The birth of quantum mechanics (partial history)

1902: Lenard's photo-electric effect (basis of photo-detector)

varied the intensity of carbon arc light by a factor of 1000 and observed NO effect on the electron energy

1905: Einstein's light quanta hypothesis, $E=h\nu$

explains photo-electric effect, but couldn't explain the phenomena of interference and diffraction

1923: de Broglie's matter wave hypothesis

while trying to explain diffraction using light-quanta,

realized that material particles might have wave property.

proposed $\lambda=h/p$ (from $E=h\nu$ and $E=cp$ for photons),

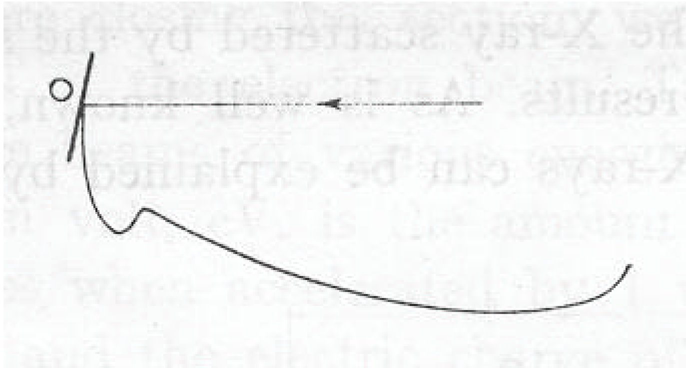
easily explains the formula $L=n\hbar$ in Bohr's model.

Dept of Phys

M.C. Chang

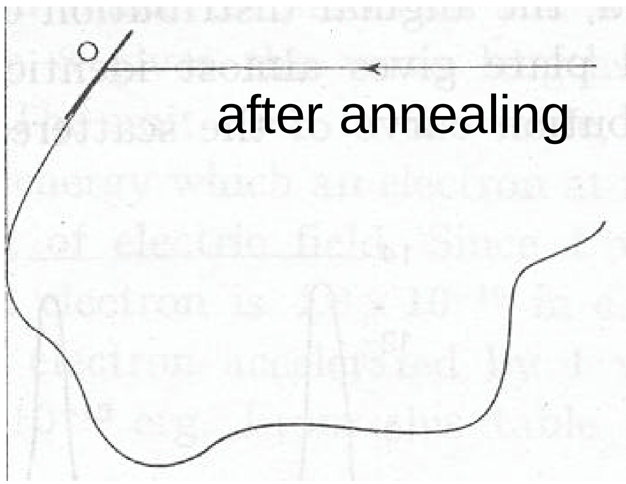
1925: Davisson and Germer (Ref: Quantum Mechanics, by Tomonaga)

Study the pattern of electron scattering off Ni target to determine the electric field in the atom

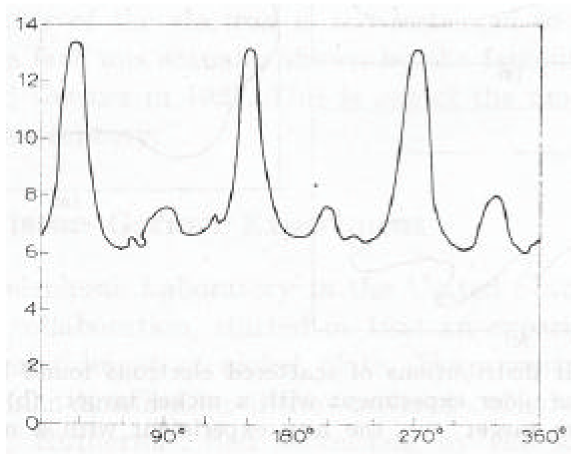


before annealing

(variation due to electronic shells?)



