Phase transitions in
Bi-layer quantum Hall systems

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Landau levels

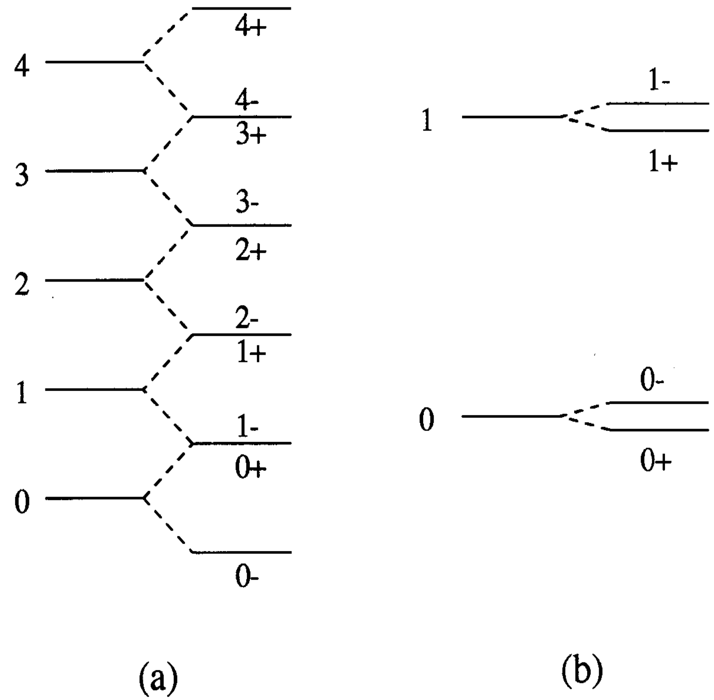


Figure 1.22: (a) Landau energy levels for an electron in free space. Numbers label the Landau levels and $+(-)$ refers to spin up (down). Since the g factor is 2, the Zeeman splitting is exactly equal to the Landau level spacing, $\hbar\omega_c$ and there are extra degeneracies as indicated. (b) Same for an electron in GaAs. Because the effective mass is small and $g \approx -0.4$, the degeneracy is strongly lifted and the spin assignments are reversed.

Ferromagnetism near integer filling factor ν=1

Zeeman energy	2 K
Exchange energy	200 K

$\rightarrow$ spontaneous ferromagnetic ordering

The wave function is simply the ν=1 Laughlin wave function

: the world's best understood ferromagnet

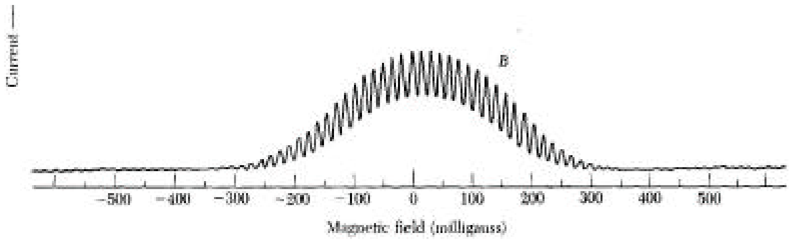
(an itinerant ferromagnet with quantized Hall resistances)

Double layer quantum Hall ferromagnet

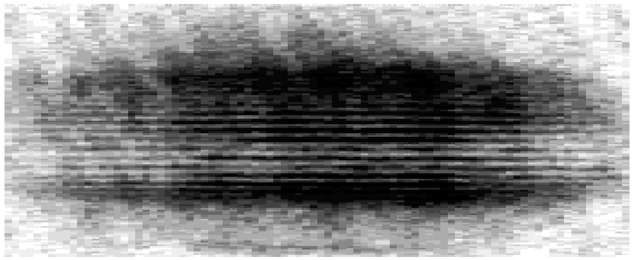
An urge to put two macroscopic quantum coherent states

next to each other

Superconductors: Josephson effect



Bose-Einstein condensation: matter wave interference



Quantum Hall effect:

?

