Quantum Mechanics final exam
1/15/2002 (10:10 – 12:00)

1. (30) The coherent state of a one-dimensional simple harmonic oscillator is defined to be $|\lambda\rangle = \exp(-|\lambda|^2/2)\exp(\lambda a^+)|0\rangle$ (λ is an arbitrary complex number).
   (a) Show that $|\lambda\rangle$ is an eigenstate of a: $a|\lambda\rangle = \lambda|\lambda\rangle$
   (b) The coherent state is not an energy eigenstate. Therefore it does not have a definite energy. Calculate the energy expectation value of the coherent state.
   (c) Solve the eigenstate equation (see (a)) in the coordinate basis and find out $\langle x|\lambda\rangle$.
   [hint: $a = (m\omega/2\hbar)^{1/2}(x + ip/m\omega)$]

2. (20) (a) Briefly explain the difference between the *canonical quantization* method and the *path integral quantization* method.
   (b) Briefly explain the relation between the path integral formalism and the least action principle in classical mechanics.
   (use equations or figures to help you clarify the answers)

3. (30) Consider two particles (each with a mass m) moving in *one dimension* and interacting with each other via a harmonic potential $V(x_1-x_2)=k(x_1-x_2)^2/2$.
   (a) Write down the Schrodinger equation for such a system using the center of mass coordinate X and the relative coordinate x.
   (b) Find out the eigen-energies and eigenstates for this two-particle system. Choose appropriate quantum numbers to express your answers. (The n-th eigenstate for a simple harmonic oscillator can simply be represented by f_n)
   (c) Will the answer in (b) be changed if these two particles are fermions? If no, explain why; if yes, explain the difference.

4. (20) (a) Show that if the Hamiltonian is invariant under space reflection, then the energy eigenstates have either even parity or odd parity.
   (b) Assuming G is the generator of a Lie group and the Hamiltonian is invariant under this Lie group transformation. Show that any **nondegenerate** energy eigenstate of this system is also an eigenstate of G.