Spin-related physics
in
integer quantum Hall system

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Outline

Multi-component quantum Hall system

spin, layer, or valley (for Si) degrees of freedom

Spin: Quantum Hall ferro-magnet

Collection excitation and quasi-particle in QHFM

Spin wave, skyrmion

Layer: ν=1 Bilayer system as a QH pseudo-FM

Collective excitation and quasi-particle in in QHpFM

Pseudo-spin wave, meron

Josephson-like effect in bilayer system

ν=2 Three different quantum phases

Ferromagnet, canted antiferromagnet, and spin-singlet

The effect of in-plane magnetic field

Layout of a quantum Hall system

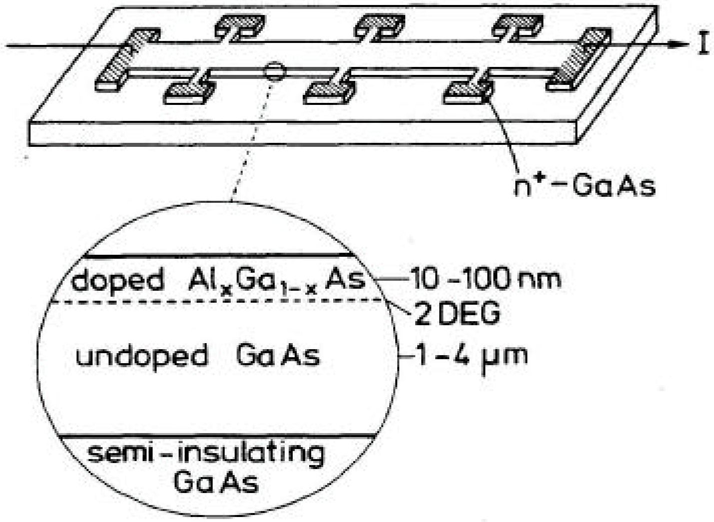
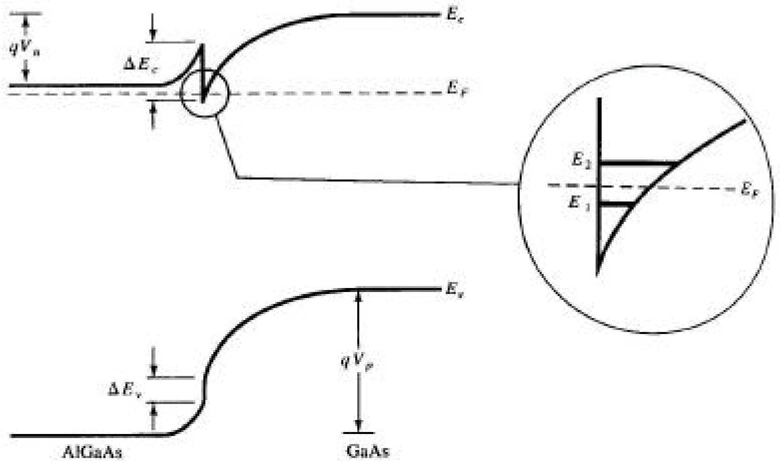
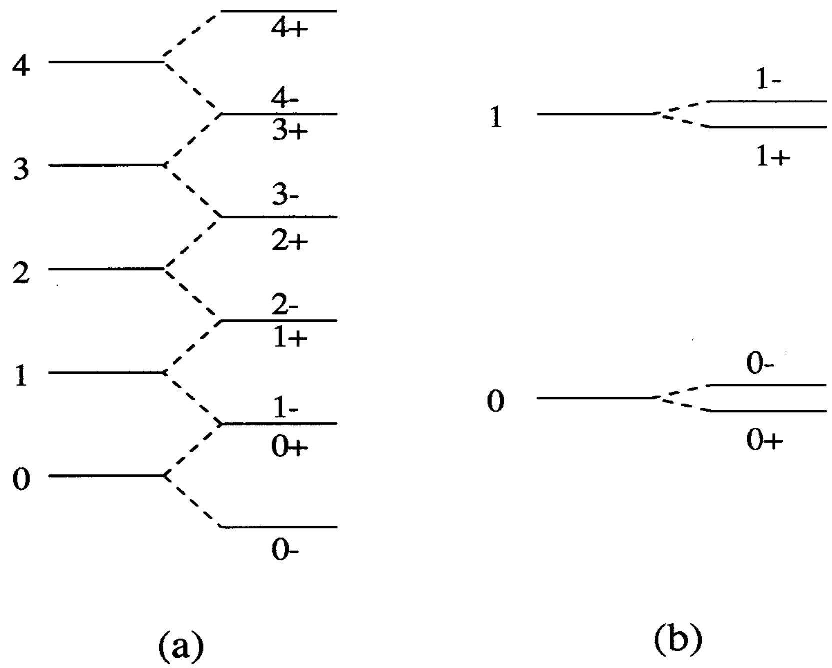


FIG. 3. Typical shape and cross section of a GaAs-$Al_xGa_{1-x}As$ heterostructure used for Hall-effect measurements.