Bi-layer system （Neglect electron spin for the moment）

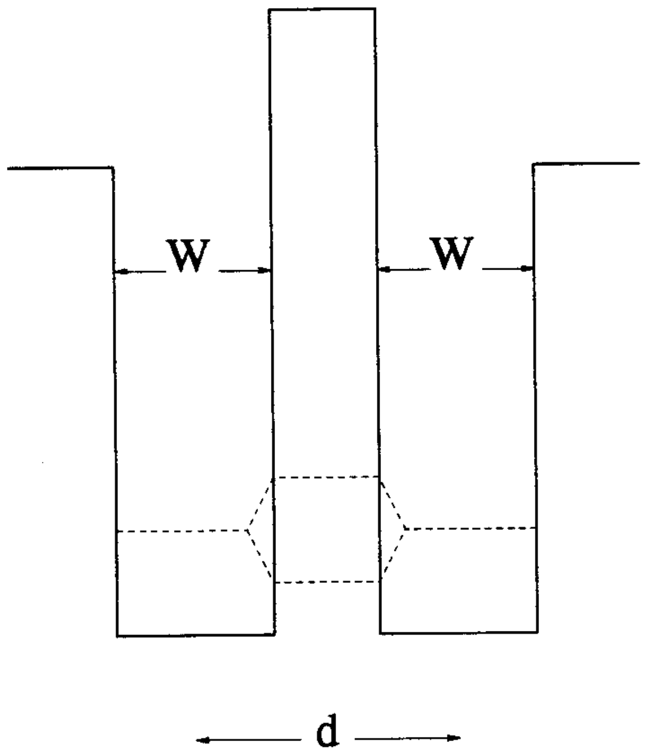


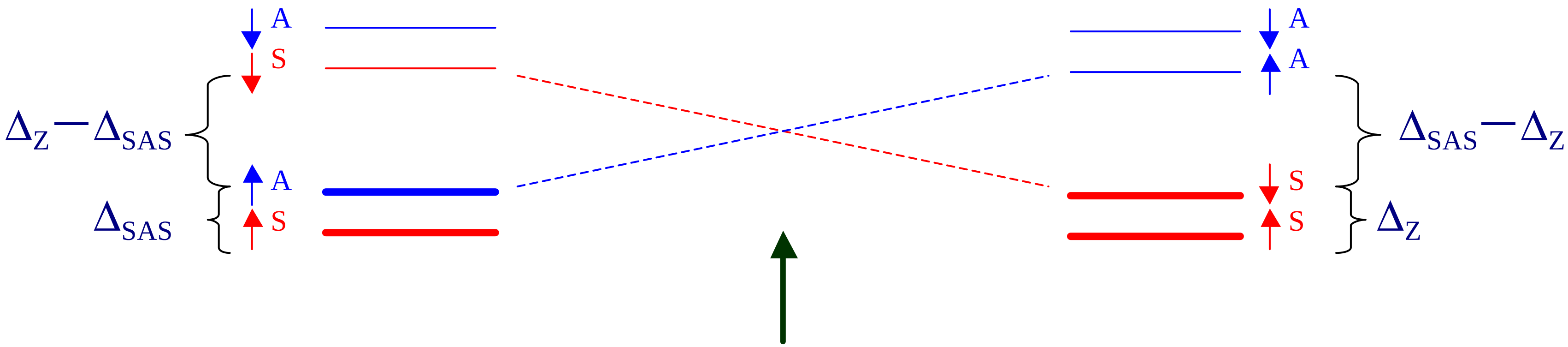
Figure 1.34: Schematic conduction band edge profile for a double-layer two-dimensional electron gas system. Typical widths and separations are $W \sim d \sim 100\text{Å}$ and are comparable to the spacing between electrons within each inversion layer.

Bi-layer quantum Hall system with real spin ($\nu = 2$)

Single particle picture

$\Delta_{SAS} << \Delta_Z$ (ferromagnet, F phase)

$\Delta_{SAS} >> \Delta_Z$ (spin singlet, S phase)



$\Delta_{SAS} = \Delta_Z$

Level-crossing

1st-order phase transition ?

L. Zheng, R. J. Radtke, and S. Das Sarma, Phys. Rev. Lett. **78,** 2453 (1997)：

(unrestricted Hartree-Fock mean field theory)

The level crossing and the associated 1st -order transition does not happen around the $\Delta_Z = \Delta_{AS}$ regime！

Instead a new purely *interaction-driven* quantum phase, the so-called canted anti-ferromagnetic (C) phase, is stabilized