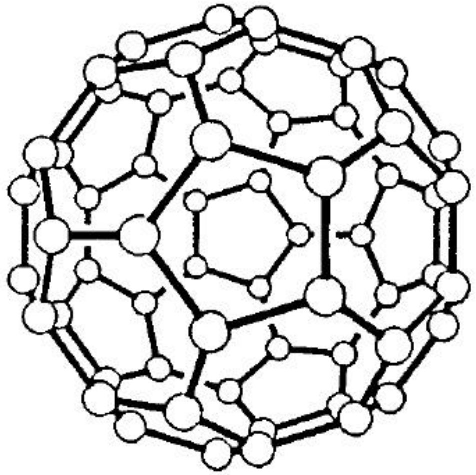
after annealing



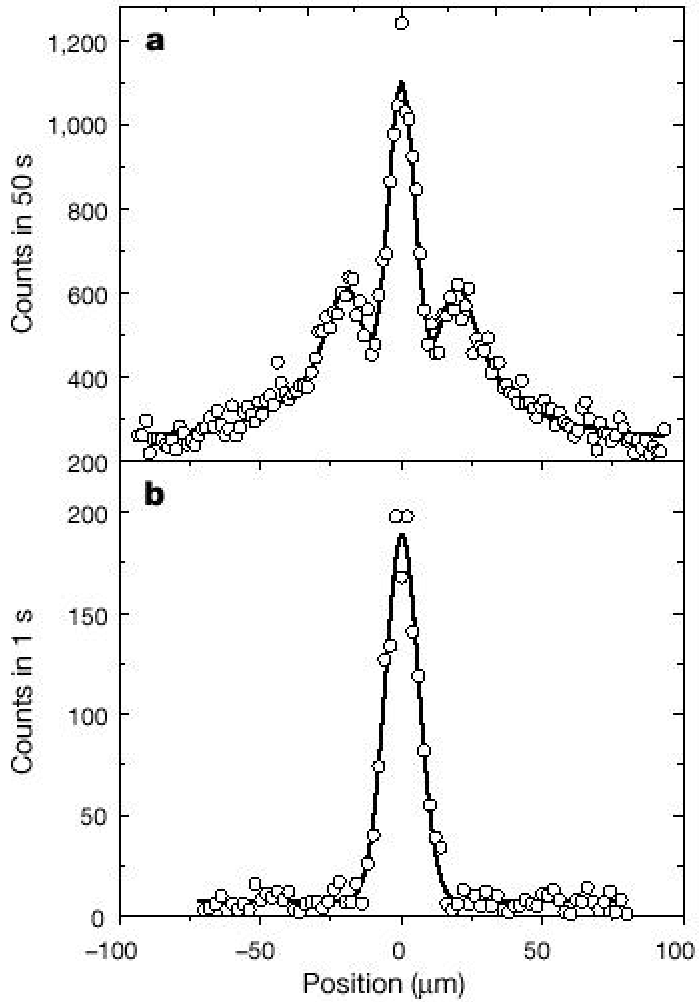
Upon a colleague's (Elasser) advice, they realized that the angular variation of the scattering maybe due to electron wave diffraction.

1999: Zeilinger's Vienna group

2-slit interference using C_{60}



The velocity of 210 m/s corresponds to a de Broglie wavelength for C_{60} of $\lambda = h/p =$ 2.5 pm.



1925: Schrodinger (Ref: Schrodinger: life and thought, by Moore)

During Nov, 1925, Schrodinger gave a seminar on de Broglie's work. One audience (Debye) suggested that there should be a wave equation. During the Christmas, Schrodinger started from the usual wave eq.

$$\nabla^2\Psi + k^2\Psi = 0 \quad (k = 2p \,/\, l\,)$$

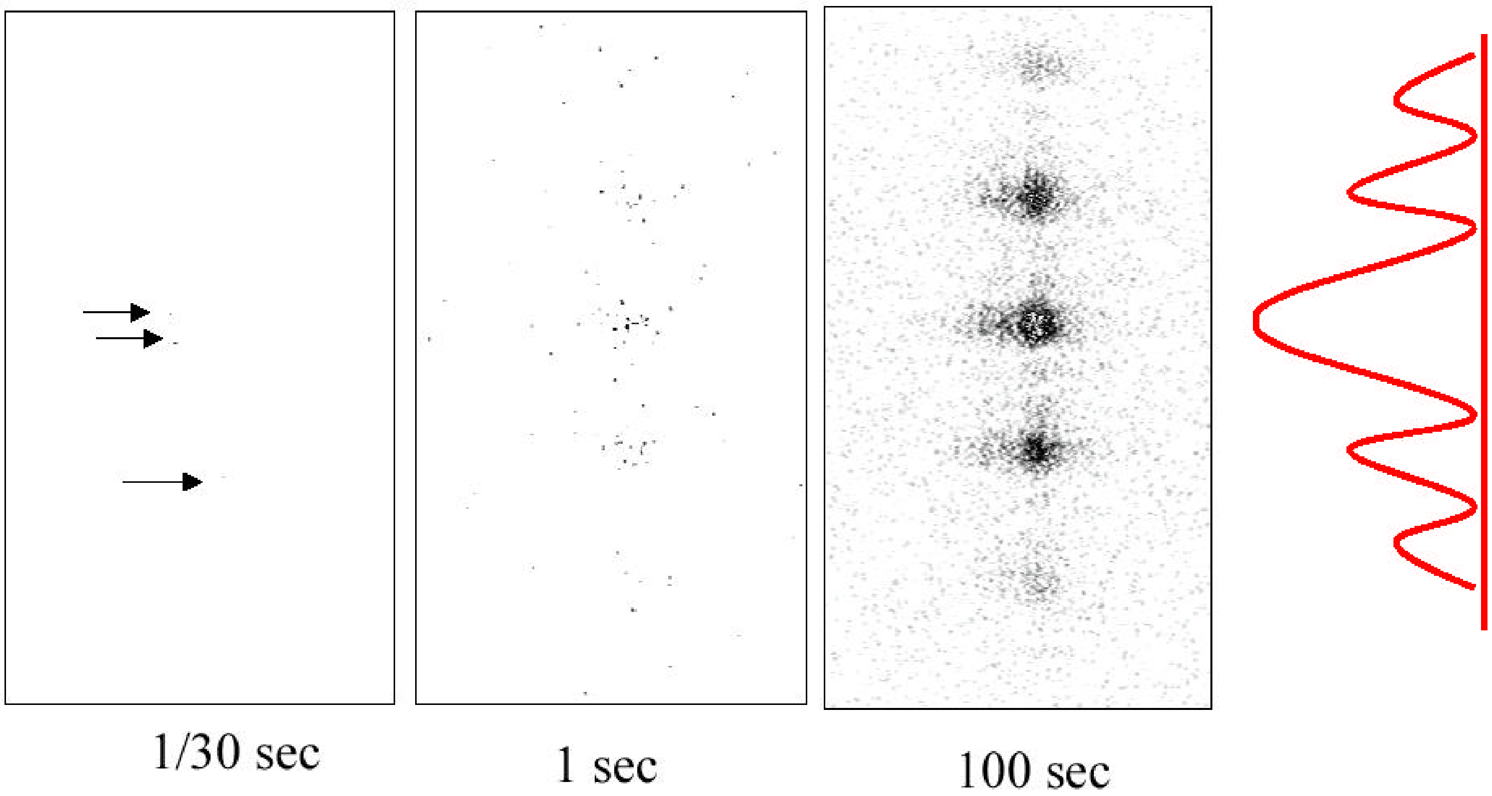
When $\lambda=h/p$ and $p^2/2m=E-V$ are used,