Landau levels



Strong magnetic field (> 1 T)

Low temperature (< 4 K)

Figure 1.22: (a) Landau energy levels for an electron in free space. Numbers label the Landau levels and $+(-)$ refers to spin up (down). Since the g factor is 2, the Zeeman splitting is exactly equal to the Landau level spacing, $\hbar\omega_c$ and there are extra degeneracies as indicated. (b) Same for an electron in GaAs. Because the effective mass is small and $g \approx -0.4$, the degeneracy is strongly lifted and the spin assignments are reversed.

Some important parameters

Typical length scale: magnetic length $\ell \equiv \sqrt{\hbar/eB}$ ≈ 128 A at 4 T

Material dep.

GaAs/AlGaAs

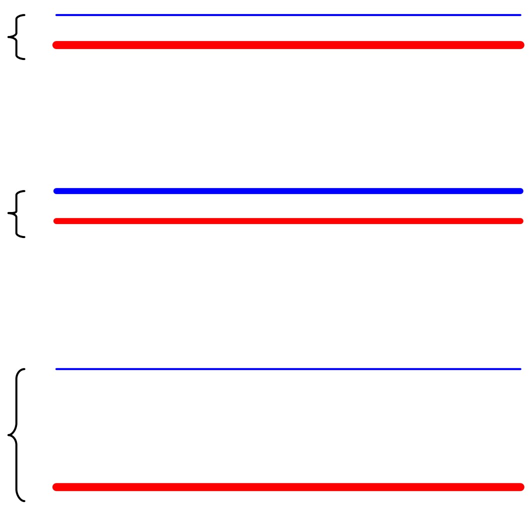
- Dielectric constant: 12.8 — $e^2/\epsilon l \approx 100$ K at 4 T
- Effective mass: $0.068\ m_e$ — $\hbar\omega_c \approx 80$ K at 4 T
  - $0.38\ m_e$ (LH), $0.6\ m_e$ (HH)
- g factor: -0.44, spin-orbit effect — $g\mu_B B \approx 1$ K at 4 T

Sample dep.

- Landau level degeneracy: $D_{LL} = B\times(\text{sample area})/\Phi_0$
- mobility ($10^4 - 10^6$ cm^2/Vs), electron density (10^{11} /cm^2)

Physics in the Lowest Landau Level (LLL): integer case

Filling factor $\nu = 1$



T=0

T > Ez

plus e-e interaction

spontaneous ferromagnetic ordering

single spin flip costs $\Sigma = (\pi/2)^{1/2}\, e^2/\varepsilon l =$ 125 K at 4 T

QHFM: an itinerant ferromagnet with quantized Hall resistances

The wave function is simply the m=1 Laughlin wave function

→ the world's best understood ferromagnet

Manybody effect on the “Zeeman splitting”

$\nu_\uparrow = 1$: $g^* \mu_B B = g \mu_B B + \Sigma$, $\Sigma = (\pi/2)^{1/2} e^2/\varepsilon l$

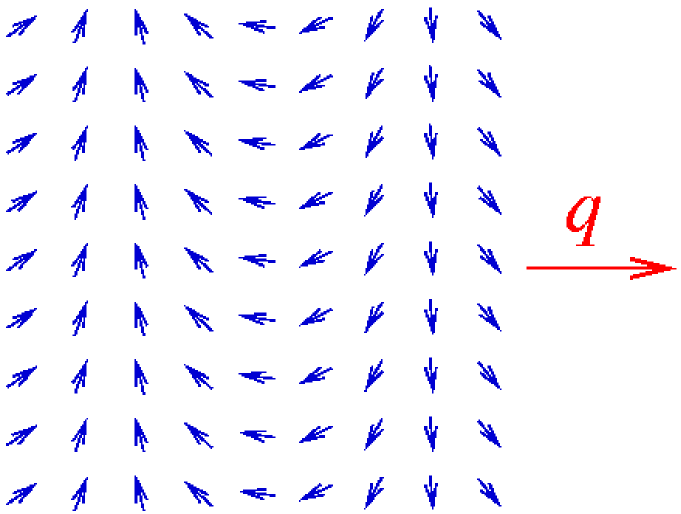
$\nu_\uparrow < 1$: $g^* \mu_B B = g \mu_B B + \Sigma_\times [n_\uparrow(g^*) - n_\downarrow(g^*)]$,