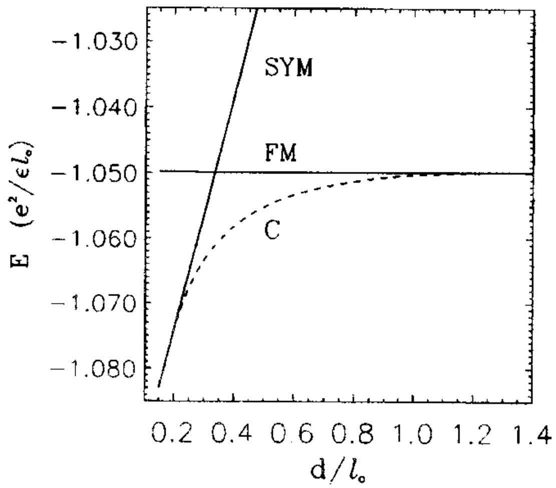
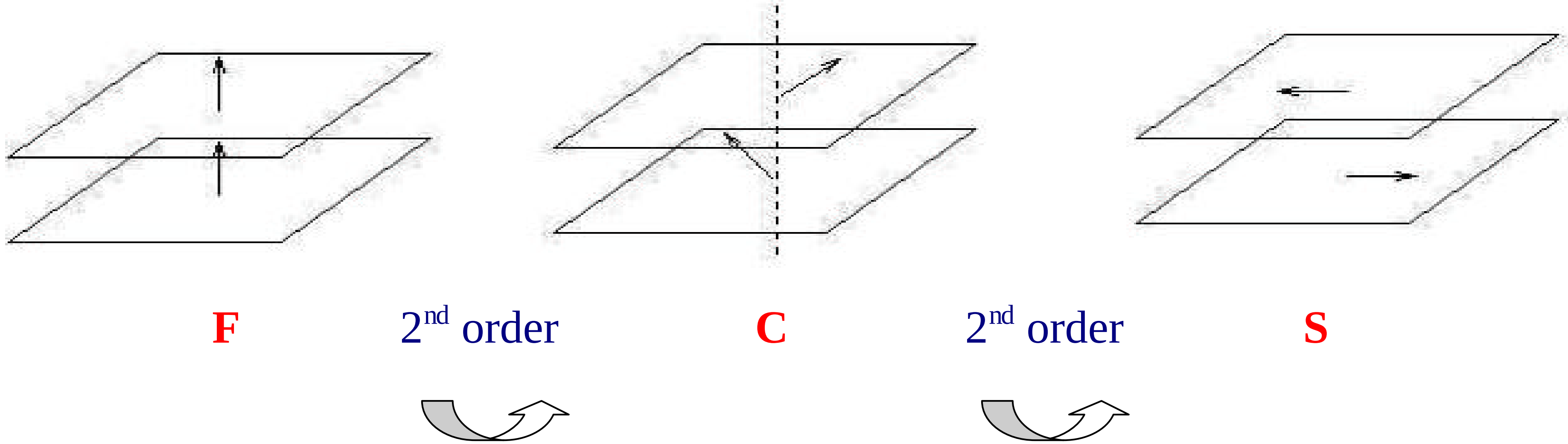
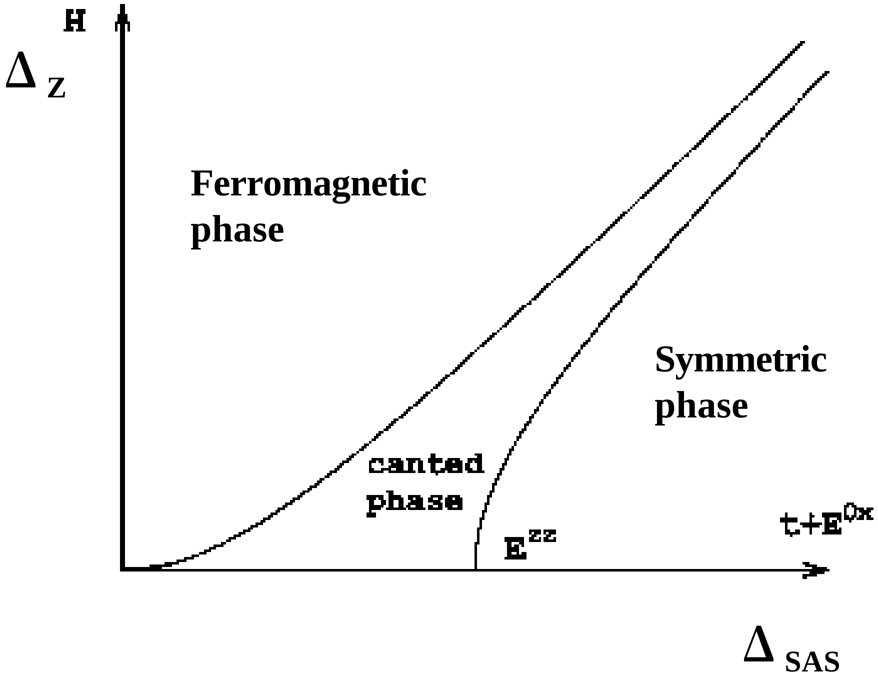


FIG. 1. The energy per magnetic flux in the SYM state, the spin-polarized FM state, and the C state for a $\nu=2$ double-layer system with $\Delta_{sas}=0.07e^2/\epsilon l_o$, $\Delta_z=0.01e^2/\epsilon l_o$, and well thickness $d = 0.8l$

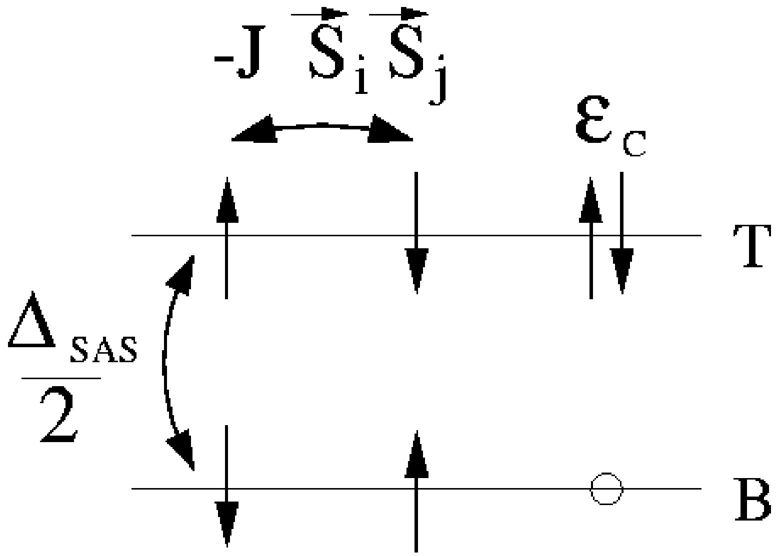

F
2nd order
C
2nd order
S

Zero-temperature phase diagram (ν=2)

Hartree-Fock approximation



- **physical reason for the canting：spin-bond theory**



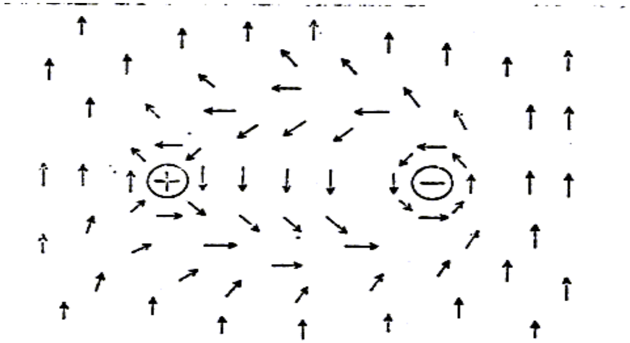
1. intra-layer Coulomb exchange interaction favors the spins of electrons within the **same** layer to align in the **same** direction
2. tunneling between the layers favors the formation of **spin singlet** states from the pairs of electrons in the **opposite** layers！