the wave eq. becomes $$-\frac{h^2}{2m}\nabla^2\Psi + V\Psi = E\Psi$$

He then obtained the correct energy spectrum for the hydrogen atom (chap 13), and studied the spectrum of SHO (chap 7), the Stark effect (chap 17), the absorption and emission of radiation by an atom (chap 18), all within 6 months of his discovery. The radiation problem led him to write down the time-dependent Schrodinger eq.

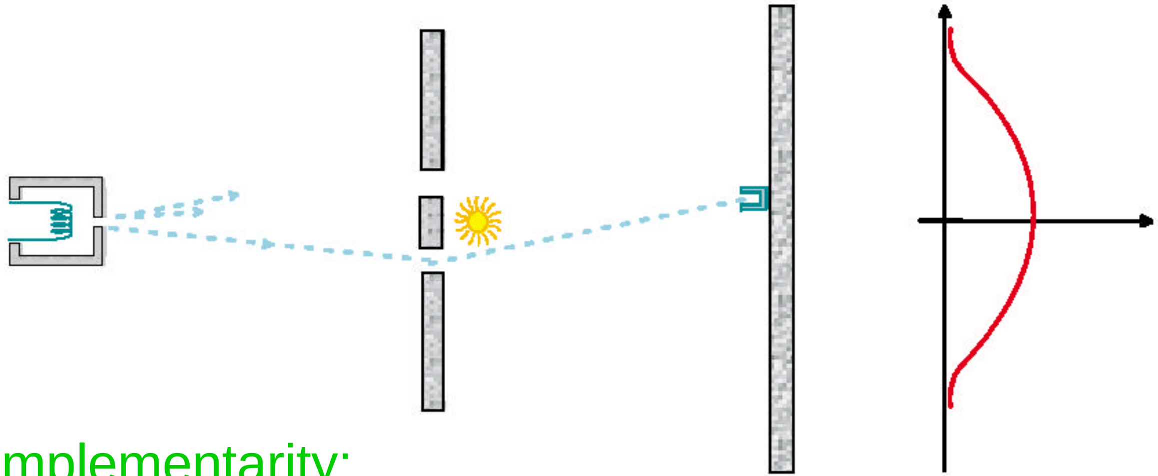
$$-\frac{h^2}{2m}\nabla^2\Psi + V\Psi = i\hbar\frac{\partial}{\partial t}\Psi$$

Low intensity photon interference



Duality (the same applies to electrons)

“Which-path” measurement



Complementarity:

Once we know the path, the interference disappears.
Particle and wave properties are like 2 sides of a coin.

The formalism of QM says that we won’t and shouldn’t be able to determine the path and keep the interference.

If we can, then QM is like statistical mechanics, and a deeper theory (hidden variable theory) is required.

Can we know the path but keep the interference?

A. Einstein's thought experiment, 1927

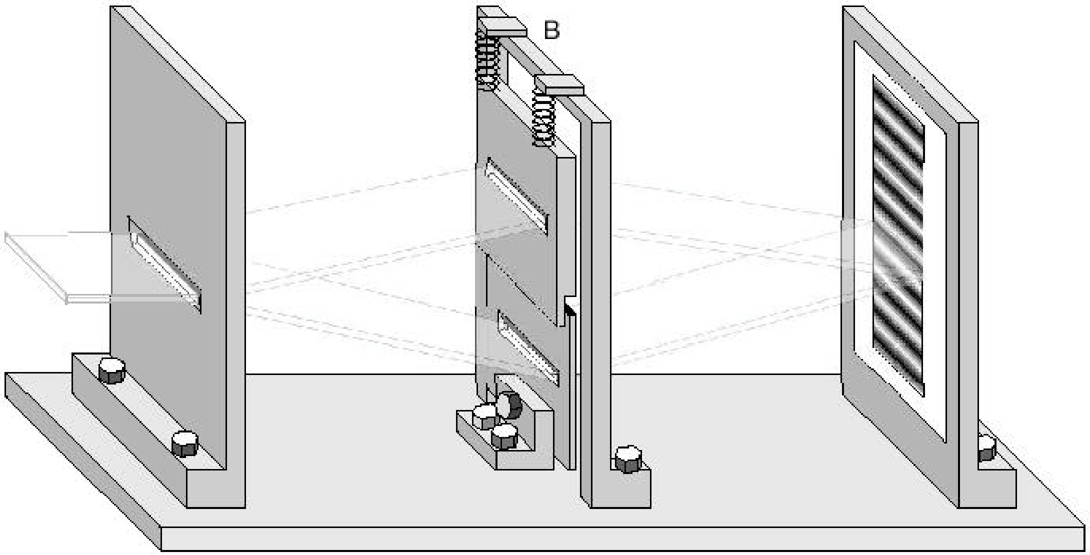
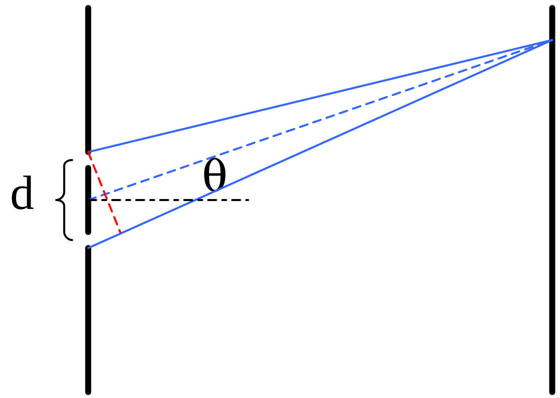


