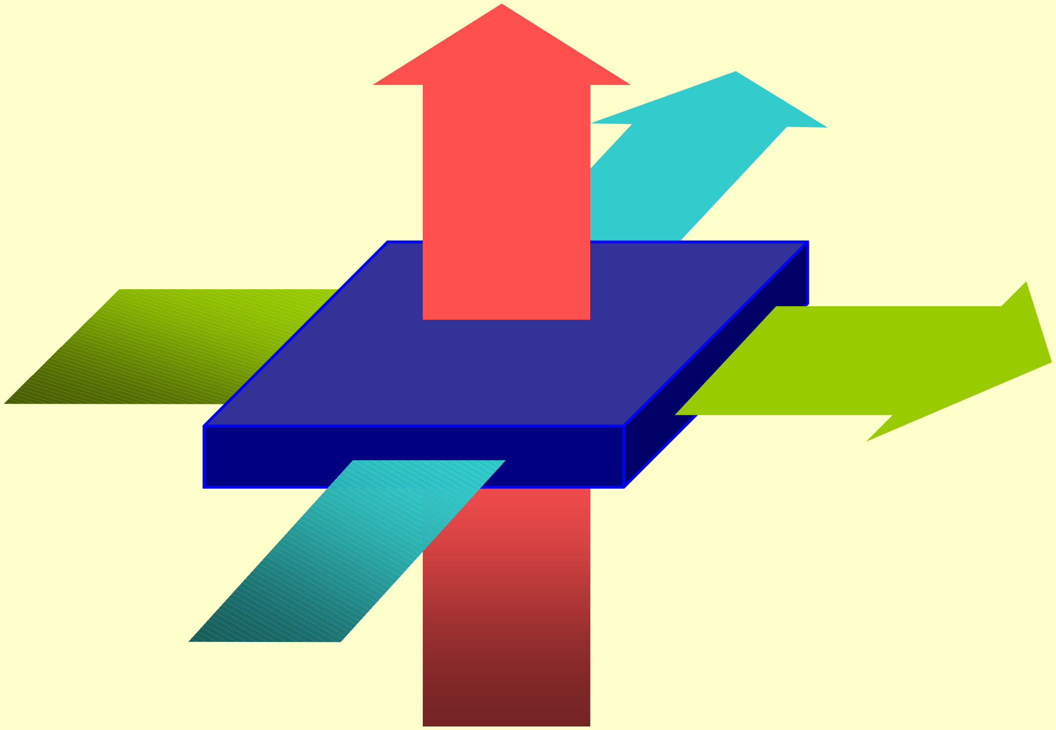
Spin Hall effect and related issues



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8/22/2005

spintronics

past/now	• magnetic memory, GMR, TMR
goal	• generation, manipulation, and detection of spins in metals, semiconductors…
on-going effort	• FM/semiconductor spin injection not easy • magnetic semiconductor not easy
wish for	• integration with existing semiconductor technology • control via electric field, instead of magnetic field
➡	• more researches on the spin-orbit coupling in semiconductors