→ their competition leads to a canted anti-ferromagnetic phase！

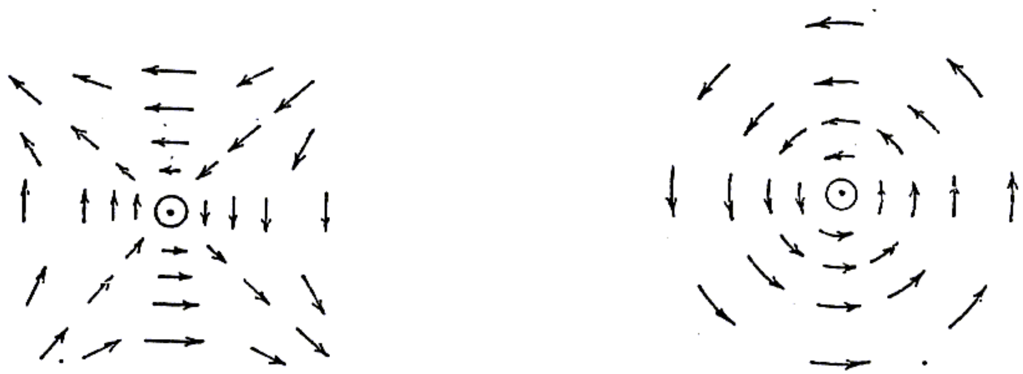
- ∵ spontaneously in-plane rotational symmetry breaking in the C phase

  ∴ Kosterlitz-Thouless transition at finite temperatures！

  （vortex-antivortex unbinding transition）



$T < T_{KT}$



$T > T_{KT}$

- **subsequent theoretical analysis：**

1. O(3) quantum non-linear sigma model

S. Das Sarma, S. Sachdev, and L. Zheng, Phys. Rev. Lett. **79**, 917 (1997)

S. Das Sarma, S. Sachdev, and L. Zheng, Phys. Rev. B **58**, 4672 (1998)

2. more detailed Hartree-Fock calculation

S. Das Sarma, S. Sachdev, and L. Zheng, Phys. Rev. B **58**, 4672 (1998)

A. H. MacDonald, R. Rajaraman, and T. Jungwirth, Phys. Rev. B **60**, 8817 (1999)

L. Brey, E. Demler and S. Das Sarma, Phys. Rev. Lett. **83**, 168 (1999)

M. F. Yang and M. C. Chang, Phys. Rev. B **61**, R2429 (2000)

3. spin-bond theory

E. Demler and S. Das Sarma, Phys. Rev. Lett. **82**, 3895 (1999)

L. Brey, E. Demler and S. Das Sarma, Phys. Rev. Lett. **83**, 168 (1999)

K. Yang, Phys. Rev. B **60**, 15578 (1999)

M. F. Yang and M. C. Chang, Phys. Rev. B **60**, R13985 (1999)

4. Chern-Simons field theory

E. Demler, E. Kim, and S. Das Sarma, Phys.Rev.B **61**, R10567 (2000)

5. exact numerical diagonalization calculation on a small system

J. Schliemann and A. H. MacDonald, Phys. Rev. Lett **84**, 4437 (2000)

- **experimental support：**

1. inelastic light scattering experiment

V. Pellegrini *et.al.*, Phys. Rev. Lett **78**, 310 (1997)

V. Pellegrini *et.al.*, Science **281**, 799 (1998)

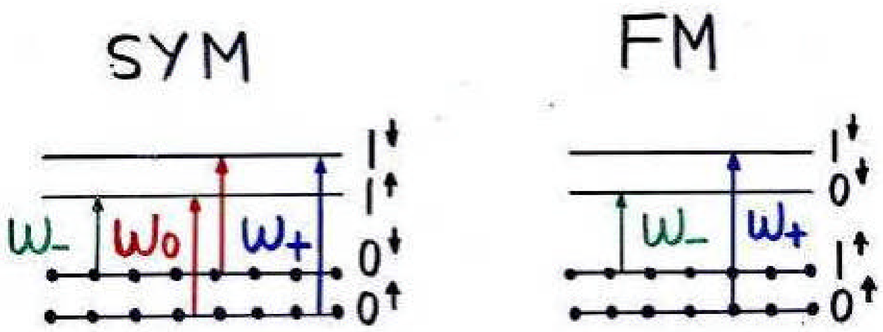
V. Pellegrini *et.al.*, Nature **402**, 638 (1999)

2. transport measurements of quantum Hall activation energies

A. Sawada *et.al.*, Phys. Rev. Lett **80**, 4534 (1998); Phys. Rev. B **59**, 14888 (1999)

V. S. Krapai *et.al.*, Phys. Rev. Lett **84**, 725 (2000)

Inter-subband spin excitations



The $\Delta S^z = 0$ mode (ω_0)

- FM Phase: forbidden
- AF Phase: small spectral weight
- SYM Phase: (TD-HFA or SMA)

$$\omega_\pm = \omega_0 \pm \Delta_z$$

Softening (collapse) of the $\delta S_Z = 0$ spin density excitations (SDE)

(V. Pellegrini, Pinczuk, Dennis, Plaut, Pfeiffer, and West, PRL 1997)

