Quantum Mechanics Midterm Exam (9:10-11:00, 4/26/2005)

1. [20%] In general, the operator U of a continuous (unitary) transformation can be written in the form U=exp($-iGa$), where G is the generator of the transformation and a is some parameter. (For simplicity, let's assume there is only one parameter.) Show that, if the Hamiltonian H is invariant under the transformation U, then G is a conserved quantity.

2. [30%]

(a) Consider a system with angular momentum j=1. Explicitly write $<j=1,m'|\, J_x\, |j=1,m>$ as a 3×3 matrix.

(b) Expand exp($-i?J_x$/h) as a polynomial of J_x with finite number of terms.

(c) Use (a) and (b) to express exp($-i?J_x$/h) as a 3×3 matrix.

$(J_{\pm} | j,m >= \sqrt{(j \mp m)(j \pm m+1)}\hbar | j,m \pm 1 >)$

3. [30%] For a spin-1/2 particle, the spin-up state (along z-axis) is $|+>=(1,0)^T$.

(a) With the help of the rotation operators U_y and U_z, find out the state $|\hat{n},+>$ that is spin-up along the direction $\hat{n}$=(sin$q$cos$f$, sin$q$sin$f$, cos$q$).

(b) Following (a), find out the expectation values $<S_x>$ and $<S_y>$ of such a state.

(c) Assume the initial state of a particle is $|\hat{n},+>$ at t=0. We then apply an uniform magnetic field B along the z direction. Calculate the expectation value $<S_x>$ at time t.

4. [20%] Consider a system made up of two spin 1/2 particles. Observer A specializes in measuring the spin components of particle-1, while observer B measures the spin component of particle-2. Suppose the system is in a spin-singlet state with S_{tot}=0.

(a) What is the probability for A to obtain $S_{1z} = \hbar/2$ when B makes no measurement?

Repeat the same problem for $S_{1x} = \hbar/2$.

(b) B determines the spin of particle-2 to be $S_{2z} = \hbar/2$. What can we conclude about the outcome of A's measurement if A measures S_{1z}? Repeat the same problem, what if A measures S_{1x}?