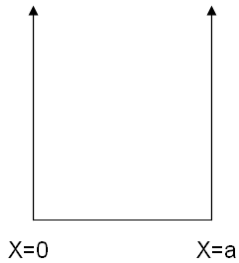
Quantum Mechanics 11/18/2013 (9:30-11:30)

1. [30 %] A particle with mass *m* is inside an *infinitely deep square well* with width *a*.

(a) Find out the normalized wave function $\psi_1(x)$ for the ground state.

(b) Find out the normalized wave function $\psi_2(x)$ for the first-excited state.

(c) At *t*=0, the state of the particle is $\psi(x,0)=\frac{1}{\sqrt{2}}\left(\psi_1(x)+\psi_2(x)\right)$. Find out the value of $\left|\psi\left(a/2,t\right)\right|^2$ at position *x*=*a*/2 and time *t*.



2. [40%] The raising and lowering operators of a harmonic oscillator can be written as

$$a_{\pm}=\frac{1}{\sqrt{2\hbar m\omega}}\left(m\omega x\mp ip\right),\ \text{and}\quad \begin{cases} a_{+}\psi_n=\sqrt{n+1}\,\psi_{n+1} \\ a_{-}\psi_n=\sqrt{n}\,\psi_{n-1} \end{cases}.$$

(a) Consider a particle in the *ground state* of the harmonic oscillator. Find out the expectation values $\langle x\rangle$ and $\langle p\rangle$ of its position and momentum.

(b) Follow (a), find out $\langle x^2\rangle$ and $\langle p^2\rangle$ for the particle in the ground state.

(c) Follow (b), what is the value of the product of standard deviations $\sigma_x\sigma_p$?

(d) The eigen-state of a_- is called a *coherent state* and satisfies $a_-\psi=\lambda\psi$.

One can expand ψ in terms of the energy eigen-states ψ_n, $\psi=\sum_{n=0}^{\infty}c_n\psi_n$.

Find out all of the coefficients c_n (assuming ψ is normalized).

3. [30%] A system is described by the following Hamiltonian,

$$H=\begin{pmatrix} a & b \\ b & a \end{pmatrix},\quad a,b\in R.$$

(a) Find out its normalized eigen-states.

(b) If the initial state of the system is $\psi(0)=\begin{pmatrix} 0 \\ 1 \end{pmatrix}$, then what is $\psi(t)$?

(c) Follow (b), a physical observable *Q* is described the following operator,

$$Q=\begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}.$$

When an experimentalist measures *Q* at time *t*=0, what are the possible values one can observe, each with what probabilities?