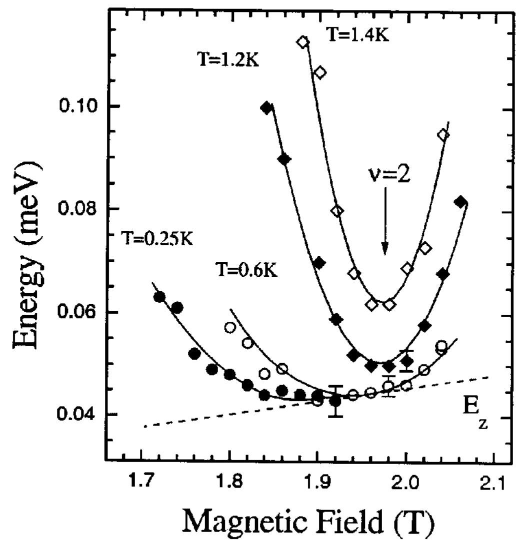


FIG. 3. Energy position of the $q = 0$, $\delta S_Z = 0$ spin-density excitations (SDE) for the higher electron density sample as a function of magnetic field and temperature. Solid lines are guides for the eyes. The dashed line represents the values of the Zeeman energy ($g = -0.4$) in the displayed range of magnetic field.

Indicate the existence of unstable spin-flip excitations with $\delta S_Z = 1$

→ Emergence of a new phase (the C phase)

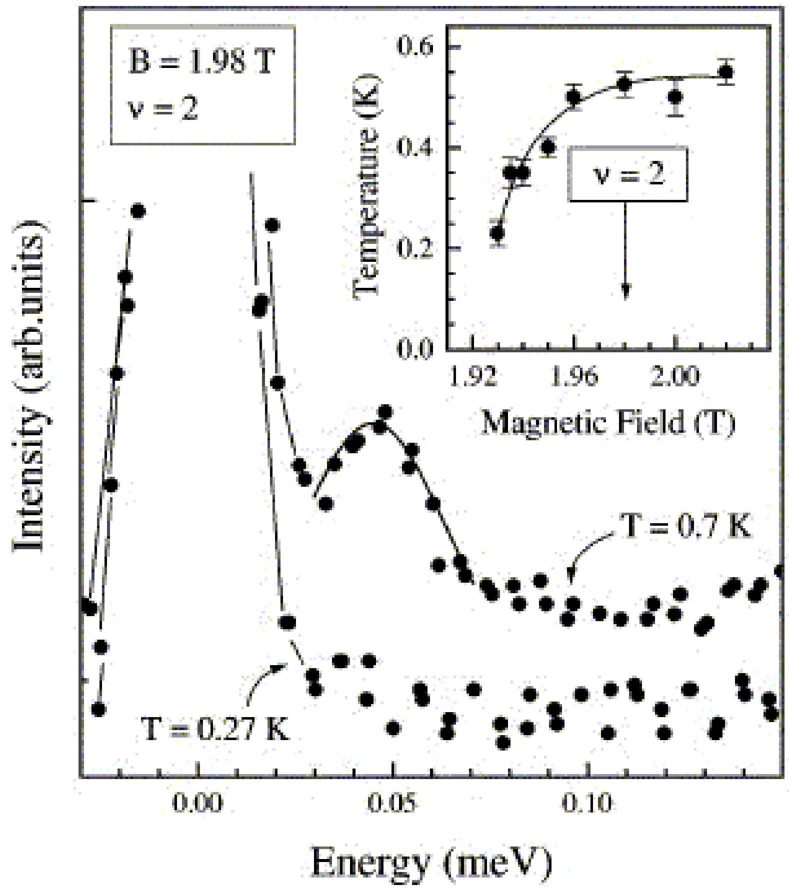
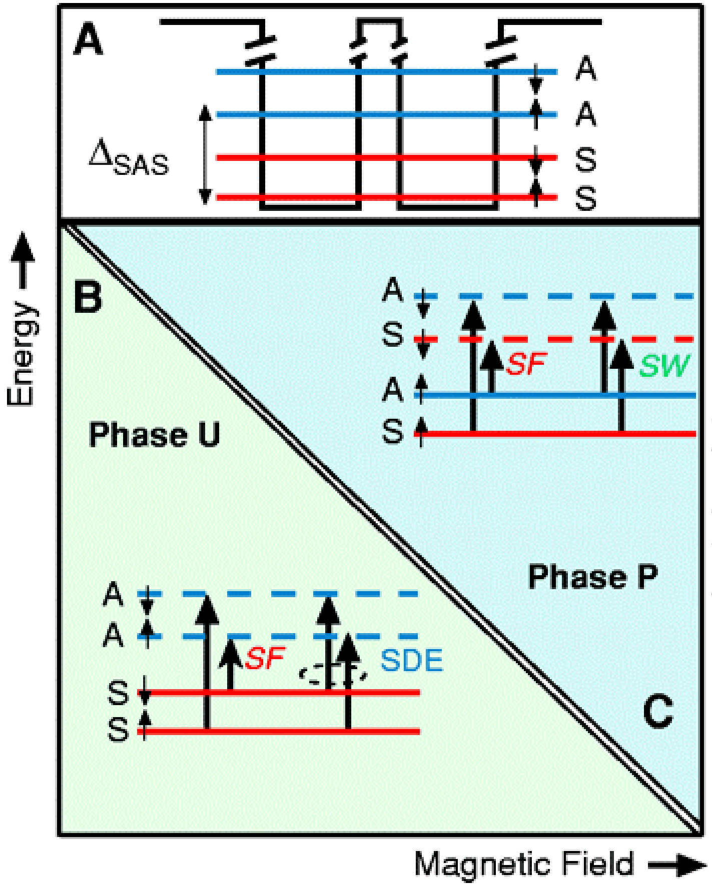
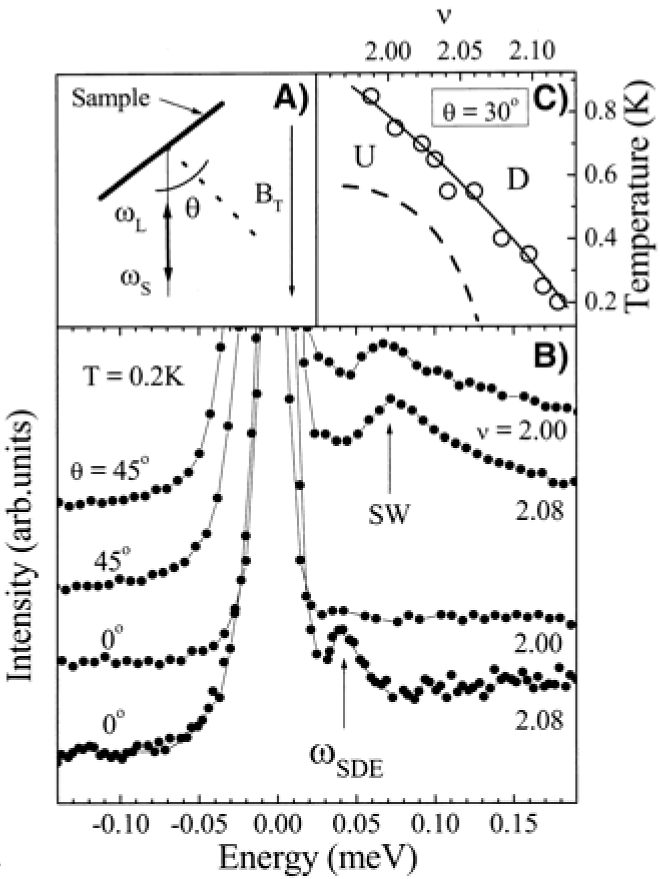