Figure is from Bertet et al, Nature 2001



To observe inteference, we need

$$\Delta d \sin\theta < \lambda$$

However, from uncertainty relation,

we know $\Delta d \Delta p_{screen} > h$

$\therefore \Delta d > h / \Delta p_{screen} = \lambda / \sin\theta$

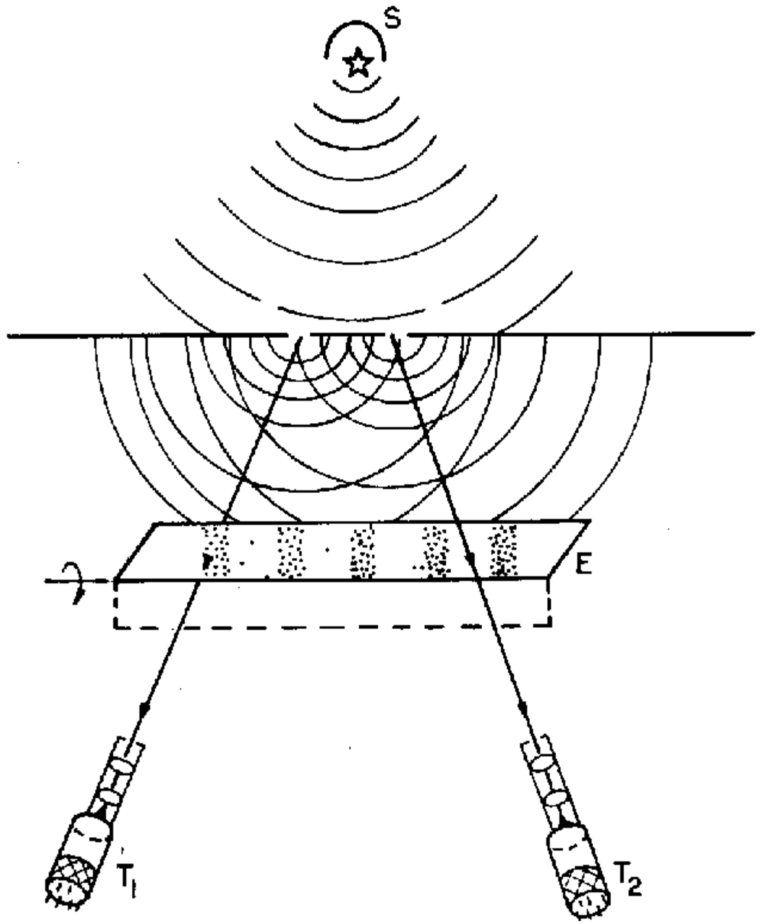
where we have used

$$\Delta p_{screen} = p \sin\theta = \frac{h}{\lambda} \sin\theta$$

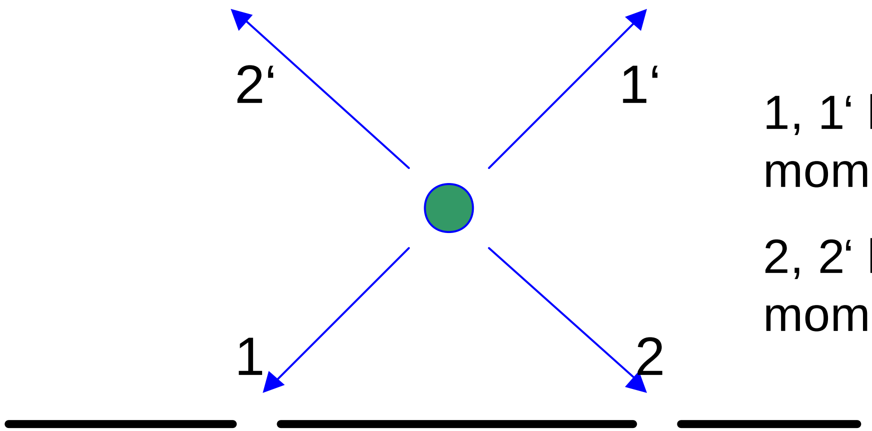
Can the photons be cheated?

Delayed choice experiment (proposed by J.A. Wheeler, 1978)

We decide whether to determine their paths only after the particles passed the slit (but before they hit the target)



Interference using particle pairs (Dopfer, 1998)



1, 1‘ have antiparallel momenta

2, 2‘ have antiparallel momenta

- If we register the positions of 1’ and 2’, then no interference
- If we register in such a way that destroys the info about the positions of 1 and 2, then interference appears
- If we don’t register? Again NO interference!

You don’t need to touch the particles to destroy the interference!

- QM is really more weird than particles showing wave property, or the existence of uncertainty relation. (esp. after we learned about multi-particle systems)
- Einstein understood this serious conflict with traditional physics long time ago and thought there is something wrong with the theory.
- Till now, all experiments are consistent with the predictions of QM. Everybody knows how to use it, but nobody can give a satisfying picture of the quantum phenomena.
- In the following, we start to learn the basic rules of QM. (slightly more general then Shankar's.)

