Spin-related physics
in
integer quantum Hall system

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Outline

Multi-component quantum Hall system
  spin, layer, or valley (for Si) degrees of freedom

Spin:  Quantum Hall ferro-magnet
  Collection excitation and quasi-particle in QHFM
    Spin wave, skyrmion

Layer:  \( v=1 \)  Bilayer system as a QH pseudo-FM
  Collective excitation and quasi-particle in in QHpFM
    Pseudo-spin wave, meron
    Josephson-like effect in bilayer system

  \( v=2 \)  Three different quantum phases
    Ferromagnet, canted antiferromagnet, and spin-singlet
    The effect of in-plane magnetic field
FIG. 3. Typical shape and cross section of a GaAs-Al\(_x\)Ga\(_{1-x}\)As heterostructure used for Hall-effect measurements.
Landau levels

(a)
(b)

Strong magnetic field (> 1 T)
Low temperature (< 4 K)

Figure 1.22: (a) Landau energy levels for an electron in free space. Numbers label the Landau levels and +(-) refers to spin up (down). Since the $g$ factor is 2, the Zeeman splitting is exactly equal to the Landau level spacing, $\hbar \omega_c$ and there are extra degeneracies as indicated. (b) Same for an electron in GaAs. Because the effective mass is small and $g \approx -0.4$, the degeneracy is strongly lifted and the spin assignments are reversed.
Some important parameters

Typical length scale: magnetic length \( \ell \equiv \sqrt{\hbar/eB} \approx 128 \text{ A at 4 T} \)

Dielectric constant: 12.8 \( \varepsilon^2/e\ell \approx 100 \text{ K at 4 T} \)

Material dep.

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GaAs/AlGaAs

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Effective mass: 0.38 \( m_e \) (LH), 0.6 \( m_e \) (HH)

G factor: -0.44, spin-orbit effect \( g\mu_B B \approx 1 \text{ K at 4 T} \)

Landau level degeneracy: \( D_{\text{LL}} = B \times \text{(sample area)}/\Phi_0 \)

Sample dep.

mobility \( 10^4 - 10^6 \text{ cm}^2/\text{Vs} \), electron density \( 10^{11} \text{ /cm}^2 \)
Physics in the Lowest Landau Level (LLL): integer case

Filling factor $v = 1$

\[
\{ \text{spontaneous ferromagnetic ordering} \}
\]

\[
\begin{cases}
\{ & \text{T}=0 \\
\{ & \text{T} > E_z \\
\{ & \text{plus e-e interaction}
\end{cases}
\]

\[
\text{single spin flip costs} \quad \Sigma = (\pi/2)^{1/2} e^2/\epsilon l = 125 \text{ K at 4 T}
\]

QHFM: an itinerant ferromagnet with quantized Hall resistances

The wave function is simply the $m=1$ Laughlin wave function

$\rightarrow$ the world’s best understood ferromagnet
Manybody effect on the “Zeeman splitting”

\[ \nu_{\uparrow} = 1: \quad g^* \mu_B B = g \mu_B B + \Sigma, \quad \Sigma = (\pi/2)^{1/2} e^2/\epsilon l \]

\[ \nu_{\uparrow} < 1: \quad g^* \mu_B B = g \mu_B B + \Sigma \times [n_{\uparrow}(g^*) - n_{\downarrow}(g^*)], \]

need to be solved self-consistently, Ando+Uemura 1974

- oscillatory behavior
Spin-related elementary excitations in QHFM

Spin wave

coherent superposition of e-h pairs

LLL projection

\[ |\psi_p\rangle = \sum_i e^{ip \cdot r_i} c_{i\uparrow}^\dagger c_{i\downarrow} |\psi_0\rangle \]
Semiclassical Picture:
\[ \frac{Pl^2}{\hbar} \rightarrow P \]

Dispersion Relation \((\nu = 1)\) \[= (\pi/2)^{1/2} \frac{e^2}{\epsilon l} \]

\[P l / \hbar \rightarrow \infty \rightarrow \text{quasielectron/quasihole pair.}\]

Are these the lowest-energy charged excitations?

different forms of spin texture in 2D:

- easy plane, vortex
- easy axis, antiskyrmion / skyrmion
Formation of a skyrmion

Noninteracting

Plus e-e interaction

Plus Zeeman energy

\[ g \ll 1: \text{larger extent of distorted spins is better} \]

\[ \rightarrow \text{rapid depolarization by adding one skyrmion} \]

\[ g \gg 1: \text{only one spin is flipped} \]
The skyrmion is charged

Spin texture determines charge density profile

Topological density

Topological charge of spin texture

\[ Q_{\text{top}} \equiv \frac{1}{8\pi} \int d^2r \, \epsilon^{\alpha\beta} \tilde{m} \cdot \partial_\alpha \tilde{m} \times \partial_\beta \tilde{m} = \text{integer} \]

\( Q_{\text{top}} \) is the wrapping number of the \( S^2(r) \rightarrow S^2(m) \) mapping

stable against smooth continuous distortion of \( m(r) \)

Electric charge \( Q = \nu \, e \, Q_{\text{top}} \)

NMR


magnetotransport

FIG. 3. Dependence of $K$ on filling factor $\nu$ for $B = 7.05$ T (open circles) at 1.55 K. As explained in the text, both fits are given by Eq. (1), but the solid line has $A = S = 1$ (non-interacting electrons), while the dashed line has $A = S = 3.6$ (finite-size Shklovskii).

