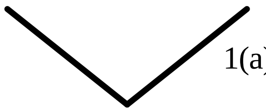
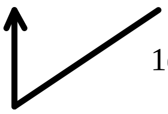
Quantum Mechanics 6/11/2002 (10:10-12:00)

1. (30 points) A particle with mass m and energy E>0 is subjected to a potential of the form V(x)=a|x| in one dimension, where a>0. (see the figure 1(a) below)
   (a) Find out the positions of the (classical) turning points.
   (b) Use the Bohr-Sommerfeld quantization condition to find out the eigen-energies.
   (c) What would the answer in (b) be changed if the potential for x<0 becomes infinite? (see figure 1(b) below)



1(a)



1(b)

2. (40 points) A two-dimensional harmonic potential has the Hamiltonian

$$H = \frac{1}{2m}\left(p_x^2 + p_y^2\right) + \frac{mw^2}{2}\left(x^2 + y^2\right) = \left(a^+a + b^+b + 1\right)\hbar w,$$ where

$$a = \frac{1}{\sqrt{2}}\left(\sqrt{\frac{mw}{\hbar}}x + \frac{i}{\sqrt{mw\hbar}}p_x\right), \quad b = \frac{1}{\sqrt{2}}\left(\sqrt{\frac{mw}{\hbar}}y + \frac{i}{\sqrt{mw\hbar}}p_y\right).$$

   (a) Describe the energy spectrum of the lowest 5 energy levels. If any energy level is degenerate, calculate its degeneracy.
   (b) The particle is subjected to a weak, uniform magnetic field aligned along the z-axis. This gives the following perturbing Hamiltonian ($B^2$ term neglected)

$$H_1 = \frac{eB}{2mc}\left(xp_y - yp_x\right).$$

   Find the energy shift of the ground state.
   (c) Find the energy shift of the first excited states.
   (d) Construct the "good" zeroth-order first-excited eigenstates for the perturbed problem.

3.(30 points) A one-dimensional harmonic oscillator with the Hamiltonian

$$H_0 = \frac{p^2}{2m} + \frac{mw^2}{2}x^2$$

is subjected to a weak perturbation $\lambda x\cos(\Omega t)$ from time –T/2 to T/2.
   (a) If T is quite large, use the Fermi golden rule to find out the transition **rate** for the electron to be excited from the ground state to the first excited state.
   (b) If T is finite, calculate the transition **probability** from the ground state to the first excited state after the perturbation has been turned off.
   (c) In general, $\hbar\Omega \neq \hbar\omega$. Explain briefly, but clearly, why the energy of the system appears not to be conserved before and after the transition in (b).

   Formulas:
$$d_f^1(t) = \frac{1}{i\hbar}\int_{t_0}^{t} dt' < f|H_1(t')|i > e^{iw_{fi}t'}$$

$$R_{i\to f} = \frac{2p}{\hbar}|< f|V|i >|^2 d(e_{fi} \mp \hbar\Omega)$$