Quantum Mechanics 4/14/2003 (10:10-12:00)

1. [30 point] The spin of an electron is up along the z-axis at time t=0. It is subject to an uniform magnetic field **B**=B**x,** where **x** is the unit vector along the x-axis.
   (a) Find out the probability that the electron remains at the spin-up state at time t.
   (b) Find out the expectation value of S_z as a function of time.
   (Hint: the Hamiltonian is H = –**μ·B**, the magnetic moment **μ** = (–e/mc)**S**, and **S** is the spin operator.)

2. [40 points] A particle is described by the wave function $\psi(\mathbf{r}) = A(x+y+2z)e^{-r}$.
   (a) When the angular momentum L_z of the particle is measured, we obtain only one of the three values $\hbar$, 0, and $-\hbar$. Explain why.
   (b) Find out the probabilities P_m of getting the result with $L_z=m\hbar$.
   (c) After $\psi(\mathbf{r})$ has been rotated counter-clockwise around the z-axis by 90 degrees, the new wave function is written as $\psi'(\mathbf{r})$. Find out the explicit form of $\psi'(\mathbf{r})$.

$$Y_0^0 = (4\pi)^{-1/2}$$
$$Y_1^{\pm1} = \mp(3/8\pi)^{1/2}\sin\theta\, e^{\pm i\phi}$$
$$Y_1^0 = (3/4\pi)^{1/2}\cos\theta$$
$$Y_2^{\pm2} = (15/32\pi)^{1/2}\sin^2\theta\, e^{\pm 2i\phi}$$
$$Y_2^{\pm1} = \mp(15/8\pi)^{1/2}\sin\theta\cos\theta\, e^{\pm i\phi}$$
$$Y_2^0 = (5/16\pi)^{1/2}(3\cos^2\theta - 1)$$

3. [30 points] Assume the Hamiltonian of a two-electron system is as follows:

$$H = J\vec{S}_1\cdot\vec{S}_2 + a\frac{\hbar}{4}(S_{1z}+S_{2z}),$$

   where J and α are constants.
   (a) Write the Hamiltonian as a 4×4 matrix using the |+,+>, |+,–>, |–,+> and |–,–> basis.
   (b) Find the energy eigenvalues and their corresponding eigenstates.