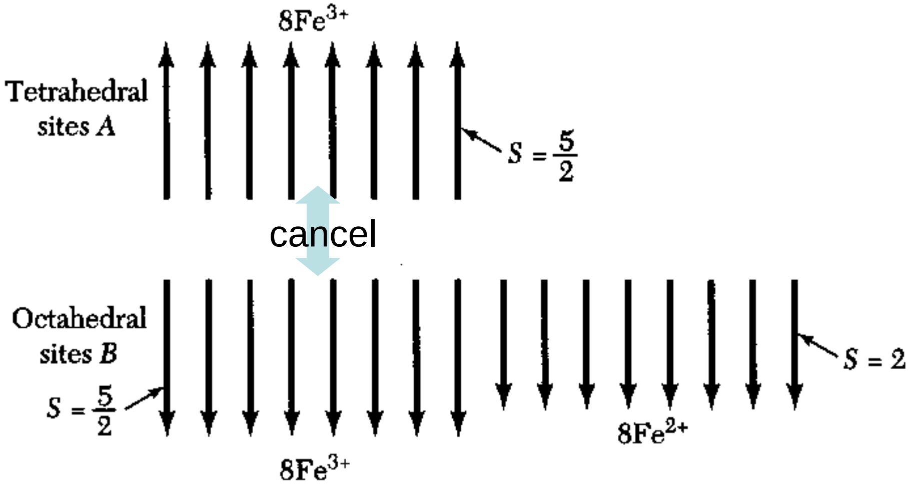
Linear dispersion at small k
(Cf, FM spin wave)

Ferrimagnetic materials

磁鐵礦 Magnetite (Fe_3O_4 or $FeO \cdot Fe_2O_3$) Hematite 赤鐵礦

• Curie temperature 585 C

• belong to a more general class of ferrite $MO \cdot Fe_2O_3$ (M=Fe, Co, Ni, Cu, Mg…) 磁性氧化物



Iron garnet 鐵石榴石

e.g., • **Yttrium iron garnet** (**YIG**) $Y_3Fe_2(FeO_4)_3$, or $Y_3Fe_5O_{12}$ 釔鐵石榴石 is a ferrimagnetic material with Curie temperature 550 K.

• YIG has high degree of Faraday effect, high Q factor in microwave frequencies, low absorption of infrared wavelengths up to 600 nm … etc (wiki)

- ferromagnetism (FM)
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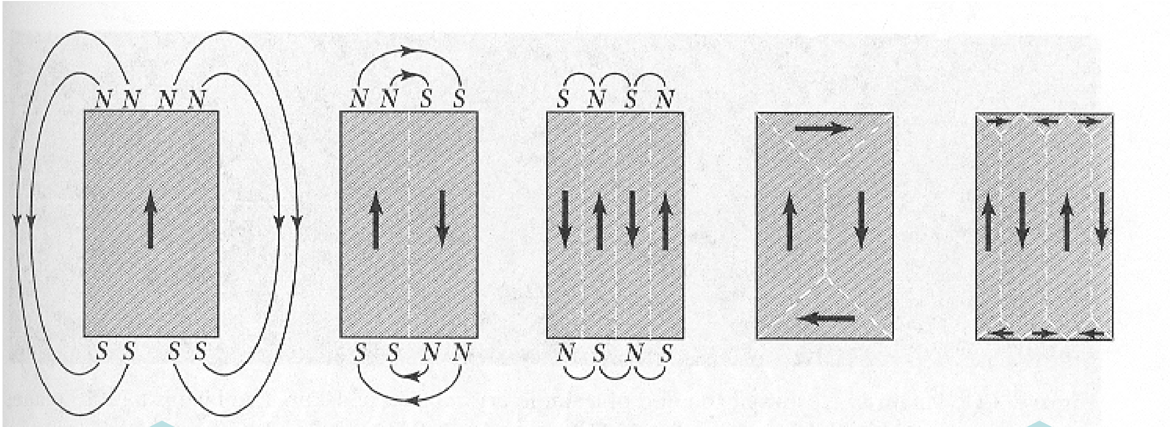
Magnetic domains (proposed by Weiss 1906)

Why not all spins be parallel to reduce the exchange energy?