need to be solved self-consistently, Ando+Uemura 1974

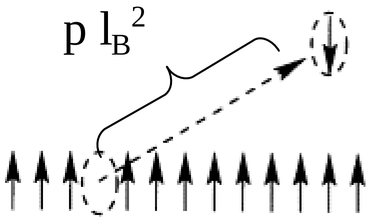
→ oscillatory behavior

Spin-related elementary excitations in QHFM

Spin wave



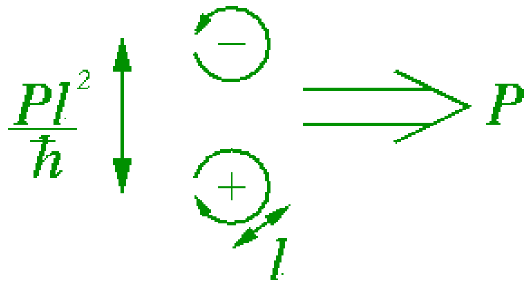
coherent superposition of e-h pairs



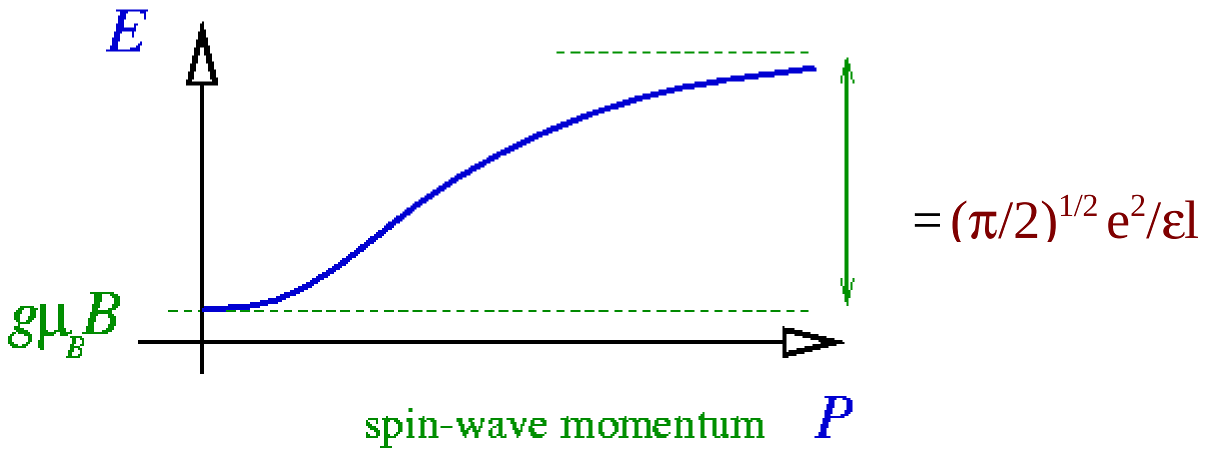
LLL projection

$$|\psi_{\mathbf{p}}\rangle = \sum_i \overline{e^{i\mathbf{p}\cdot\mathbf{r}_i}}\; c^\dagger_{i\downarrow}\, c_{i\uparrow}\, |\psi_0\rangle$$

Semiclassical Picture:



Dispersion Relation $(\nu = 1)$ [Lerner & Lozovik, 1980]



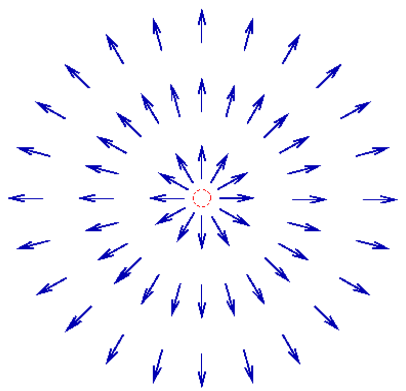
$P\ell/\hbar \to \infty \Longrightarrow$ quasielectron/quasihole pair.

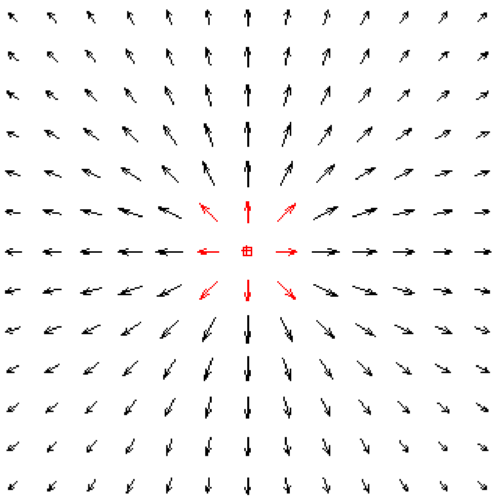
Are these the lowest-energy charged excitations?

