Quantum Mechanics 4/29/2004 (2:10-4:00)

1. [20 %] A system is invariant under space inversion, that is, $H(-\vec{x},-\vec{p}) = H(\vec{x},\vec{p})$. Prove that if the initial state of the system is an eigenstate of parity, then it remains a parity eigenstate as time evolves.

2. [30 %] A particle in a spherically symmetric potential is known to be in an eigenstate of L^2 and L_z with eigenvalues $\ell(\ell+1)\hbar^2$ and $m\hbar$ respectively.
   (a) Find out the expectation values of L_x and L_y in the $|\ell m>$ state.
   (b) Calculate the uncertainties of L_x and L_y in the $|\ell m>$ state. Recall that
   $\Delta L_x = \sqrt{< L_x^2 > - < L_x >^2}$, similarly for ΔL_y.

3. [30 %] The spin of an electron at t=0 is pointed at +x axis. From time t=0 to t=T, it is influenced by an uniform magnetic field $\vec{B} = B\hat{z}$.
   (a) Find out the state of the spin at time T.
   (b) At time T, the direction of the magnetic field is suddenly changed from +z to +y. The electron is under the influence of the new magnetic field $\vec{B} = B\hat{y}$ from t=T to t=2T. What is the probability of finding the measured value of S_x to be $\hbar/2$ at t=2T?

4. [20 points] The Hamiltonian of a two-electron system is as follows:

$$H = J_1\vec{S}_1 \cdot \vec{S}_2 + J_2\left(\vec{S}_1 \cdot \vec{S}_2\right)^2,$$

   where J_1 and J_2 are constants.
   (a) Find out all the spin-related operators that commute with the Hamiltonian.
   (b) Find the energy eigenvalues of this system.