→ it would cost “stray field” energy

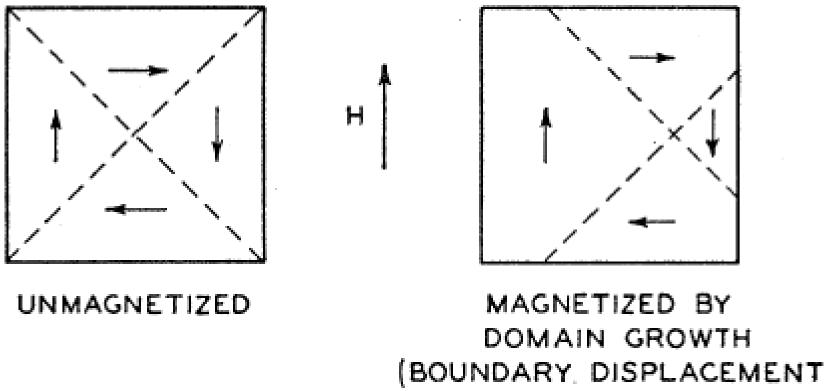


magnetic energy $B^2/8\pi$

$\approx 10^6$ erg/cm^3

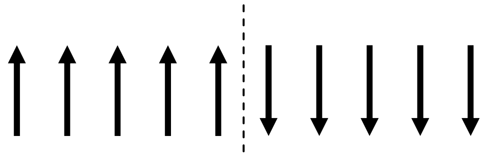
Little leaking field

• Magnetization and domains



Transition between domain walls

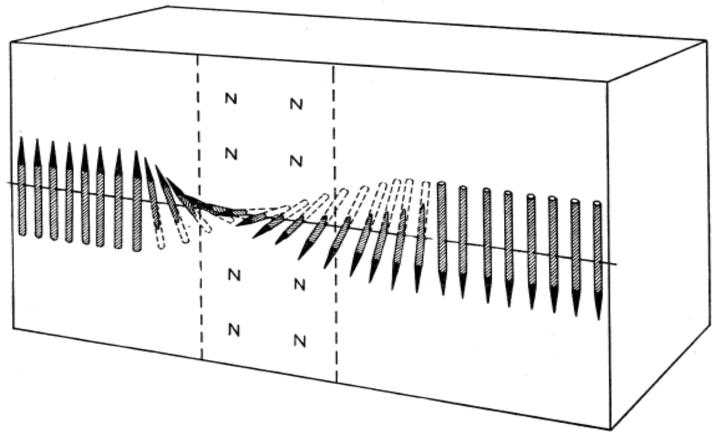
Why not just



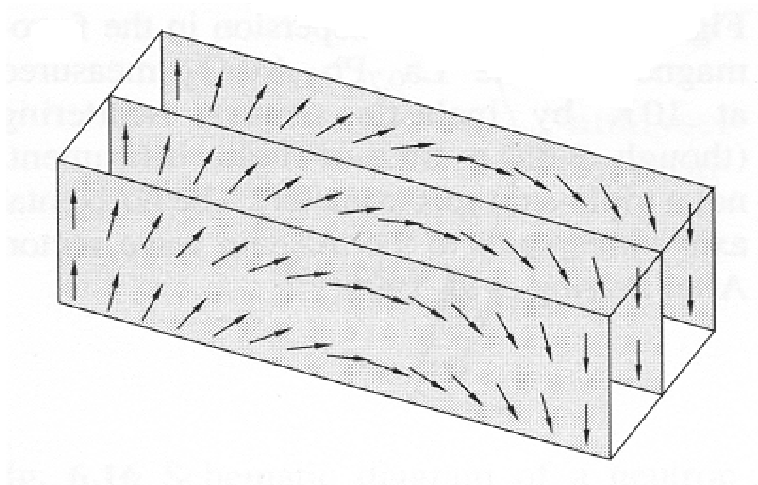
→ Would cost too much exchange energy

(not so in ferroelectric materials)