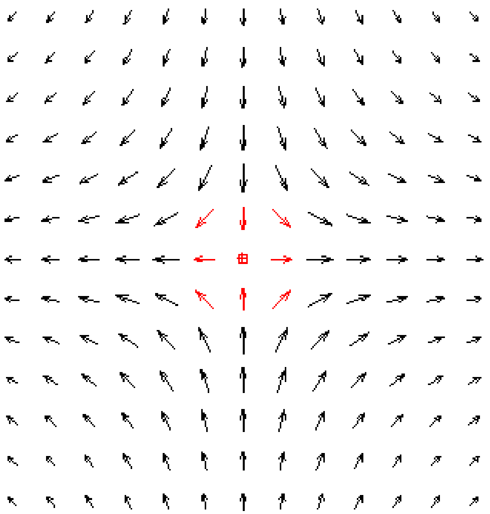
skyrmion – charged spin-texture excitation [Sondhi et al, 1993] –

different forms of spin texture in 2D:

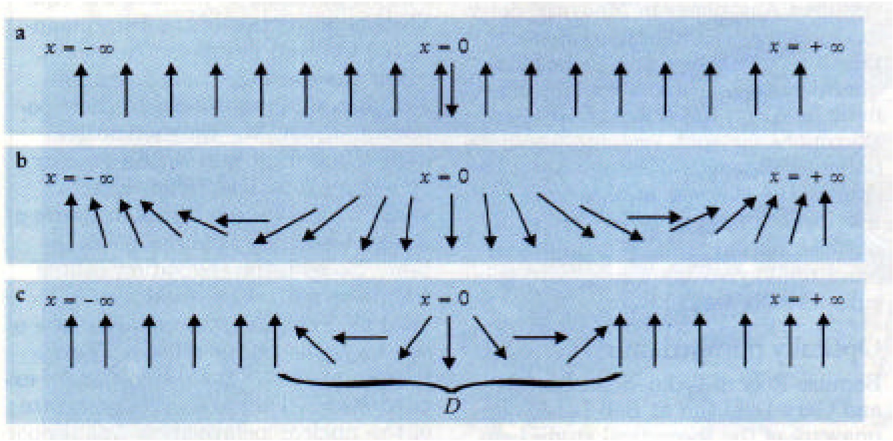
easy plane, vortex   easy axis, antiskyrmion / skyrmion







Formation of a skyrmion



Noninteracting

Plus e-e interaction

Plus Zeeman energy

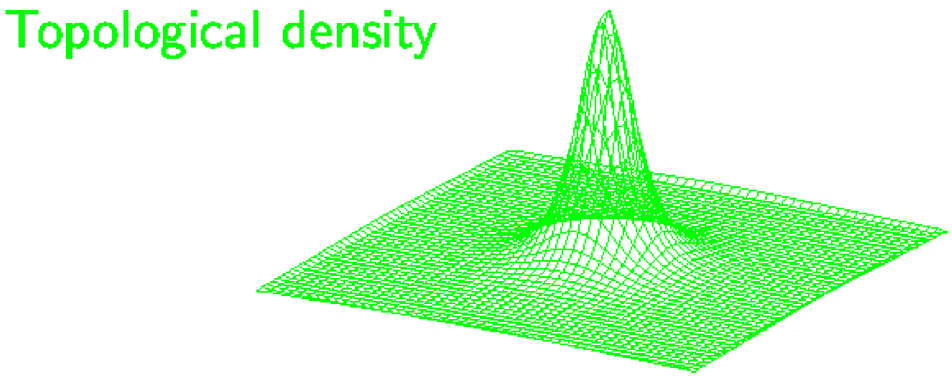
g << 1: larger extent of distorted spins is better

→ rapid depolarization by adding one skyrmion

g >> 1: only one spin is flipped

The skyrmion is charged

Spin texture determines charge density profile



Topological charge of spin texture

$$Q_{\mathrm{top}} \equiv \frac{1}{8\pi}\int d^2r\,\epsilon^{\alpha\beta}\vec{m}\cdot\partial_\alpha\vec{m}\times\partial_\beta\vec{m} \quad = \text{integer}$$

Q_{top} is the wrapping number of the $S^2(\mathbf{r}) \rightarrow S^2(\mathbf{m})$ mapping

stable against smooth continuous distortion of $m(\mathbf{r})$

Electric charge $Q = \nu\, e\, Q_{top}$

Barrett et al, 1995.