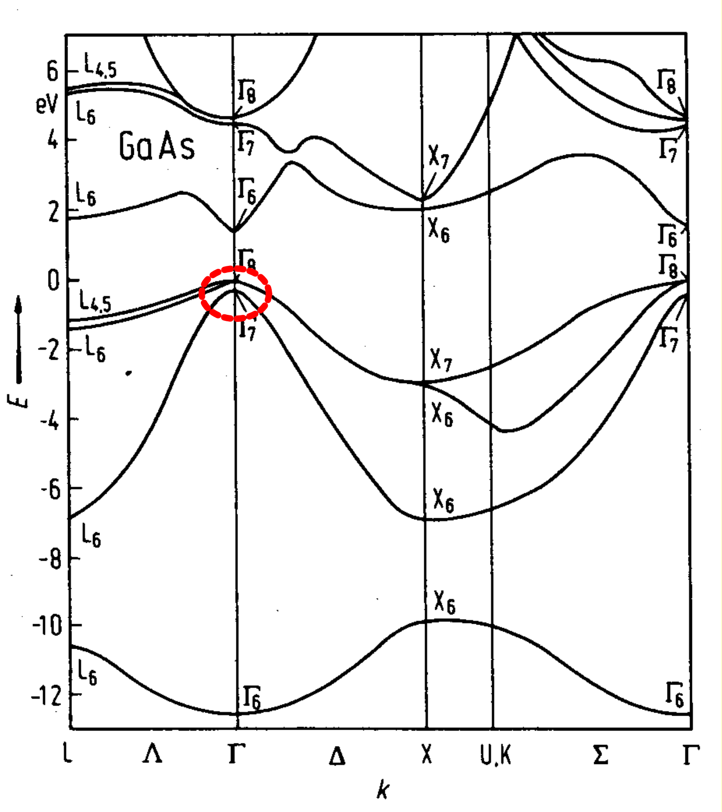
Spin-orbit interaction in semiconductor

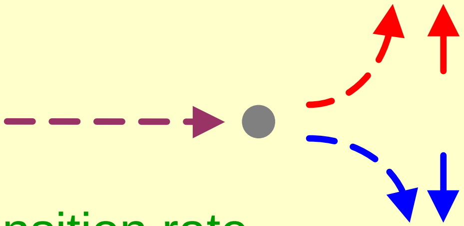
(Kittel, Quantum Theory of Solids)

$$H_{SO}=\frac{1}{2mc^2}\vec{S}\cdot\nabla V(\vec{x})\times\vec{v}$$

(*V*(*x*) is the lattice potential energy)

- splitting of valence bands (GaAs, Δ=0.34 eV)
- change of g-factor (GaAs, $g^*=-0.44$)
- for materials without inversion symmetry, lift the spin degeneracy of energy bands (Dresselhaus, Rashba)
- skew scattering from impurities





transition rate,

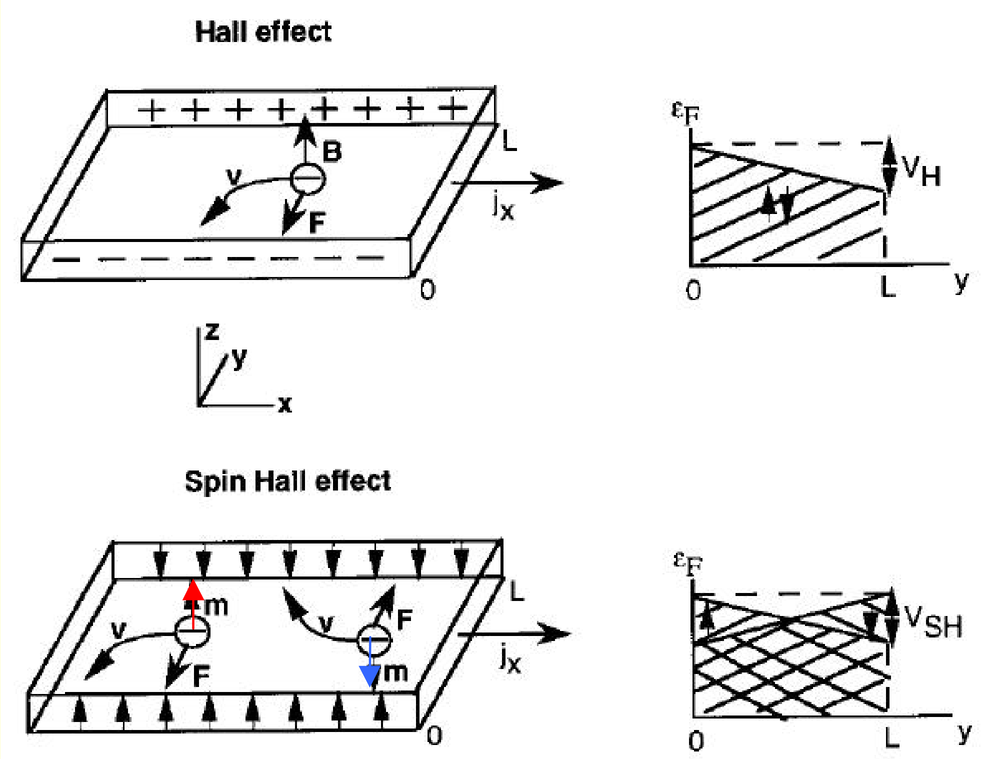
$$W_{\vec{k}s\to\vec{k}'s'}\approx l_{SO}\,\vec{s}_{s's}\cdot\vec{k}'\times\vec{k}$$

For strong SO couplings, choose low-symm, narrow-gap materials formed from heavy elements ($g^*\approx-50$ in InSb) *(Rashba, cond-mat/0309441)*

Generation of spin in semiconductor using SO coupling (Rashba PRB 2004)

