Quantum Mechanics 1/6/2014 (9:30-11:30)

1. [20 %] (a) The ground state of the hydrogen atom is $\psi_{100} = \frac{1}{\sqrt{\pi a^3}} e^{-r/a}$, a is the Bohr radius. $P(r)dr = |\psi_{100}|^2 4\pi r^2 dr$ is the probability distribution of the electron to be within a thin shell of thickness dr. $P(r)$ reaches a maximum at a radius r_0. Find out r_0.

   (b) Find out the expectation value $\left\langle \frac{1}{r^2} \right\rangle_{100}$ for the ground state.

2. [30%] (a) Calculate the commutator $[L_z, L_\pm]$, where $L_\pm = L_x \pm iL_y$.

   (b) If the eigenvalue of L_z for a state ψ is $m\hbar$, then find out "the eigenvalue of L_z" for the state $L_\pm \psi$.

   (c) A particle (without spin) is moving along a *one-dimensional* circular ring with radius R, which lies on the *x-y* plane. If the angular momentum $L_z = \frac{\hbar}{i}\frac{d}{d\phi}$ of the particle is $m\hbar$, then what is the *normalized* wave function $\psi_m(\phi)$ of the particle?

3. [30%] (a) A spin-1/2 particle is in the state $\psi = \frac{1}{\sqrt{3}}\begin{pmatrix} 1 \\ 1+i \end{pmatrix}$.

   Calculate the expectation values $\langle S_x \rangle, \langle S_y \rangle, \langle S_z \rangle$.

   (b) What are the probabilities of getting $\hbar/2$ and $-\hbar/2$ when you measure S_z?

   (c) When you measure S_x, what possible values will you get?
   Each with what probabilities?

4. [20%] (a) A system has two spin-1/2 particles. Write down the *singlet* state and the *triplet* states in term of the spin states $|\uparrow\rangle$, $|\downarrow\rangle$ of a particle.

   [Just write down the answer. No derivation is required]

   (b) Two particles have spin quantum numbers $(s_1, m_1) = (1,1)$, $(s_2, m_2) = (1,-1)$, what are the possible total spin quantum numbers S and m of the two particles?
   [No derivation is required]