Quantum Mechanics 1/8/2004 (2:10-4:00)

1. (40 points) A coherent state is defined as an eigenstate of the lowering (or annihilation) operator a: $a|\lambda>=\lambda|\lambda>$, where λ is a complex number.
   (a) Show that $|\lambda>=\exp(-|\lambda|^2/2)\exp(\lambda a^+)|0>$ satisfies the definition above.
   (b) Find out the inner product of two coherent states $|\lambda>$ and $|\lambda'>$ (notice that λ' is a complex number different from λ).
   (c) Calculate the average number of the energy quanta of the coherent state $<\lambda|N|\lambda>$, where N is the number operator.
   (d) Calculate the probability P_n to find n energy quanta in the coherent state (n=0,1,2…).
   (Hint: $e^A e^B = e^{A+B} e^{[A,B]/2}$ if [A,B] commutes with both A and B.)

2. (30 points)
   (a) The propagator is defined as $K(x,t;x',0)=<x,t|x',0>$ and can be calculate by using $<x|U(t)|x'>$, in which the evolution operator depends on the Hamiltonian of the system being considered. For the simplest case, calculate the propagator for *a free particle in one dimension*.
   (b) In the spirit of path integral method, explain how could we calculate the propagator without using the unitary operator? Use equations or figures to clarify your explanation if necessary.
   (Hint: $\int_{-\infty}^{\infty} e^{-ax^2} dx = \sqrt{p/a}$)

3. (30 points) There are 3 non-interacting particles in a one-dimensional simple harmonic potential $V(x)=(1/2)m\omega^2 x^2$. In the ground state *of the system*, the *total energy* of the 3 particles is as low as it can be. Answer the following questions (there is *no need* to consider the spin degree of freedom, also *no need* to derive your result).
   (a) What is the ground state energy *of the system* if these 3 particles are bosons? What if they are fermions?
   (b) Write down the normalized ground state wave function $|\Psi_G>$ for the 3-particle system. First consider the boson case, then the fermion case. You can simply write $\phi_n(x)$ for the (normalized) single particle wave function at level n (n=0,1,2…), *without* writing the explicit form involving Hermit polynomials.
   (c) What is the normalized wave function of these 3 fermions when they are at the lowest *excited* state (of the system) above the ground state?