Three postulate of quantum mechanics

Ψ Ω ω

Ψ

I. The state of a system is represented by a vector $|\Psi\rangle$ in a Hilbert space

Note:

- In the $|x\rangle$ basis, $\langle x|\Psi\rangle = \Psi(x)$ is the familiar wave function in (1-dim) real space, which is complex-valued.
- In the $|k\rangle$ basis, $\langle k|\Psi\rangle = \Psi(k)$ is the Fourier-transformed wave function in k-space.
- $|\Psi\rangle$ and $c|\Psi\rangle$ (c is a constant) describe the same state for a physical system, so we always choose the normalized state.

Ω

II. To every physical observable, there is a corresponding Hermitian operator Ω

Note: For example, the operators

x and $p=\hbar/i\,(d/dx)$

are the position and momentum operators (in 1-dim)

- An operator can have very different forms on different bases.

E.g.,

$$< x|\hat{x}|\Psi >= x < x|\Psi >$$

$$< p|\hat{x}|\Psi >= i\hbar \frac{d}{dp} < p|\Psi >$$

- It's not always easy to know the operator for an observable, some can be constructed from **x** and **p** (e.g. $\mathbf{L}=\mathbf{x}\times\mathbf{p}$), some cannot (e.g. spin **S**)

- Hermitian operator $\leftrightarrow$ physical observable

  Unitary operator $\leftrightarrow$ transformation of a state (space rotation, time evolution...)

  Neither of the above: anti-unitary operator for time reversal, creation/annihilation operators a, a^+ (chap 7)… etc

ω

III. Assume $\{|\omega_i>\}$ is the complete set of eigenstates of the the physical observable Ω. The state of the system can be expanded as

$$|\Psi>=\sum_i |w_i><w_i|\Psi> \qquad (w_\mathrm{i} \text{ can be discrete or continuous})$$

(a) Born's rule: When we measure Ω experimentally, we'll get one of the eigenvalues ω_i , with the probability

$$P(\omega_i)=|<\omega_i|\Psi>|^2 \qquad (\Sigma_i P(\omega_i)=1)$$

(b) The state of the system is changed from $|\Psi>$ to the eigenstate $|\omega_i>$ as a result of the measurement !!

Note:

- The expectation value of the physical observable Ω after many measurements:

$$<\Omega>=\sum_i w_i P(w_i)=\sum_i w_i |<w_i|\Psi>|^2 = <\Psi|\Omega|\Psi>$$

- The uncertainty of the measurement is defined by

$$\Delta\Omega=\sqrt{<\Omega^2>-<\Omega>^2} \qquad \text{(standard deviation)}$$

Example. (dim of Hilbert space = ∞)

- the state of an electron is described by $|\Psi>$ (postulate I)
- the position of an electron is a physical observable

  the corresponding hermitian operator is **x** (postulate II)
- its eigenvalues are x, with eigenstates $|x>$

  when we measure the position,

  we get one particular eigenvalue x_1 (a dot on the screen),

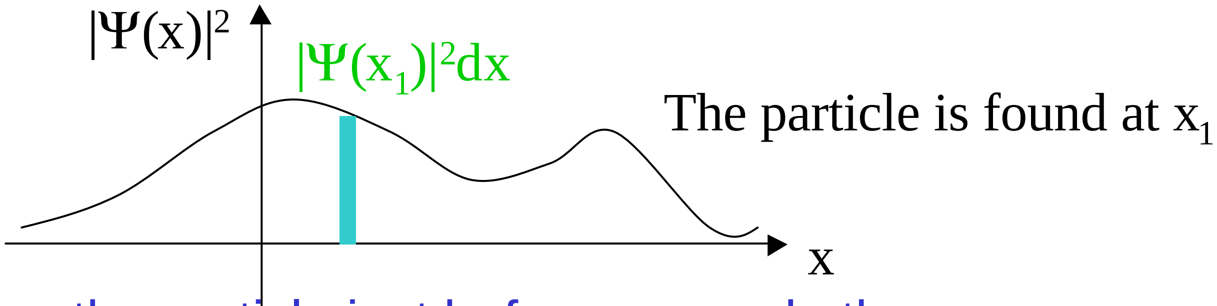
  with probability $P(x_1)=|<x_1|\Psi>|^2=|\Psi(x_1)|^2$ (postulate III a)

  and $|\Psi> \longrightarrow |x_1>$ (postulate III b)

Note that the new wave function is $<x|x_1>=\delta(x-x_1)$

("collapse" of the wave function)

A note on the wave function: (Ref: Introduction to QM, by Griffith)



Q: Where was the particle just before we made the measurement?

1. The realist view (shared by Einstein, de Broglie, Schrodinger...)

   The particle was at x.

2. The orthodox view (the "Copenhagen interpretation", shared by Bohr, Heisenberg, Born...)

   The particle wasn't really anywhere, it's the measurement that produces the result.

3. The agnostic view

   Refuse to answer. It makes no sense to talk about things before the measurement.

1964: Bell's inequality, we can distinguish 1 and 2 by experiment !!

1982: The Aspect experiment, view 2 wins, as expected.