FIG. 4. Resonant inelastic light scattering spectra for the higher electron density sample at $\nu = 2$ and at two different temperatures. The peak corresponds to the $q = 0$ spin-density excitation. The inset shows the lowest value of the temperature at which the SDE peak first appears.

→ $T_{SDE} \approx 0.5K$

V. Pellegrini *et.al.*, Science **281**, 799 (1998)





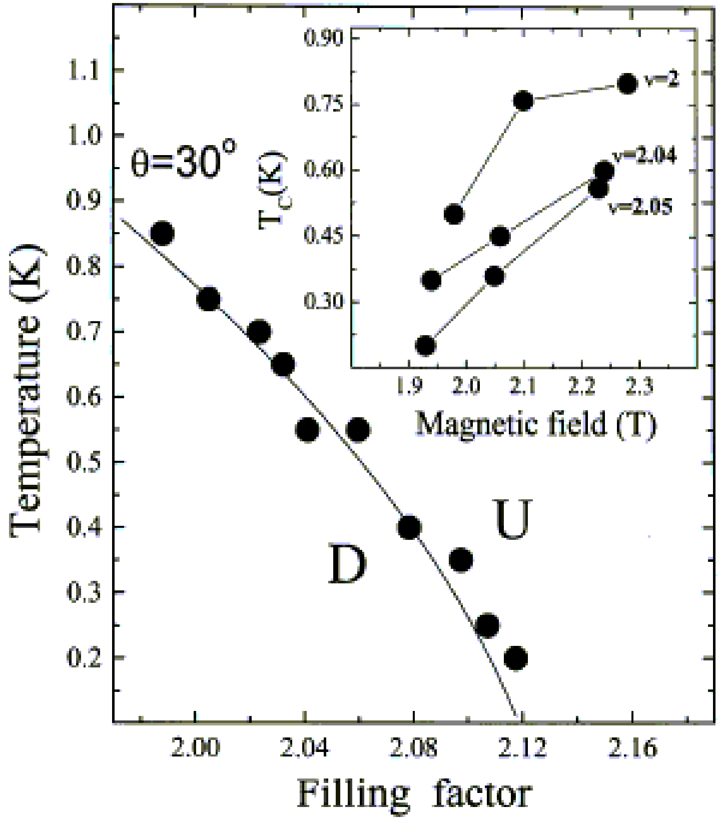


Fig. 3. Dependence of the critical temperature T_c on filling factors at tilt angle $\theta = 30^\circ$ (T_c is the lowest value of the temperature at which the spin-density excitation modes reappear in the spectra of phase D). The inset shows the values of T_c at three different total magnetic fields and for different filling factors.

→ T_{SDE} increases as tilt angle Θ increases !