• Bloch wall



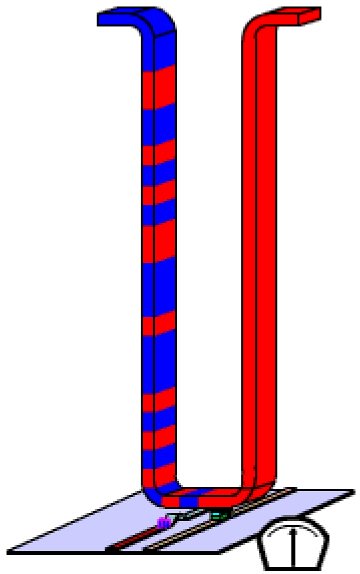
• Neel wall



Domain wall dynamics

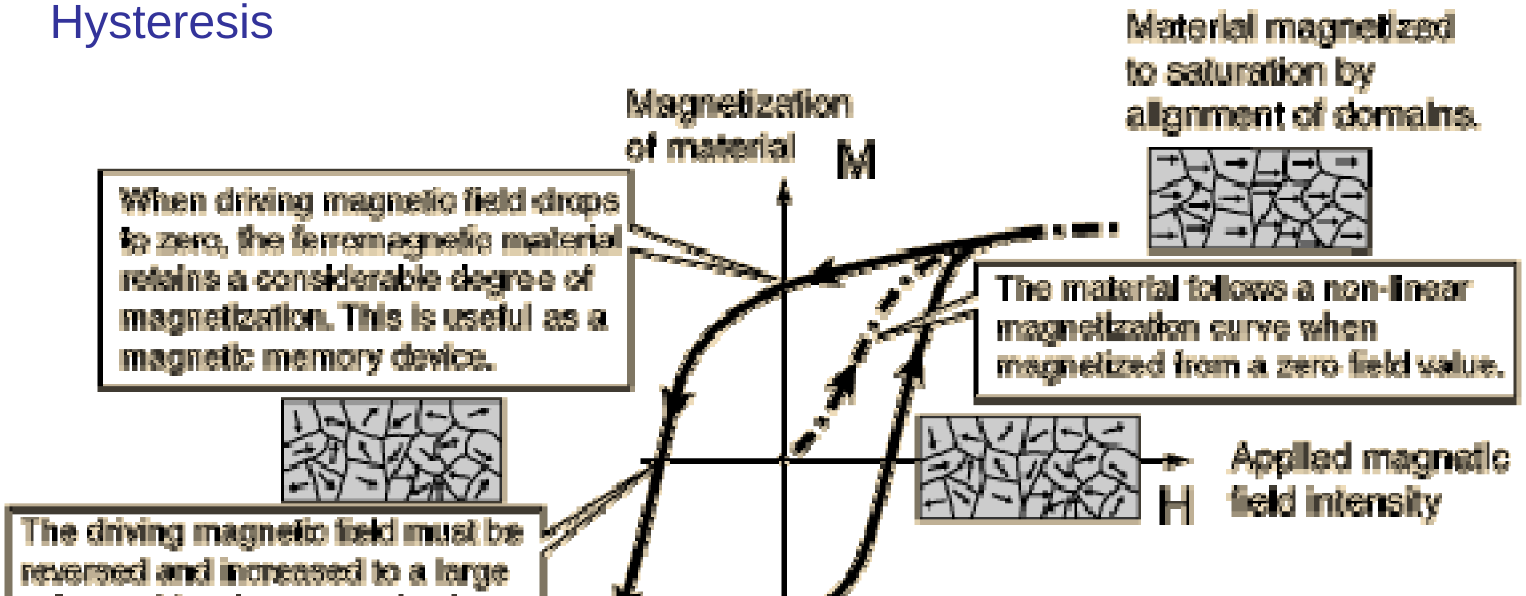
• domain wall motion induced by spin current