FIG. 2. Results of tilted-field experiments on the $\nu = 1$ QHE. The energy gaps $\Delta$ at fixed $B$ are plotted vs the Zeeman energy $g_e B \mu_B$, both in units of $e^2/\epsilon_0 c_0$. Each data set starts with $\theta = 0$ and $B_{\text{sat}} = 3$ at the lower left. On the quantum well samples we use gate electrodes to tune the electron densities [11]. From top to bottom the samples had electron densities 0.6, 1.0, 0.6, and $1.0 \times 10^{11}$ cm$^{-2}$ and mobilities 3.4, 0.32, 0.18, and $0.15 \times 10^{5}$ cm$^2$/V s, respectively. For comparison we include lines with $\Delta / (g_e \mu_B \theta) = 1$ (dashed) and 1 (dotted). The inset shows a Hartree-Fock result of Shklovskii theory (full line) [514].
Maude et al, 1996
magnetotransport

FIG. 3. The measured energy gap at filling factor $\nu = 1$ as a function of the bare $g$ factor (bottom axis) and as a function of $g'$, the ratio of the Zeeman and Coulomb energies (top axis). The solid line is the expected gap for a Skyrmion-type excitation [7], while the short-dashed line indicates the expected variation for the "bare" Zeeman dependence $E_g + \mu g |\mu_B| E$ (with $r = 1$ as predicted by the spin-wave dispersion model). Lines with slopes corresponding to $s = 7$ spin flips (long-dashed line) and $j = 33$ spin flips (long-short-dashed line) are shown for comparison.
Inter-Landau level excitations

Different ways to lift an electron from \( n=0 \) to \( n=1 \):

\( v = 1 \)

- Magnetic plasma
- Spin-flip

\( v = 2 \)

- Magnetic plasma
- Spin-flip
- Spin density excitation

(singlet exciton) (triplet exciton)
FIG. 3. Energy shifts for a spin-polarized sample with only the lowest Landau level filled, \( \nu_1 = 1 \) and \( \nu_2 = 0 \), and \( m = 1 \) are shown. The energy scale is in units of \( e^2/\ell_0 \). The solid curve denotes \( \Delta E_0^\sigma = E_0^\sigma(k) - \omega_c \), where \( \omega = E_0^\sigma(k) \) is the pole in the density response function \( \chi_\rho \). The same pole appears in \( \chi_\sigma_\sigma \) also. The dashed curve denotes \( \Delta E_{\sigma^+}^\sigma = E_{\sigma^+}^\sigma - \omega_c - |g\mu_B B| \), where \( \omega = E_{\sigma^+}^\sigma \) is the pole in the spin-response function \( \chi_{\sigma^+\sigma^+} \). The RPA energy shift, \( E_{\text{RPA}} - \omega_c \), is denoted by the dotted curve.
\( n = 2 \)

spin-polarization instability in a tilted magnetic field

[Giuliani + Quinn, 1985]

\( n = 1 \)

1\textsuperscript{st} order transition

\( n = 0 \)

paramagnetic \( \rightarrow \) ferromagnetic phase transition

\( \nu = 2 \)

\( h\omega_C \)

triplet exciton

\( g\mu_B B \)
Daneshvar et al, 1997

FIG. 3. (a) Filled symbols show the energy gaps measured at $\nu = 4$ as a function of the total magnetic field applied to the sample. $\Delta_4$ drops linearly to a gap of $3.5 \pm 0.1$ K at its turning point at 9.13 T. The solid line is a fit to the $\Delta_4$ data below 7.2 T with the slope corresponding to a $g$ factor of 1.1. Open symbols show similar data for $\nu = 6$. (b) The energy gaps $\Delta_3$ and $\Delta_5$ at odd filling factors. $\Delta_5$ exhibits distinct curvature near to its turning point at 5.7 T. The solid lines have the same gradients as the line in (a), and demonstrate that away from 6 T the data are consistent with the $g$ factor measured at $\nu = 4$. 
Summary

\( v = 1 \) Quantum Hall ferromagnet

spin-related excitations

- spin wave
  - \( \Delta n = 0 \)
    - Skyrmion, tunneling
  - \( \Delta n = 1 \)
    - spin flip

\( v = 2 \) Quantum Hall paramagnet

Giuliani-Quinn instability, PM \( \rightarrow \) FM transition

bilayer system (\( v = 2 \)), FM/CAM/SYM phases

[Das Sarma et al, 1998]