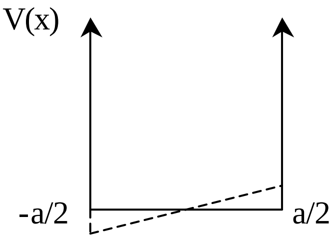
Quantum Mechanics **6/17/2004 (2:10-4:00)**

1. [30%] A particle with mass m is moving in a 1-dim potential $V(x)=(k/2)x^2$.
   (a) Assume the energy of the particle is E, find out the positions of the classical turning points.
   (b) Use the Bohr-Sommerfeld quantization condition to find out the energy E_n of the n-th bound state (do not just write down the answer directly)
   (c) If the potential for $x<0$ is replaced by a hard wall with infinite height, how would the answer for (b) be changed?

2. [30%] A particle with mass m is moving <u>freely</u> in a 1-dim region [–L/2,L/2]. Let us use the periodic boundary condition (that is, the two points –L/2 and L/2 are glued together).
   (a) Find out the normalized eigen-states ψ_n and eigen-energies E_n of this system.
   (b) Let us put a little "dimple" in the potential at x=0 by introducing a perturbation $H' = -V_0\exp(-x^2/a^2)$, where $a<<L$. Find the first order correction to E_n.
   (c) What are the "good" linear combinations of ψ_n and ψ_{-n}.
   (Hint: Since $a<<L$, you can extend the interval of some integrals from [–L/2,L/2] to $[-\infty,\infty]$ to simplify the calculation, and use $\int_{-\infty}^{\infty} dx e^{-ax^2} = \sqrt{p/a}$.)

3. [40%] A particle with mass m and charge q is moving in a square-well potential.

$$V(x)=\begin{cases}0 & x\in[-a/2,a/2]\\ \infty & x\notin[-a/2,a/2]\end{cases}$$



Beginning from t=0, a weak oscillating electric field $\vec{E}(t)=E_0\sin(wt)\hat{x}$ is applied.

   (a) Write down the normalized wave functions and energies of the lowest two *unperturbed* eigenstates.
   (b) After a long time of perturbation, find out (by using the Fermi Golden rule) the transition rate $R_{1->2}$ for the particle to jump to the first excited state.
   (c) If the perturbation lasts for only a finite time t, find out the probability $P_2(t)$ to find the particle at the first excited state after the perturbation is turned off.

Hint: $d_f^1(t)=\frac{1}{i\hbar}\int_{t_0}^{t} dt'\langle f^0|H'(t')|i^0\rangle e^{iw_{fi}t'}$; $R_{i\to f}=\frac{2p}{\hbar}|V_{fi}|^2\,d(e_{fi}-\hbar w)$