Measuring two different physical observables
(Ref: Sakurai, Modern QM, chap 1)

1) Compatible observables, $[\Omega,\Lambda] = 0$

Can have a complete set of simultaneous eigenstates $\{|\omega_i, \lambda_j>\}$

In general, we can expand $|\Psi>=\sum_{i,j} C_{i,j}|w_i, l_j>$

$$|\Psi> \xrightarrow[\text{get } w_i]{\text{measure } \Omega} \sum_j C_{i,j}|w_i, l_j> \xrightarrow[\text{get } l_j]{\text{measure } \Lambda} C_{i,j}|w_i, l_j> \xrightarrow{\text{measure } \Omega} \overset{\text{still get}}{|w_i, l_j>}$$

$$||$$

$$|\Psi> \xrightarrow[\text{get } l_j]{\text{measure } \Lambda} \sum_i C_{i,j}|w_i, l_j> \xrightarrow[\text{get } w_i]{\text{measure } \Omega} C_{i,j}|w_i, l_j>$$

2) Incompatible observables $[\Omega,\Lambda] \neq 0$

Do not have a "*complete set*" of simultaneous eigenstates

(If they do, then they will commute. Prove it!)

Note: can have a "*subset*" of simultaneous eigenstates

$$\text{In general, we can expand } |\Psi\rangle = \sum_i |w_i\rangle\langle w_i|\Psi\rangle$$

$$\text{or } = \sum_i |l_i\rangle\langle l_i|\Psi\rangle$$

$$|\Psi\rangle \xrightarrow[\text{get } w_i]{\text{measure } \Omega} |w_i\rangle\langle w_i|\Psi\rangle \xrightarrow[\text{get } l_j]{\text{measure } \Lambda} |l_j\rangle\langle l_j|w_i\rangle\langle w_i|\Psi\rangle$$

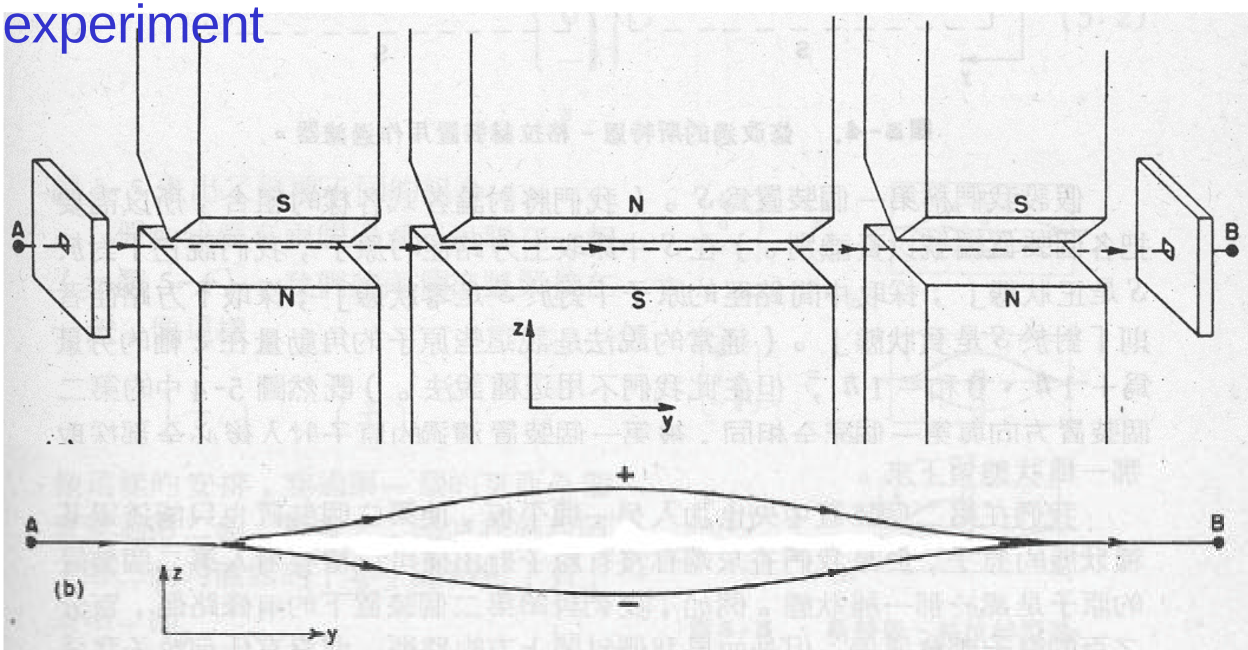
$$\neq$$

$$|\Psi\rangle \xrightarrow[\text{get } l_j]{\text{measure } \Lambda} |l_j\rangle\langle l_j|\Psi\rangle \xrightarrow[\text{get } w_i]{\text{measure } \Omega} |w_i\rangle\langle w_i|l_j\rangle\langle l_j|\Psi\rangle$$

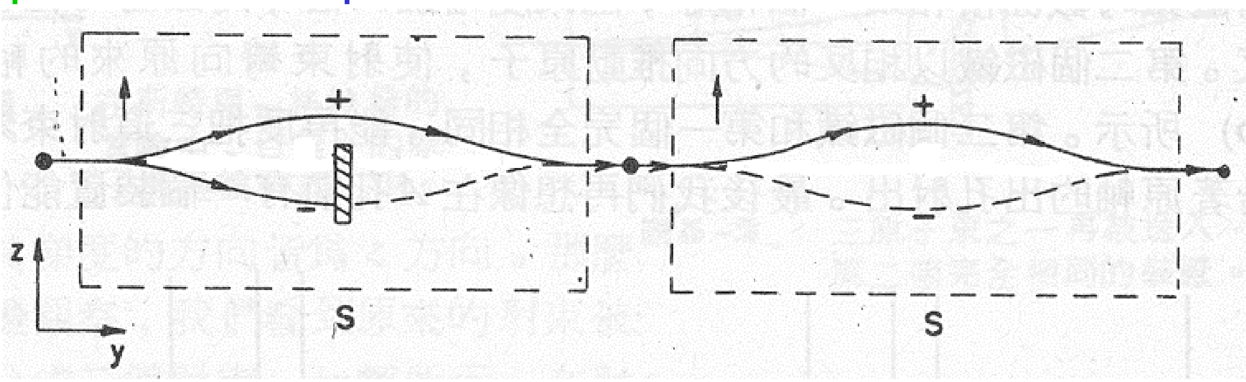
The final state depends on the order of the measurement