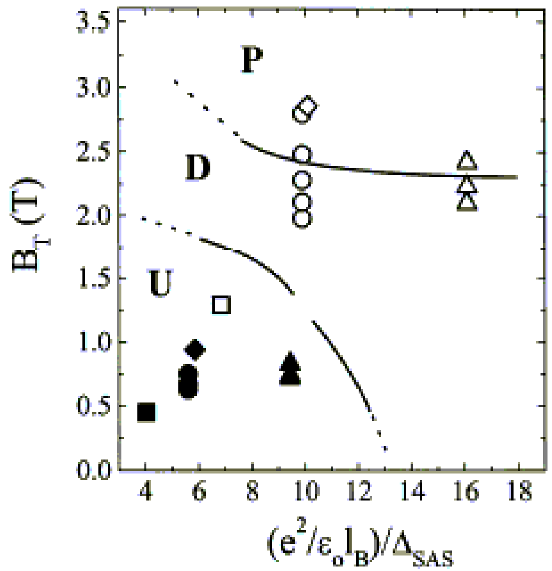


Fig. 2. Phase diagram for electron bilayers at even-integer values of filling factors and $T = 0.2$ K. Solid lines indicate the positions of phase boundaries determined from experiment. Dotted lines are introduced to give continuity to the boundaries. The total magnetic field B_T is plotted here against the reciprocal tunneling gap in units of the Coulomb interaction energy $(e^2/\varepsilon_o l_B)$. The samples are GaAs quantum wells of different densities. Squares: $n = 6.2 \times 10^{10}$ cm^{-2}; circles: $n = 9.9 \times 10^{10}$ cm^{-2}; triangles: $n = 1.05 \times 10^{11}$ cm^{-2} and diamonds: $n = 1.44 \times 10^{11}$ cm^{-2}. Open symbols are for $\nu = 2$ and full symbols for $\nu = 6$.

$\rightarrow \Theta_C \approx 37°$

Phase transitions induced by an in-plane magnetic field

Shift of phase boundaries using the effective spin model（∵ tend to be ferromagnetic）

(M.F. Yang, and M.C. Chang, PRB 1999)

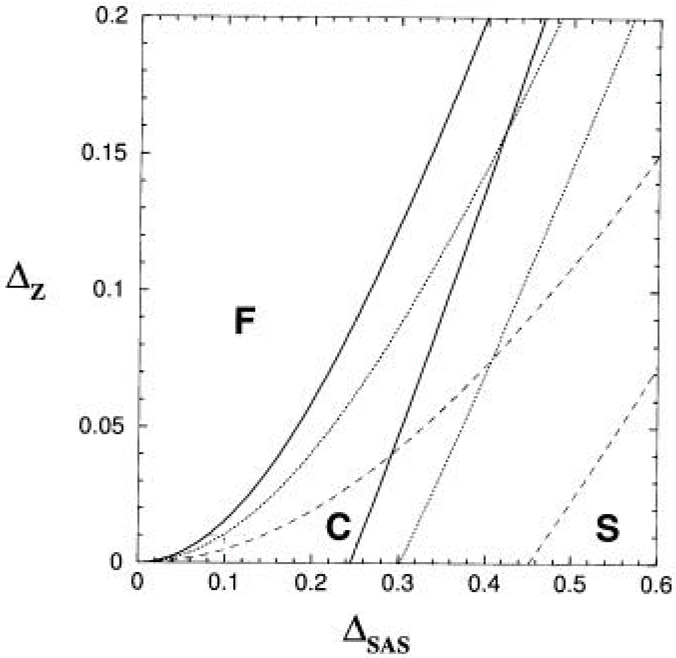


FIG. 1. ν=2 bilayer phase diagrams in the bosonic spin theory for different in-plane magnetic fields $B_{\|}$. Here $\bar{\Delta}_z$ means the Zeeman energy caused by the *total* magnetic field. Continuous, dotted, and dashed lines correspond to $B_{\|}/B_{\perp} = 0, 1/\sqrt{3}$, and 1, respectively. The width of the electron layer is 1.0 and the interlayer separation is 1.45.

Critical tilted angle for the C-F phase transition

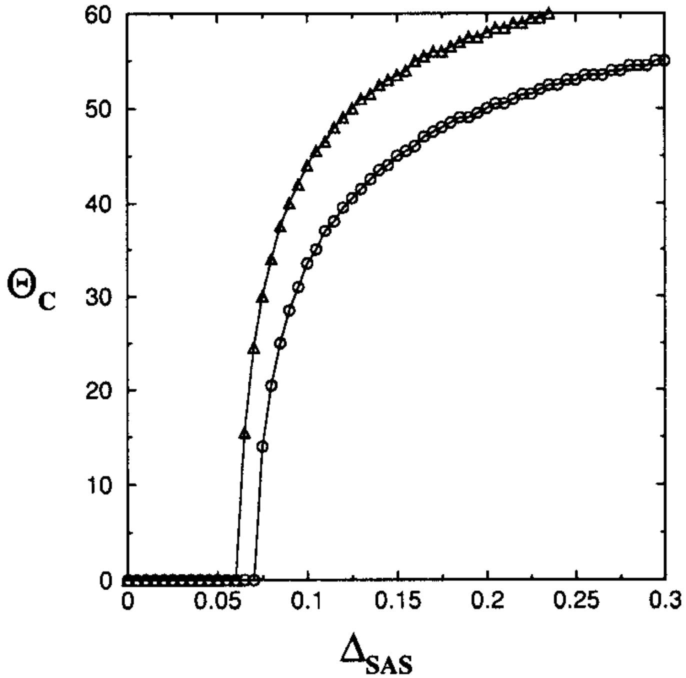


FIG. 2. Critical tilted angles for the C-F phase transition are plotted as a function of Δ_{SAS}. Circles are obtained using the parameters for the sample in Ref.[3], where $(\Delta_z, d, b) = (0.008, 1.45, 1.0)$; squares are for the sample in Ref.[5], where $(\Delta_z, d, b) = (0.00687, 1.08, 0.94)$. To compare the critical tilted angles with experimental values, choose $\Delta_{SAS} = 0.10$ for the first sample and $\Delta_{SAS} = 0.117$ for the second sample.

Is the predicted Kosterlitz-Thouless transition temperature T_{KT} related to the observed T_{SDE} in bi-layer system?