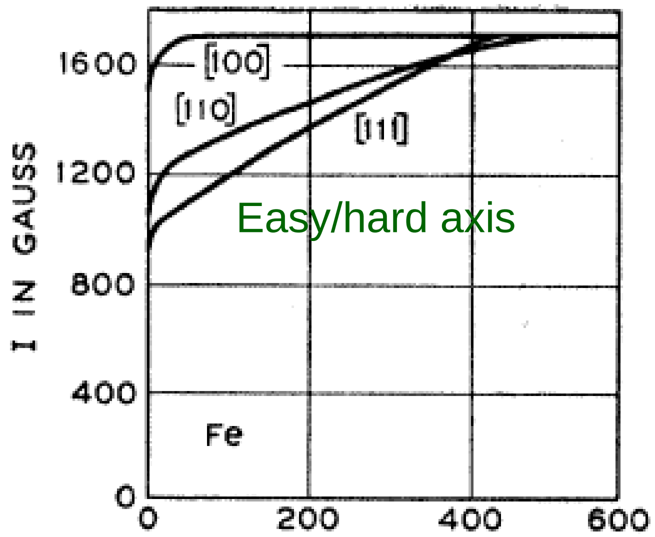
• …



Race-track memory (IBM)

Hysteresis

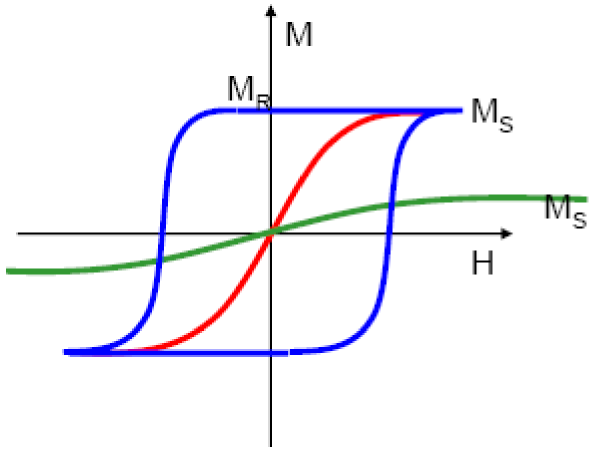




From hyperphysics

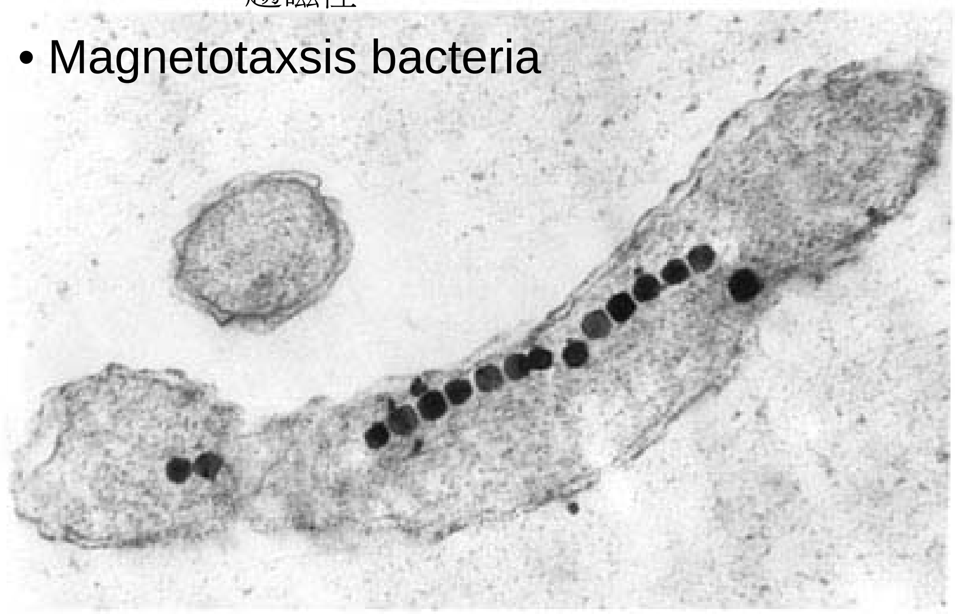
Single domain particle: ferrofluid, magnetic data storage …