NMR

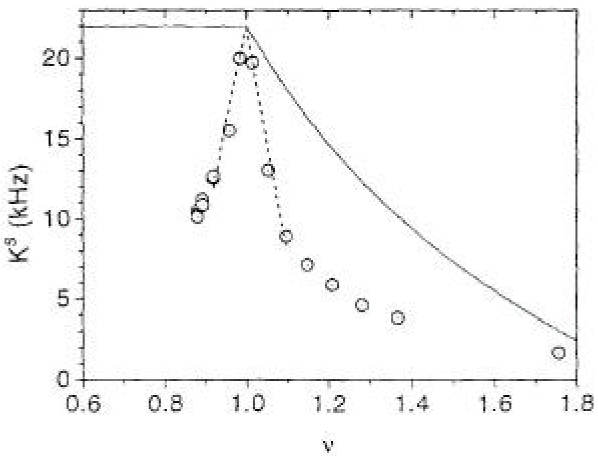


FIG. 3. Dependence of K_s on filling factor ν for $B = 7.05$ T (open circles) at 1.55 K. As explained in the text, both fits are given by Eq. (1), but the solid line has $\mathcal{A} = S = 1$ (noninteracting electrons), while the dashed line has $\mathcal{A} = S = 3.6$ (finite-size Skyrmions).

Schmeller et al, 1995.

magnetotransport

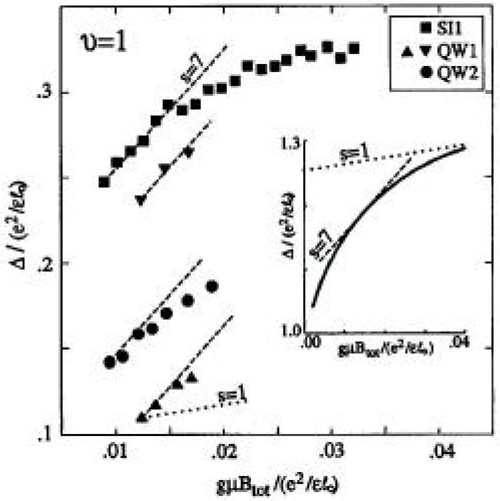


FIG. 2. Results of tilted-field experiments on the $\nu = 1$ QHE. The energy gaps Δ at fixed $B_\perp$ are plotted vs the Zeeman energy $g\mu_B B_{tot}$, both in units of $e^2/\epsilon\ell_0$. Each data set starts with $\theta = 0$ and $B_{tot} = B_\perp$ at the lower left. On the quantum well samples we use gate electrodes to tune the electron densities [11]. From top to bottom the samples had electron densities 0.6, 1.0, 0.6, and 1.0×10^{11} cm^{-2} and mobilities 3.4, 0.52, 0.18, and 0.16×10^6 cm^2/V s, respectively. For comparison we include lines with $\partial\Delta/\partial(g\mu_B B_{tot}) = s = 7$ (dashed) and 1 (dotted). The inset shows a Hartree-Fock result of Skyrmion theory (full line) [514].

