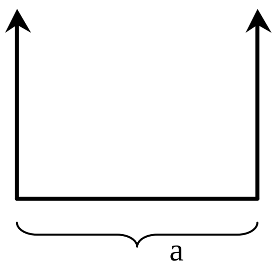
Quantum Mechanics 12/4/2001 (10:10-12:00),

1. (30 points) Consider a three-dimensional ket space. If a certain set of orthonormal kets – say, |1>, |2>, and |3> -- are used as the base kets, the operators A and B are represented by (a and b are both real)

$$A = \begin{pmatrix} a & 0 & 0 \\ 0 & -a & 0 \\ 0 & 0 & -a \end{pmatrix}, \quad B = \begin{pmatrix} b & 0 & 0 \\ 0 & 0 & -ib \\ 0 & ib & 0 \end{pmatrix}$$

   (a) Obviously A exhibits a degenerate spectrum. Does B also exhibit a degenerate spectrum?
   (b) Show that A and B commute.
   (c) Find a new set of orthonormal kets which are simultaneous eigenkets of both A and B. Specify the eigenvalues of A and B for each of the three eigenkets.

2. (20 points) Consider a particle in an infinitely deep square well (see the figure). If the wave function of the particle is $C(x^2 - a^2/4)$, where C is a constant, what is the energy fluctuation (standard deviation of the energy) ΔE of such a state?



3. (20 points) Consider a system which has only two states Ψ_1 and Ψ_2, with energies E_0 and $-E_0$. At time t=0 a perturbation V is introduced. V has the property that $\langle\Psi_1|V|\Psi_1\rangle = \langle\Psi_2|V|\Psi_2\rangle = 0$ and $\langle\Psi_1|V|\Psi_2\rangle = \langle\Psi_2|V|\Psi_1\rangle = V_0$, where V_0 is a constant. Assume that the system is in state 1 at t=0. Calculate the probability as a function of time that the system can be found in either state 1 or state 2.

4. (30 points) A coherent state is defined as an eigenstate of the lowering operator a, $a|\lambda\rangle = \lambda|\lambda\rangle$, where λ is a complex number.
   (a) Show that $|\lambda\rangle = \exp(-|\lambda|^2/2)\exp(\lambda a^+)|0\rangle$ satisfies the definition above.
   (b) Find out the scalar product of two coherent states $|\lambda\rangle$ and $|\lambda'\rangle$.
   (c) Given the Hamiltonian $H = \hbar\omega(a^+a + 1/2)$, an initial coherent state $|\lambda,0\rangle = |\lambda\rangle$ will evolve with time to another coherent state $|\lambda,t\rangle$. Assuming $a|\lambda,t\rangle = \lambda(t)|\lambda,t\rangle$, find out the function $\lambda(t)$.

   (Hint: $e^A e^B = e^{A+B} e^{[A,B]/2}$ if [A,B] commutes with both A and B;
   $e^A B e^{-A} = e^{\gamma} B$ if $[A,B] = \gamma B$.)