Quantum Mechanics 4/23/2002 (10:10-12:00)

1. [30 points]
   (a) Consider a system with angular momentum j=1. Explicitly write $<j=1,m'|\, J_y\, |j=1,m>$ as a 3×3 matrix.
   (b) Expand $\exp(-i\theta J_y/h)$ as a polynomial of J_y with finite number of terms.
   (c) Use (a) and (b) to express $\exp(-i\theta J_y/h)$ as a 3×3 matrix.
   ($J_\pm | j,m>=\sqrt{(j\mp m)(j\pm m+1)}\hbar | j,m\pm 1>$)

2. [20 points] Assume that when an electron in a central force potential is in the $|a,l,m=0>$ state, the probability amplitude of finding it at the location **r** = r **k** along the z-axis is $f_{al}(r)$.
   (a) Given this information, we can determine the probability amplitude $\psi_{alm}(r,\theta,f)$ for any value of m at any location. Express $\psi_{alm}(r,\theta,f)$ using $f_{al}(r)$ and appropriate rotation operators U_x, U_y, or U_z.
   (b) Use the answer you obtained from Problem 1 to determine $\psi_{a11}(r,\theta,f)$, $\psi_{a10}(r,\theta,f)$, and $\psi_{a1-1}(r,\theta,f)$ explicitly.
   (Hint: It's useful to know that ψ_{alm} at a point *along the z-axis* has a non-zero value only when m=0.)

3. [30 points] A spin-1/2 electron placed in a rotating magnetic field
   $$\vec{B} = B_0\hat{z} + B_\perp\left(\cos\Omega t\,\hat{x} + \sin\Omega t\,\hat{y}\right)$$
   has the following Hamiltonian
   $$H = \frac{\hbar v_0}{2} s_z + \frac{\hbar v_\perp}{2} U^+ s_x U,$$
   where $U=\exp(i\Omega t s_z/2)$.
   (a) Choose a rotating coordinate such that the Hamiltonian becomes time-independent. Write down the Hamiltonian in this rotating coordinate.
   (b) Solve the Schrodinger equation in this rotating coordinate.
   (c) Rotate the state you solved in (b) back to the laboratory frame to answer the following question: If at t=0 the electron's spin is up along the z-direction, what is the probability to find the electron with spin-down at time t?

4. [20 points] Consider a spin-1/2 electron bound to a nucleus in a state with angular momentum l =2.
   (a) Write done all possible $|j,m>_c$ states for this composite system.
   (b) Write $|3/2,3/2>_c$ as a linear combination of the states in the $|m_1,m_2>$ basis.