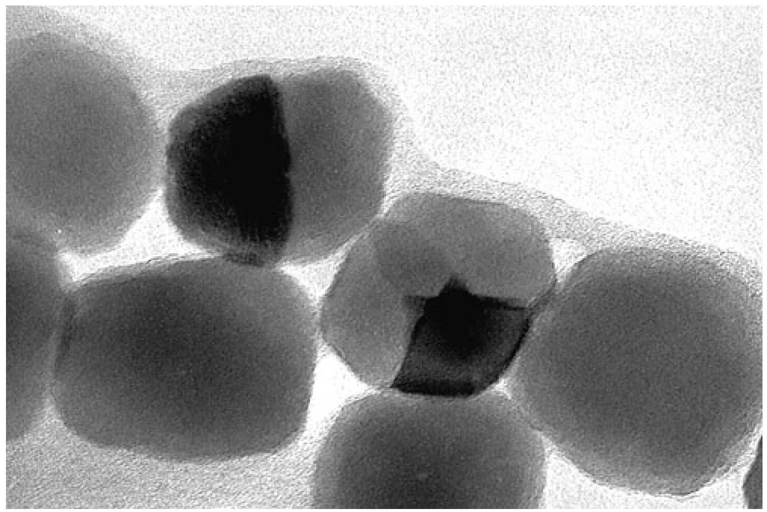
• superparamagnetism



趨磁性

• Magnetotaxsis bacteria





http://www.calpoly.edu/~rfrankel/mtbphoto.html

supplementary

The zoo of magnetoresistance (first discovered by Lord Kelvin, 1857)

- GMR (giant MR)
- CMR (colossal MR)
- TMR (tunneling MR, Julliere, Phys. Lett. 1975)
- EMR (extraordinary MR, Solin, 2000)
- …

Soh and Aeppli, Nature (2002)