Spin system (Ref: Chap 1, Modern QM, by Sakurai; Chap 5, 6 in Feynman's lectures, vol. 3)

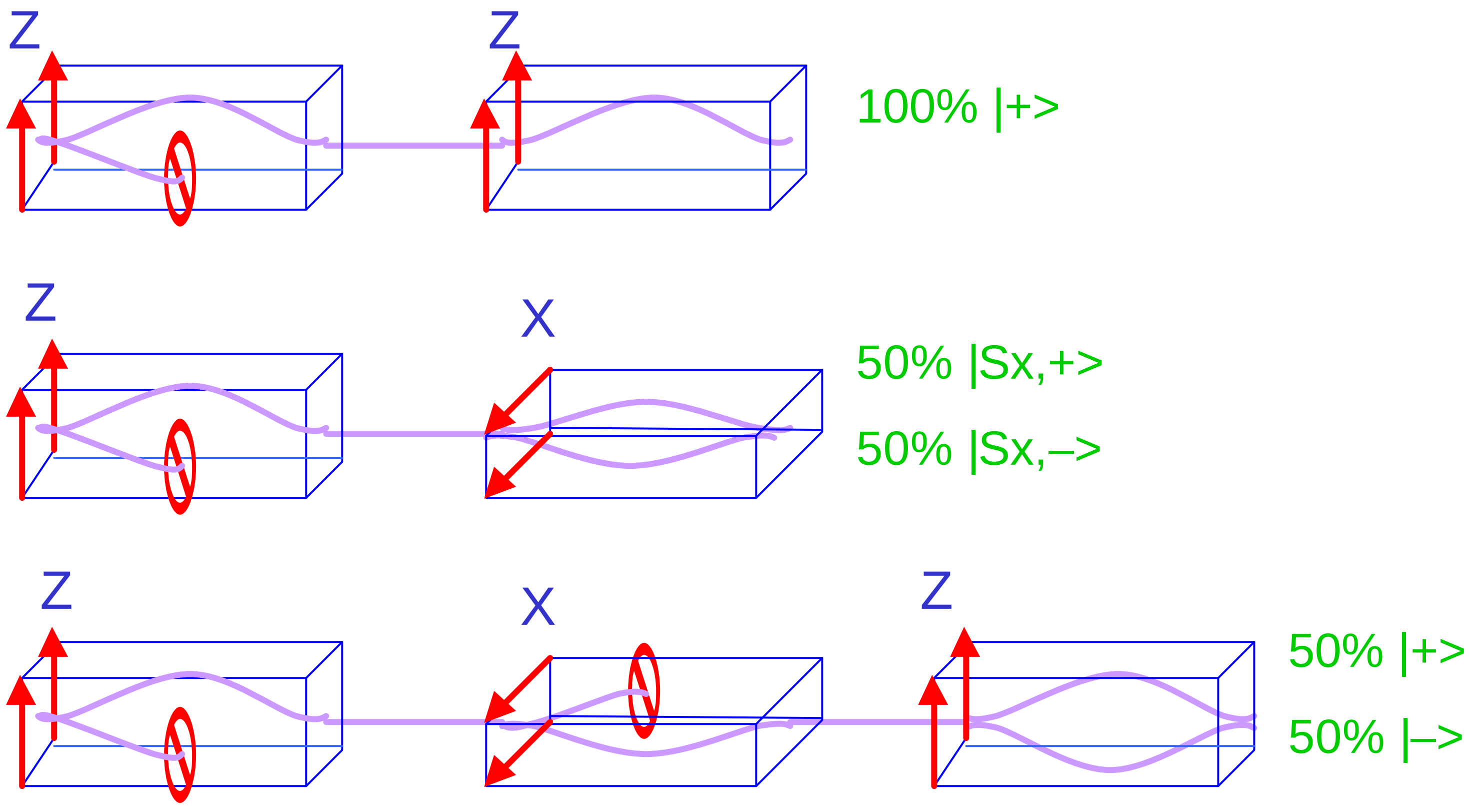
SG experiment



Sequential SG experiment



More sequential experiments and results

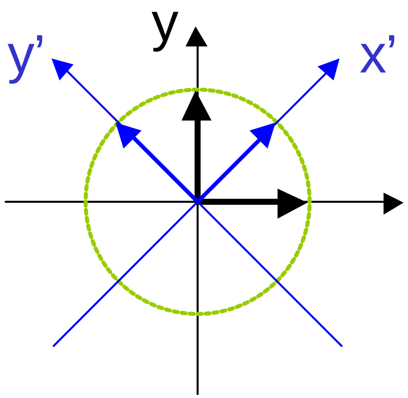


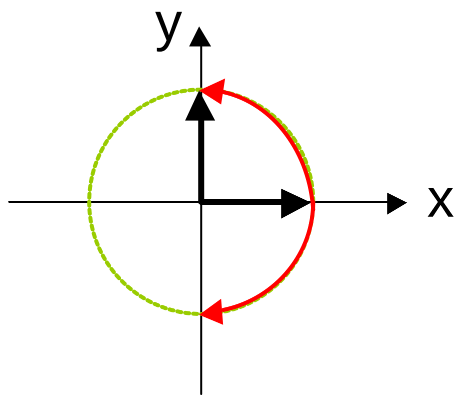
The measurement of S_x completely destroys the info about S_z

