Quantum Mechanics 11/21/2002 (1:10-3:00),

1. (30 points) Consider a Hamiltonian H and two physical observables L_x, L_y

$$H = w\begin{pmatrix} 1 & 0 & 1 \\ 0 & 0 & 0 \\ 1 & 0 & 1 \end{pmatrix},\ L_x = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix},\ L_y = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 & -i & 0 \\ i & 0 & -i \\ 0 & i & 0 \end{pmatrix}.$$

(a) Find out the eigenvalues and eigenstates of the Hamiltonian.

(b) What are the possible observed values when we measure the L_x of the first excited state of the Hamiltonian, with what probabilities?

(c) Assuming that the measured value of L_x is one, we immediately make a second measurement on L_y, what values we may observe, with what probabilities?

2. (40 points) The initial wave function of a particle in an infinitely deep square well (see the figure below) is a mixture of the ground state and the first-excited state,

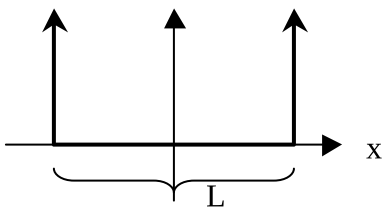
$$\psi(x,0) = A[\psi_1(x) + \psi_2(x)]$$

(a) Write down the normalized wave functions $\psi_1(x)$, $\psi_2(x)$ and find out the normalization constant A.

(b) Find out the expectation value of energy and the energy fluctuation ΔE.

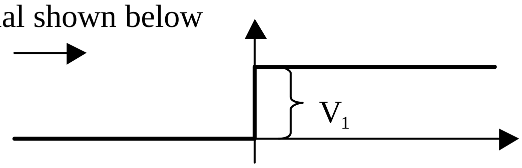
(c) Write down the state $\psi^{\perp}(x)$ that is orthogonal to the initial state and find out the time T for the initial state to evolve to its orthogonal state.

(d) Find out the expectation value of the particle's position as a function of time.



Hint: $\int_{-L/2}^{L/2} dx\, x e^{iax} = i\frac{2}{a^2}\sin\left(a\frac{L}{2}\right) - i\frac{L}{a}\cos\left(a\frac{L}{2}\right)$

3. (30 points) A plane wave $\exp(ikx)$ with energy $E>V_1$ is incident from the left to the step potential shown below



(a) Find out the reflection coefficient R.

(b) Calculate the currents on the left and on the right of the step edge.

(c) Should the two currents calculated in (b) have the same magnitude?
If yes, then what's the relation between R and T (transmission coefficient)?
If no, explain why.