[1]	• Hirsch, PRL 1999	• spin Hall effect (SHE), skew scattering	
	• Voskoboynikov et al, PRB 1999 and many others	• resonant tunneling related ideas	
	• Kiselev and Kim, APL 2001	• T-shaped filter	***device design***
	• Ioniciociu and D’Amico, PRB 2003	• Stern-Gerlach device	
	• Ramaglia et al, Euro Phys J B 2003	• quantum point contact	
	• Watson et al, PRL 2003	• adiabatic pumping (need B field)	
	• Rokhinson et al, PRL 2004	• electron focusing (need B field)	
	• Bhat and Sipe, PRL 2000	• all-optical technique	
	• Mal’shukov et al, PRB 2003	• AC gate	
[2]	• Murakami et al, Science 2003	• SHE, in bulk p-type semiconductor	
[3]	• Sinova et al, PRL 2004	• SHE, in n-type heterojunction (2DEG)	

Hall effect (E.H. Hall, 1879)



[1] Spin Hall effect

(J.E. Hirsch, PRL 1999, S Zhang, PRL 2000, Dyakonov and Perel, JETP 1971.)

skew scattering

by spinless impurities:

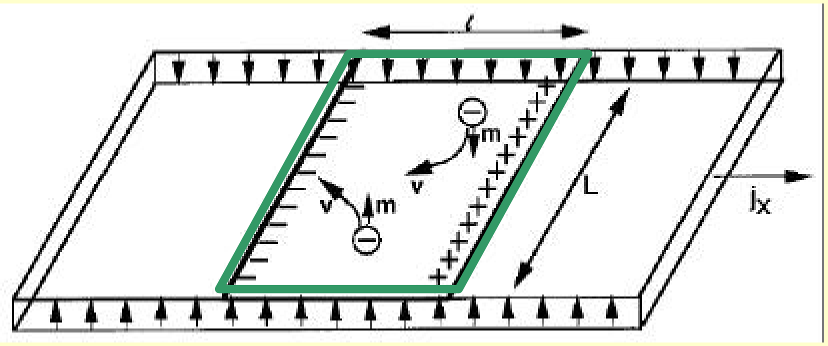
no magnetic field required

From spin accumulation to charge accumulation

L< spin coherence length δ_s

$\delta_s \approx 130\ \mu$m at 36 K for Al

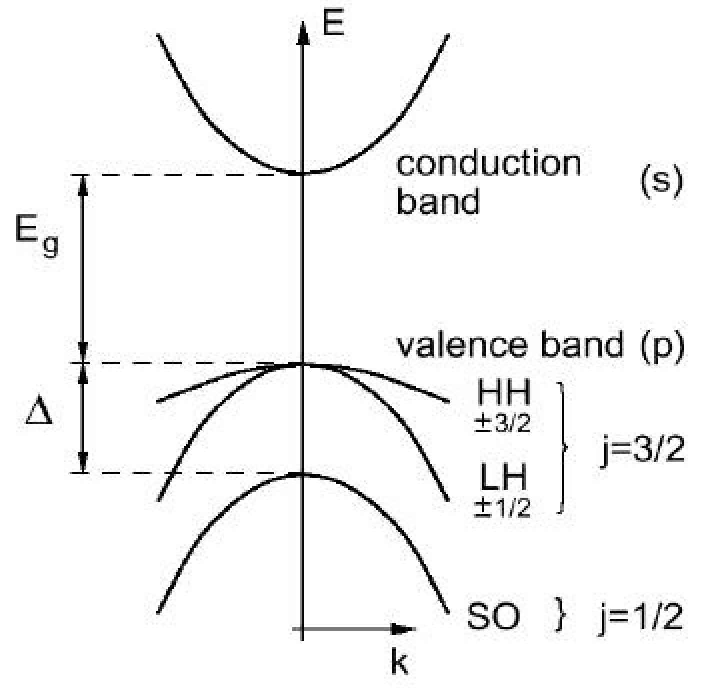
(Johnson and Silsbee, PRL 1985)



[2] Intrinsic spin Hall effect in p-type semiconductor (I)

(Murakami, Nagaosa and Zhang, Science 2003)

Valence band of GaAs:



Luttinger Hamiltonian (1956)
(for *j*=3/2 valence bands)

$$H=\frac{1}{2m}\left[\left(g_1+\frac{5}{2}g_2\right)k^2-2g_2\left(\vec{k}\cdot\vec{S}\right)^2\right]$$

$l=\hat{k}\cdot\vec{S}$ (helicity)

is a good quantum number

(Non-Abelian) gauge potential

$$A_{ll'}(\vec{k})=i\left\langle\vec{k},l\left|\frac{\partial}{\partial\vec{k}}\right|\vec{k},l'\right\rangle$$

Berry curvature,
due to monopole field in k-space

$$\vec{\Omega}_l(\vec{k})=-2l\left(l^2-\frac{7}{4}\right)\frac{\hat{k}}{k^2}$$

Intrinsic spin Hall effect in p-type semiconductor (II)