Maude et al, 1996

magnetotransport

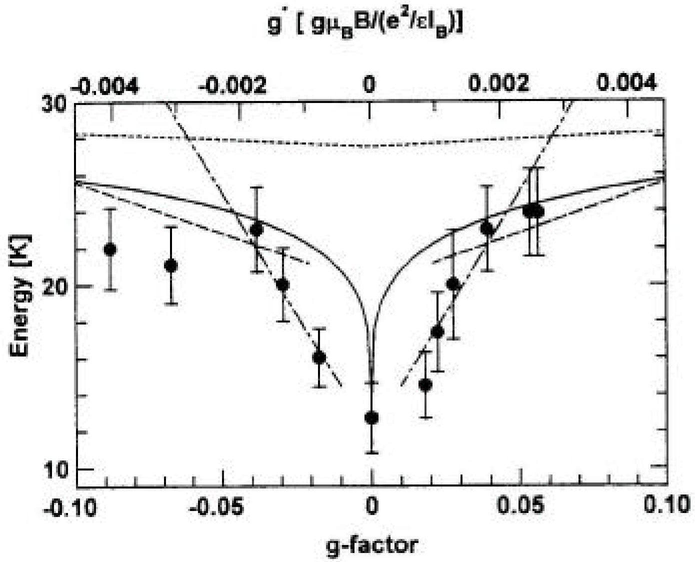
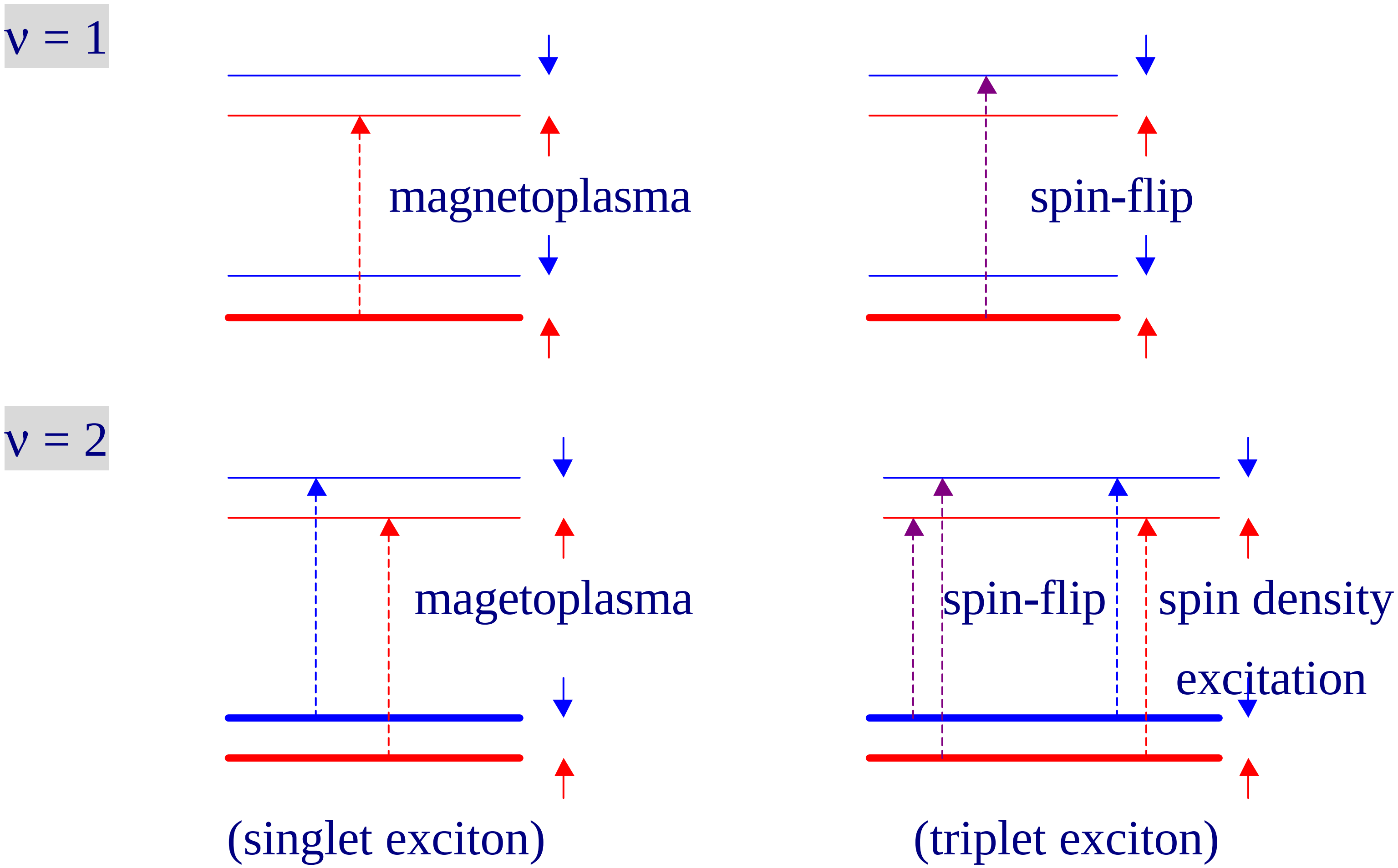


FIG. 3. The measured energy gap at filling factor $\nu = 1$ as a function of the bare g factor (bottom axis) and as a function of g^*, the ratio of the Zeeman and Coulomb energies (top axis). The solid line is the expected gap for a Skyrmion-type excitation [7], while the short-dashed line indicates the expected variation for the "bare" Zeeman dependence $E_B + s|g|\mu_B B$ (with $s = 1$ as predicted by the spin-wave dispersion model). Lines with slopes corresponding to $s = 7$ spin flips (long-dashed line) and $s = 33$ spin flips (long-short-dashed line) are shown for comparison.