$\therefore S_z$ and S_x cannot be determined simultaneously (do not commute)

Similarly for S_z and S_y (incompatible observables)

Analogy with the polarization of light





$S_z \pm$ atoms $\leftrightarrow x, y$ polarized light

$S_x \pm$ atoms $\leftrightarrow x', y'$ polarized light

$S_y +$ atoms $\leftrightarrow$ right circularly polarized

$S_y -$ atoms $\leftrightarrow$ left circularly polarized

$$E_0\hat{x}'\cos(kz-wt) = E_0 \quad \frac{1}{\sqrt{2}}\hat{x}\cos(kz-wt)+\frac{1}{\sqrt{2}}\hat{y}\cos(kz-wt) \ ,$$

$$E_0\hat{y}'\cos(kz-wt) = E_0 \ -\frac{1}{\sqrt{2}}\hat{x}\cos(kz-wt)+\frac{1}{\sqrt{2}}\hat{y}\cos(kz-wt) \ .$$

$$\text{or Re } \vec{E}_0 e^{i(kz-wt)} \ = \text{Re } \ E_0 \ \pm\frac{1}{\sqrt{2}}\hat{x}+\frac{1}{\sqrt{2}}\hat{y} \ e^{i(kz-wt)}$$

$$\vec{E}_{\substack{R\\L}}(z,t) = E_0 \ \frac{1}{\sqrt{2}}\hat{x}\cos(kz-wt)\pm\frac{1}{\sqrt{2}}\hat{y}\cos \ kz-wt+\frac{p}{2}$$

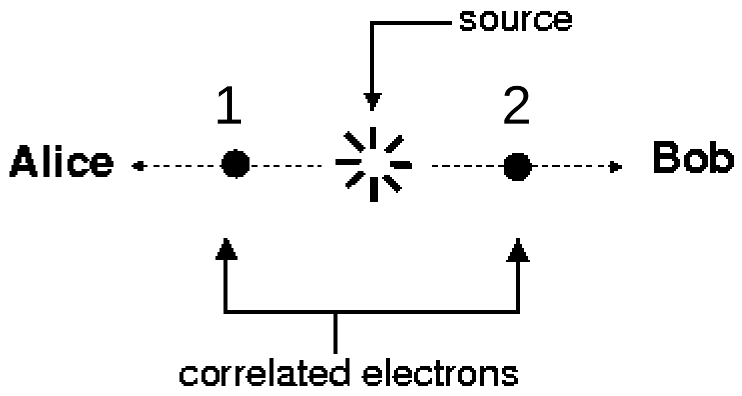
$$= \text{Re } \ E_0 \ \frac{1}{\sqrt{2}}\hat{x}\pm\frac{i}{\sqrt{2}}\hat{y} \ e^{i(kz-wt)}$$

$$|S_x;\pm\rangle = \pm\frac{1}{\sqrt{2}}|S_z;+\rangle+\frac{1}{\sqrt{2}}|S_z;-\rangle \to \frac{1}{\sqrt{2}}\begin{pmatrix}\pm1\\1\end{pmatrix}$$

$$|S_y;\pm\rangle = \frac{1}{\sqrt{2}}|S_z;+\rangle\pm\frac{i}{\sqrt{2}}|S_z;-\rangle \to \frac{1}{\sqrt{2}}\begin{pmatrix}1\\\pm i\end{pmatrix}$$

Another of Einstein's attack on the uncertainty principle:

EPR paradox (Einstein, Podolsky, and Rosen, 1935)



$$[x_1 - x_2, p_1 + p_2] = 0$$

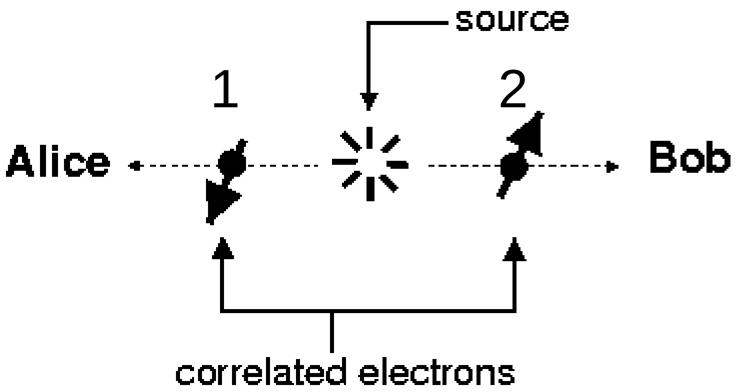
So $x_1 - x_2$ and $p_1 + p_2$ can both be measured precisely

Therefore, x_1 and p_1 (via p_2) can both be determined precisely

$$\therefore \Delta x_1 \Delta p_1 = 0 \,!$$

Spooky action at a distance is required!

Bohm's version of EPR paradox



2 electrons have zero total angular momentum

if we measure S_{1x}, then we know $S_{2x}(= -S_{1x})$

if we measure S_{2y}, then we know $S_{1y}(= -S_{2y})$

$\therefore\ \Delta S_{1x}\Delta S_{1y} = 0$!

Quantum nonlocality, or entanglement