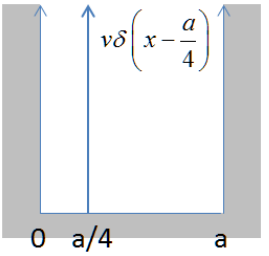
Quantum Mechanics 4/14/2014 (9:30-11:30)

1. [30%] There is a particle of mass m_0 in an infinitely deep square well potential (see Fig).

   The unperturbed eigen-states of the square well are $\psi_n^0 = \sqrt{\frac{2}{a}}\sin\left(\frac{n\pi}{a}x\right)$, ($n$=1,2…).

   The potential well is perturbed by $H' = v\delta\left(x - \frac{a}{4}\right)$.

   (a) Find out the first-order energy perturbation E_n^1 due to H' (for every n).

   (b) Calculate the perturbation on the ground state (i.e., n=1):

   $\psi_n^1 = \sum_{m \neq n} \frac{\langle \psi_m^0 | H' | \psi_n^0 \rangle}{E_n^0 - E_m^0} \psi_m^0$. You only need to calculate the *first* non-zero term in the expansion.

2. [20%] A degenerate three-state Hamiltonian H^0 is perturbed by H' as follows:

   $$H^0 = h\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}, \quad H' = v\begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & -1 \\ 0 & -1 & 0 \end{pmatrix}, \quad (v << h).$$

   (a) Find out the first-order energy corrections, E_n^1, to the degenerate energy $E_n^0 = h \quad (n = 1,2,3)$.

   (b) Find out the zero-th order *good* eigen-states $\psi_n^0 \quad (n = 1,2,3)$.

3. [30%] (a) The WKB wave function inside a potential well is of the form $\psi(x) = \frac{C}{\sqrt{p(x)}} e^{i\int_0^x dx' p(x')}$.

   Briefly give a physics explanation on why the probability density $|\psi(x)|^2 \propto \frac{1}{p(x)}$.

   (b) The WKB quantization condition is given by $\int_{-x_c}^{x_c} p(x)dx = \left(n + \frac{1}{2}\right)\pi\hbar$ (n=0, 1, 2…),

   where x_c is the classical turning point. Apply this formula to a simple harmonic potential $V(x) = \frac{m\omega^2}{2}x^2$ and find out the bound state energies E_n.

4. [20%] In the time-dependent perturbation theory, we need to solve equation of the form: $\frac{d\vec{v}(t)}{dt} = \underset{\sim}{M}(t)\vec{v}(t)$, in which $M(t)$ is a time-*dependent* matrix. In the method of iteration (疊代法),

   the zero-th order solution is $\vec{v}^0(t) = \vec{v}_0$ (initial value of $\vec{v}(t)$).

   (a) Find out the first-order correction $\vec{v}^1(t)$ (which is accurate to the first order of M).
   (b) Find out the second-order correction $\vec{v}^2(t)$ (which is accurate to the second order of M).