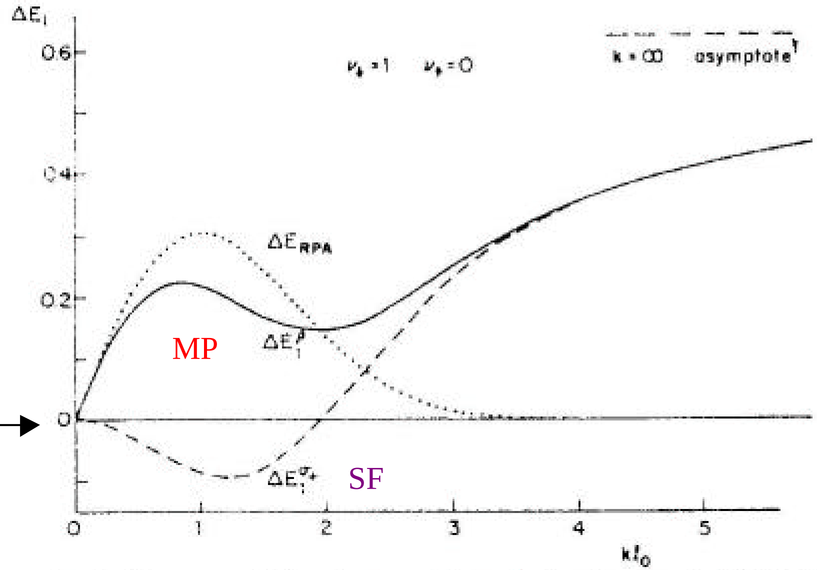
Inter-Landau level excitations

Different ways to lift an electron from n=0 to n=1:



$\nu = 1$ [Kallin + Halperin, 1984]

Kohn's theorem



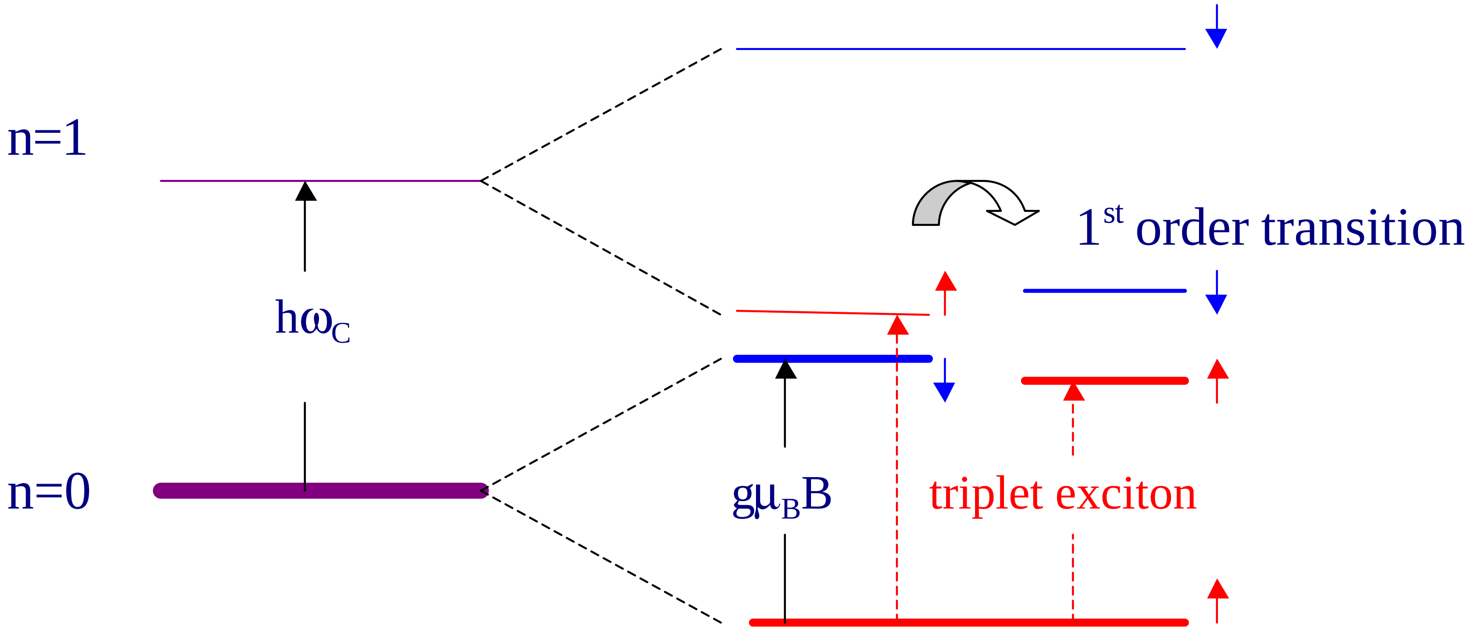
$(\pi/2)^{1/2}\, e^2/\varepsilon l$

FIG. 3. Energy shifts for a spin-polarized sample with only the lowest Landau level filled, $\nu_\downarrow = 1$ and $\nu_\uparrow = 0$, and $m = 1$ are shown. The energy scale is in units of $e^2/\epsilon l_0$. The solid curve denotes $\Delta E_1^\rho = E_1^\rho(k) - \omega_c$, where $\omega = E_1^\rho(k)$ is the pole in the density response function χ_ρ. The same pole appears in χ_{σ_z} also. The dashed curve denotes $\Delta E_1^{\sigma_+} = E_1^{\sigma_+} - \omega_c - |g\mu_B B|$, where $\omega = E_1^{\sigma_+}$ is the pole in the spin-response function χ_{σ_+}. The RPA energy shift, $E_{RPA} - \omega_c$, is denoted by the dotted curve.