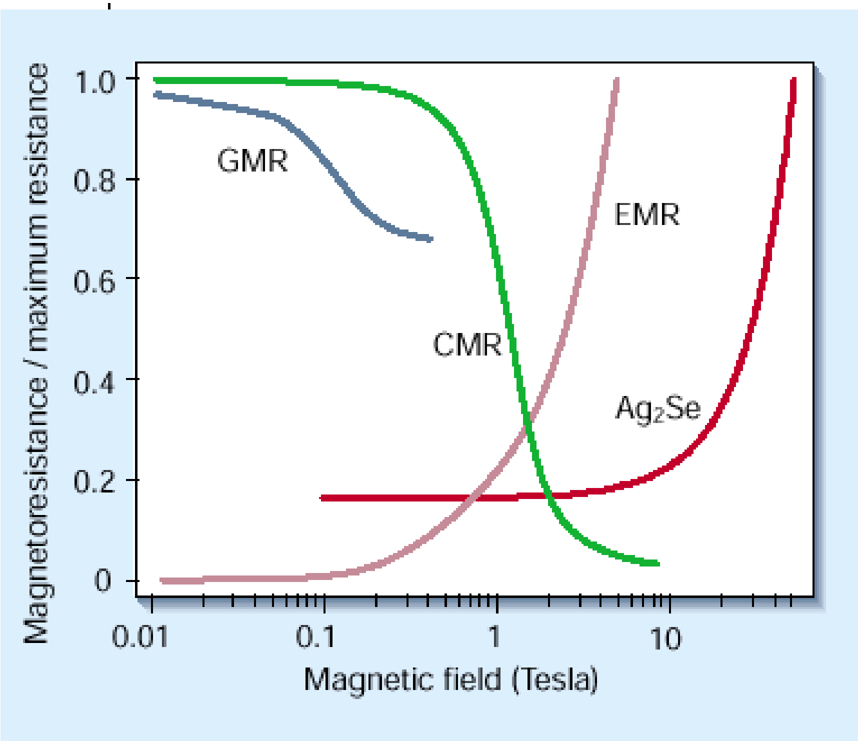


Figure 1 **Performance of magnetic-field sensors.** The dependence of magnetoresistance on applied magnetic field is shown for typical devices based on giant magnetoresistance (GMR) at a temperature of 295 K, colossal magnetoresistance (CMR) at 220 K and extraordinary magnetoresistance (EMR) at 300 K. Husmann *et al.*[8] have devised a new sensor based on the silver chalcogenide Ag_2Se, which can be used to measure magnetic-field strengths as high as 50 Tesla. The data shown here were measured at a temperature of 290 K, but the device performs just as well over a wide temperature range, even down to just a few degrees above absolute zero. (Data derived from refs 3, 6, 8 and A. Biswas, personal communication.)

Getzlaff and Mathias - Fundamentals of Magnetism, p.259

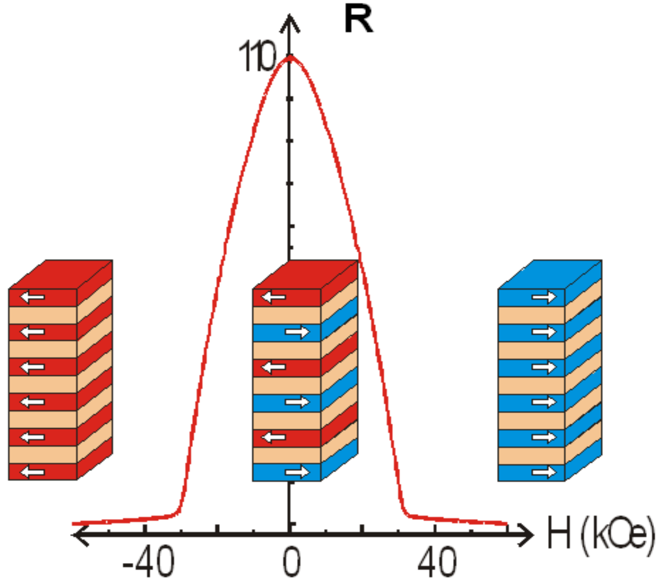
supplementary

Giant MR

(Gruenberg JAP; Fert PRL, 1988)

- In 1988, GMR was discovered
- In 1996, GMR reading heads were commercialized
- Since 2000: Virtually all writing heads are GMR heads