Das Sarma et.al.：Yes！

∵ $T_{KT} \approx 1.8$ K（Hartree-Fock calculation）is close to the observed $T_{SDE} \approx 0.52$ K

Yang and Chang：No！

∵ the dependence of T_{SDE} on tilt angle Θ is different from that of T_{KT} even qualitatively！

∴ inelastic light scattering experiment might not really probe the C phase！

Kosterlitz-Thouless transition in bi-layer system?

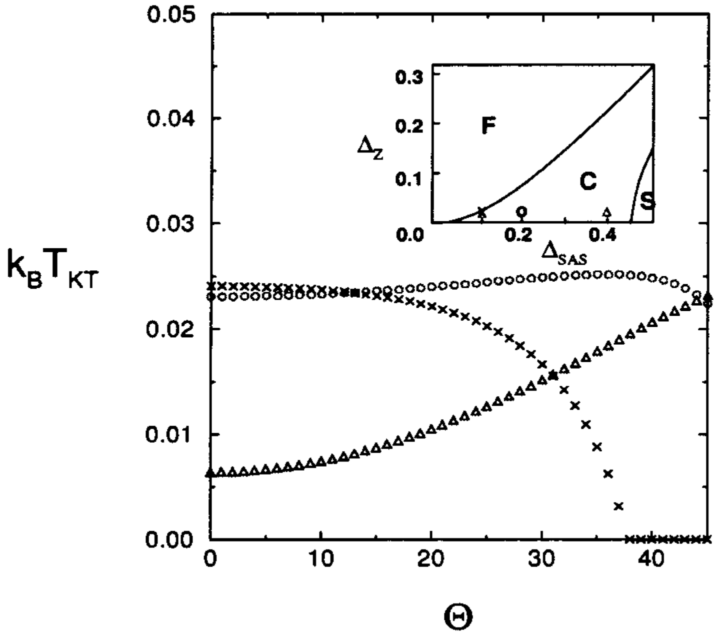


FIG. 1. $k_B T_{KT}$ as a function of the tilted angle Θ (in unit of degree) of the applied magnetic field with a fixed $B_\perp$. The energy unit is the intralayer Coulomb energy $e^2/\epsilon l$. The Zeeman energy caused by $B_\perp$ is $\Delta_z = 0.008$. The interlayer separation is $d = 1.45$ and the layer thickness is $b = 1.0$. Crosses, circles, and triangles correspond to $\Delta_{\mathrm{SAS}} = 0.1$, 0.2, and 0.4, respectively. Their locations in the $\Delta_z - \Delta_{SAS}$ quantum phase diagram calculated by the Hartree-Fock theory with $\Theta = 0°$ are shown in the inset. Notice that the cross symbol represents the experimental sample of Ref.[9], and its location is very close to the F-C phase boundary in the quantum phase diagram.

crossover temperature by level-crossing !

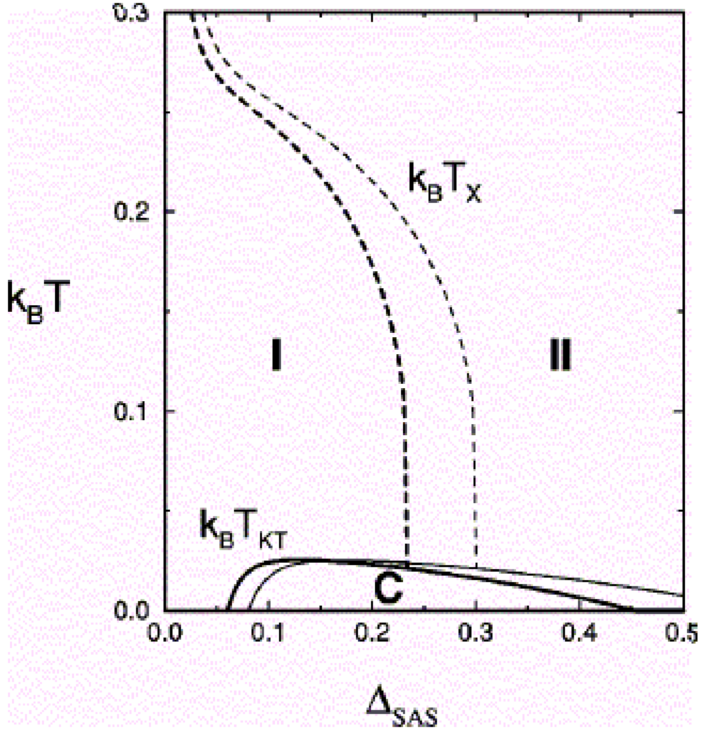


FIG. 2. $\nu=2$ bilayer phase diagrams at finite temperatures. The energy unit is $e^2/\epsilon l$. The sample parameters, Δ_z, d, and b are the same as in Fig. 1. The continuous lines are the phase boundaries for the KT transitions. The dashed lines are for the crossovers. The boundaries of the KT transitions and the crossovers under a magnetic field tilted by $\Theta=30°$ with a fixed $B_\perp$ are shown with thinner continuous and dashed lines, respectively.

- **further development：**

  1. C phase in multilayer superlattice

  L. Brey, PRL **81**, 4692 (1998)

  2. C phase in double quantum dots

  L. Martin-Moreno et.al., cond-mat/0006294

  3. the effects of spontaneous symmetry breaking in bilayer quantum Hall systems on the edge states

  A. Lopez and E. Fradkin, cond-mat/0008219