Quantum mechanics final exam (10:10 – 12:00, 6/9/2003)

1. [20] Which of the following matrix elements of the hydrogen atom vanishes and which does not? If it vanishes, explain why; if not, give a rough estimate of its magnitude.
   (a) <211|p_z|200>, where the |nlm> basis has been used.
   (b) <211|x^2|200>

2. [20] A particle is moving in a 1-dim potential V(x)=αx^4.
   (a) Use the quantization condition below to show that the energy E_n of the n-th bound state is proportional to $(n+1/2)^{4/3}$.

$$\oint p(x)dx = (n+1/2)h$$

   (b) This result is more accurate for small n or large n? What is the physical reason?

3. [30] Consider a spin-1 particle with the following Hamiltonian

   $H=aS_z^2+b(S_xS_y+S_yS_x)$, where a and b are constants and a>>b.

   Treating the b term as a perturbation, find the normalized eigenstates of $H_0=aS_z^2$ that are stable under perturbation and calculate the energy shifts to first order in b. (**DO NOT** use the exact diagonalization method!)

$$S_x = \frac{\hbar}{\sqrt{2}}\begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}, S_y = \frac{\hbar}{\sqrt{2}}\begin{pmatrix} 0 & -i & 0 \\ i & 0 & -i \\ 0 & i & 0 \end{pmatrix}$$

4. [30] A hydrogen atom at the ground state is placed between the plates of a capacitor with time-dependent but spatial uniform electric field **E**(t)=$\mathbf{E_0}$ exp(−t/τ) *along z-axis* (the electric field is zero at t<0). Find out the probabilities that the atom to be found at the 2s and 2p states after t>>τ.

$$\Psi_{100}(\vec{r}) = \left(\frac{1}{a_0}\right)^{3/2} \frac{1}{\sqrt{p}} e^{-r/a_0},$$

$$\Psi_{200}(\vec{r}) = \left(\frac{1}{a_0}\right)^{3/2} \frac{1}{2\sqrt{2p}} \left(1-\frac{r}{2a_0}\right) e^{-r/2a_0},$$

$$\Psi_{210}(\vec{r}) = \left(\frac{1}{a_0}\right)^{3/2} \frac{1}{4\sqrt{2p}} \frac{r}{a_0} e^{-r/2a_0} \cos q, \Psi_{21\pm1}(\vec{r}) = \left(\frac{1}{a_0}\right)^{3/2} \frac{1}{8\sqrt{p}} \frac{r}{a_0} e^{-r/2a_0} \sin q \, e^{\pm i f}.$$

$$d_f(t) = \frac{1}{i\hbar}\int_0^t dt' < f^0|H'(t')|i^0 > e^{iw_{fi}t'},$$

$$R_{i\to f} = \frac{2p}{\hbar} |< f|V|i >|^2 d(e_{fi} \mp \hbar\Omega).$$