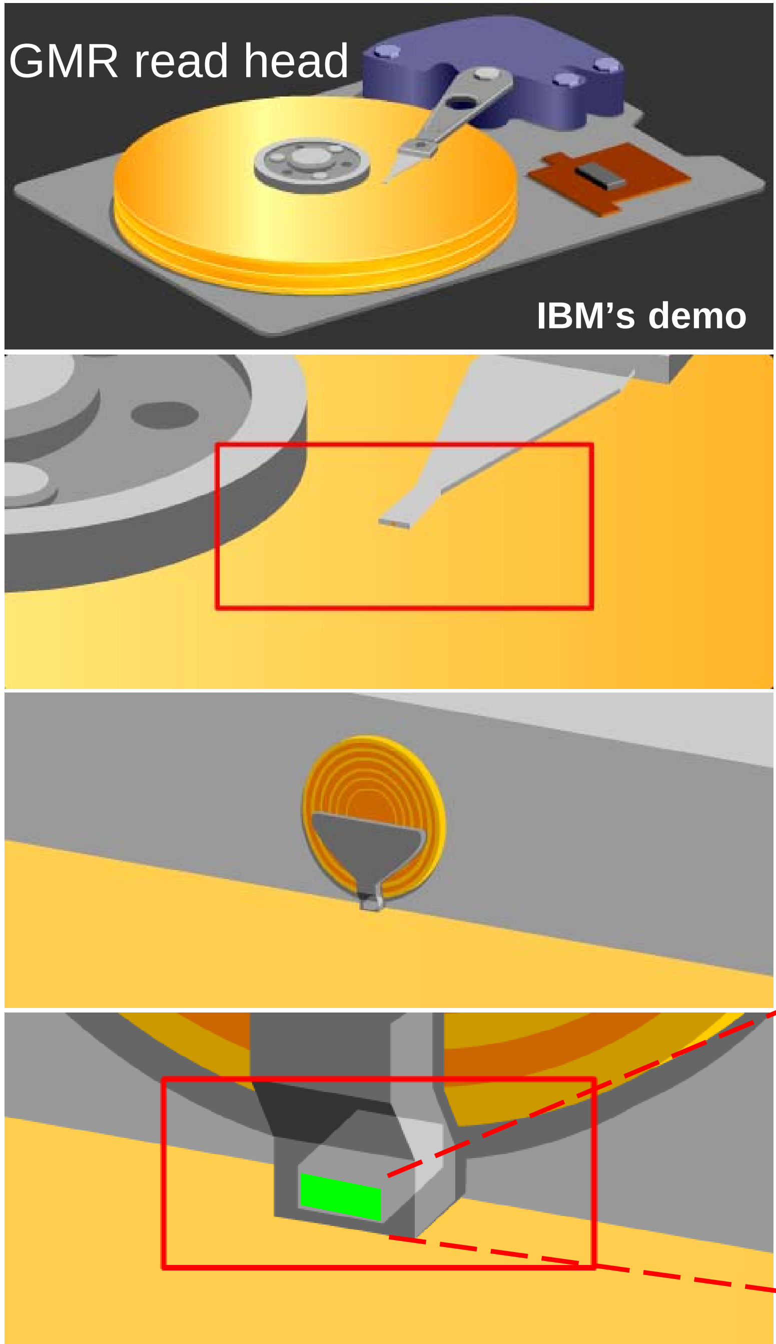
A. Fert and
P. Grünberg



TYPE OF MR EFFECT USED	MR AT 300 KELVINS (percent)	DATA DENSITY (Gb/in^2)	SIGNAL-TO-NOISE RATIO (decibels) (larger is better)	TIME CONSTANT (nanoseconds) (smaller is faster)	MAGNETIC FIELD NEEDED (teslas) (smaller is better)
Target	4–10	100–1,000	30–40	0.01–0.1	0.005–0.05
EMR	> 35	> 300	43	< 0.001	0.05
GMR	10	125	29	0.1	0.005
TMR	15	200 estimated	34	0.1	0.001
CMR	0.4	100 estimated	–17	1.0	0.05
BMR	3,000	> 1,000	10	0.1	0.03

Solin, Scientific American, 2004

supplementary



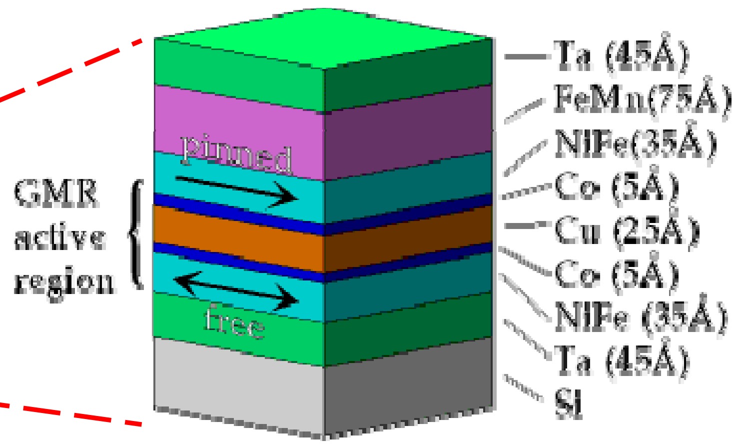


Fig from http://www.stoner.leeds.ac.uk/research/