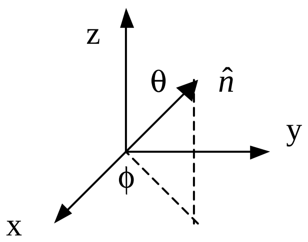
Quantum Mechanics Midterm Exam **12/03/2003 (9:10-11:10)**

1. A Hilber space has the following basis |j> (j=1,2,3). Two operators Ω_+ and Ω_- in the Hilbert space are defined by the following mappings: $\Omega_\pm$: |j> $\rightarrow$ |j±1>.
   That is, $\Omega_\pm$|j>=|j±1>. (We will assume that Ω_+|3>=0 and Ω_-|1>=0.)
   (a) Write the operators Ω_+ and Ω_- in matrix forms using the basis above.
   (b) Define $\Omega=\Omega_+ + \Omega_-$. Is it a hermitian operator? Find out its eigenvalues and (normalized) eigenstates.
   (c) Find out the expectation value of Ω in the state $|\psi\rangle=(|1\rangle+|2\rangle+|3\rangle)/\sqrt{3}$

2. The states $|\pm\rangle$ are the eigenstates of spin-1/2 operator S_z. Let the eigenstates of the operator $\vec{S}\cdot\hat{n}$ be denoted by $|\hat{n},\pm\rangle$, where $\hat{n}$ is a unit vector shown below.



   (a) Write down the matrix form of the operator $\vec{S}\cdot\hat{n}=S_x n_x+S_y n_y+S_z n_z$.
   (b) Find out its eigenvalues and show that the "+" eigenstate can be chosen as
   $$|\hat{n},+\rangle=\begin{pmatrix}\cos q/2\\ e^{if}\sin q/2\end{pmatrix}.$$
   (c) If we measure the values of S_x for the eigenstate $|\hat{n},+\rangle$, what result will we get, with what probabilities?
   $$[S_x=\frac{\hbar}{2}\begin{pmatrix}0&1\\1&0\end{pmatrix}, S_y=\frac{\hbar}{2}\begin{pmatrix}0&-i\\i&0\end{pmatrix}, S_z=\frac{\hbar}{2}\begin{pmatrix}1&0\\0&-1\end{pmatrix}]$$

3. A particle with mass m is moving freely in a one-dimensional system with length L with the *periodic boundary condition* $y(0)=y(L)$.
   (a) Find out the energy eigenvalues and (normalized) eigenstates of this system. Notice that the Hamiltonian is simply $H=p^2/2m$.
   (b) Assume the initial wave function for the particle is $y_0(x)=A\cos^2(2px/L)$, where A is a constant. What is the probability to find it in the lowest energy eigenstate (with nonzero momentum)?
   (c) When we measure its momentum p, what possible results will we get? With what probabilities?
   (d) Find out the uncertainty of momentum Δp for the state at time t.