$\nu = 2$

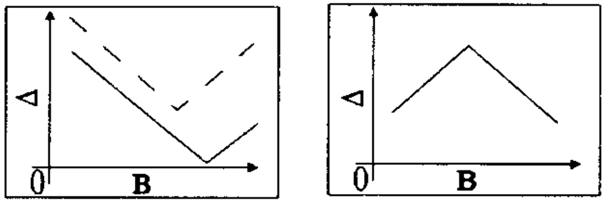
spin-polarization instability in a tilted magnetic field

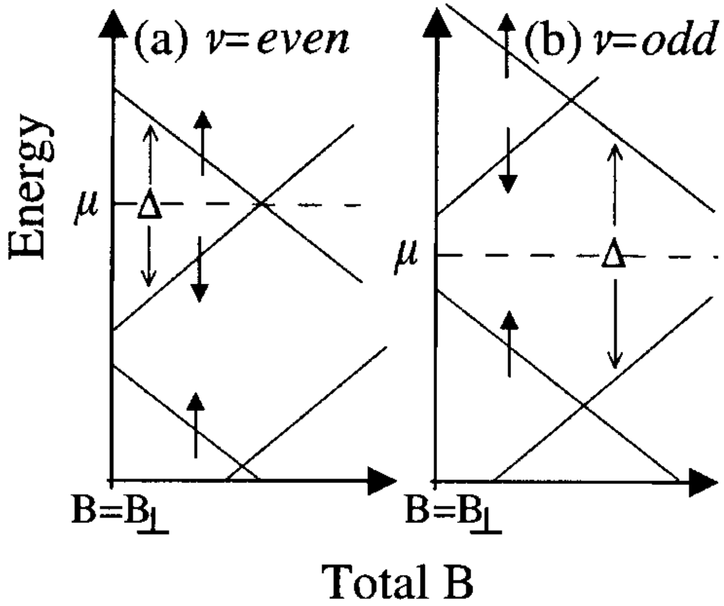
[Giuliani + Quinn, 1985]



paramagnetic → ferromagnetic phase transition

Daneshvar et al, 1997





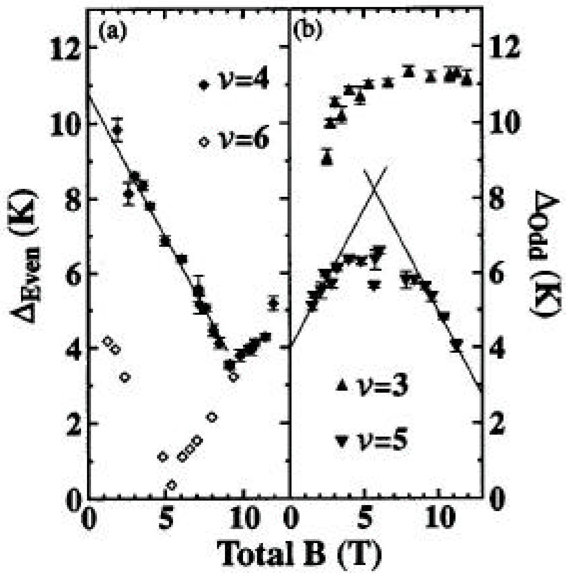


FIG. 3. (a) Filled symbols show the energy gaps measured at $\nu = 4$ as a function of the total magnetic field applied to the sample. Δ_4 drops linearly to a gap of 3.5 ± 0.1 K at its turning point at 9.13 T. The solid line is a fit to the Δ_4 data below 7.2 T with the slope corresponding to a g factor of 1.1. Open symbols show similar data for $\nu = 6$. (b) The energy gaps Δ_3 and Δ_5 at odd filling factors. Δ_5 exhibits distinct curvature near to its turning point at 5.7 T. The solid lines have the same gradients as the line in (a), and demonstrate that away from 6 T the data are consistent with the g factor measured at $\nu = 4$.

Summary

$\nu = 1$ Quantum Hall ferromagnet

spin-related excitations

$\Delta n=0 \begin{cases} \text{spin wave} \\ \text{Skyrmion, tunneling} \end{cases}$

$\Delta n=1 \begin{cases} \text{magnetoplasma} \\ \text{spin flip} \end{cases}$

$\nu= 2$ Quantum Hall paramagnet

Giuliani-Quinn instability, PM $\rightarrow$ FM transition

bilayer system ($\nu = 2$), FM/CAM/SYM phases

[Das Sarma et al, 1998]