Semiclassical EOM

$$\begin{cases} \hbar \dfrac{d\vec{k}}{dt} = e\vec{E} \\ \dfrac{d\vec{x}}{dt} = \dfrac{\partial E_l(\vec{k})}{\hbar \partial \vec{k}} - \boxed{\dfrac{d\vec{k}}{dt} \times \vec{\Omega}_l(\vec{k})} \end{cases}$$

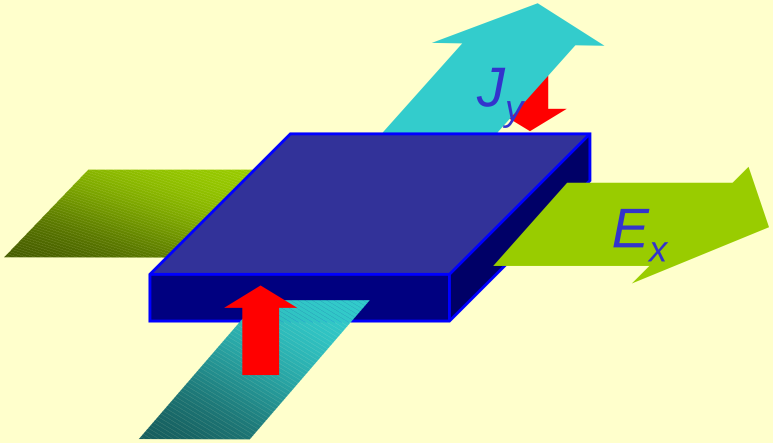
Anomalous velocity due to Berry curvature

(Chang and Niu, PRL 1995
Sundaram and Niu, PRB 1999)

Spin current

$$\text{HH} \quad J_y^z = \frac{1}{3} \sum_{l=\pm 3/2, \vec{k}} \dot{y} S^z n_l(\vec{k}) = -\frac{k_F^H}{4p^2} eE_x ,$$

$$\text{LH} \quad J_y^z = \frac{1}{3} \sum_{l=\pm 1/2, \vec{k}} \dot{y} S^z n_l(\vec{k}) = +\frac{k_F^L}{12p^2} eE_x ,$$



Spin Hall conductivity $\quad J_y^z = s^z_{yx} E_x$

$$|s^z_{yx}| = \frac{e}{12p^2}\left(3k_F^H - k_F^L\right) \quad \text{(semiclassical)}$$

$$-\frac{e}{12p^2}\left(k_F^H + k_F^L\right) \quad \text{(Q correction)}$$

$$= \frac{e}{6p^2}\left(k_F^H - k_F^L\right)$$

No magnetic field required

Applies to Si as well

[3] Intrinsic spin Hall effect in 2 dimensional electron gas (2DEG)

(Sinova, Culcer, Niu, Sinitsyn, Jungwirth, and MacDonald, PRL 2004)

Semiconductor heterojunction

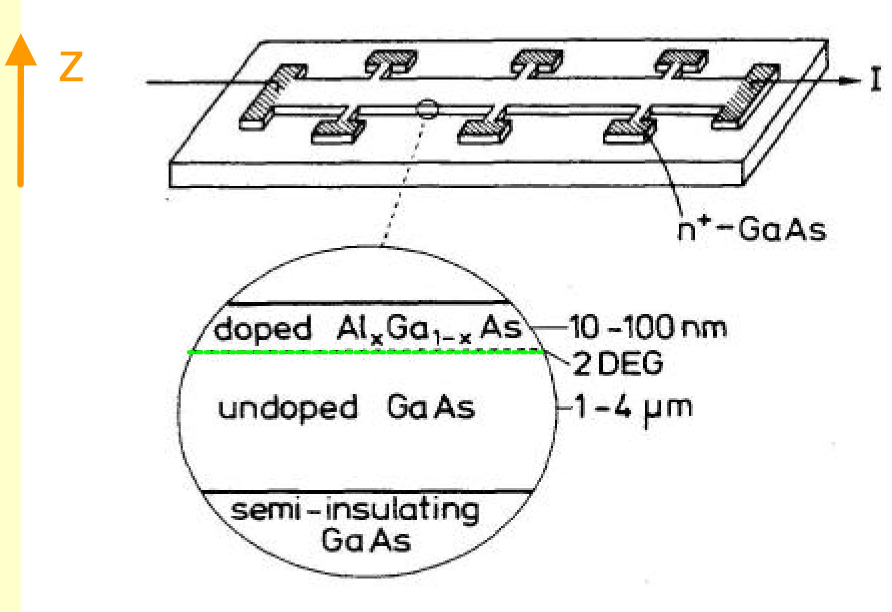
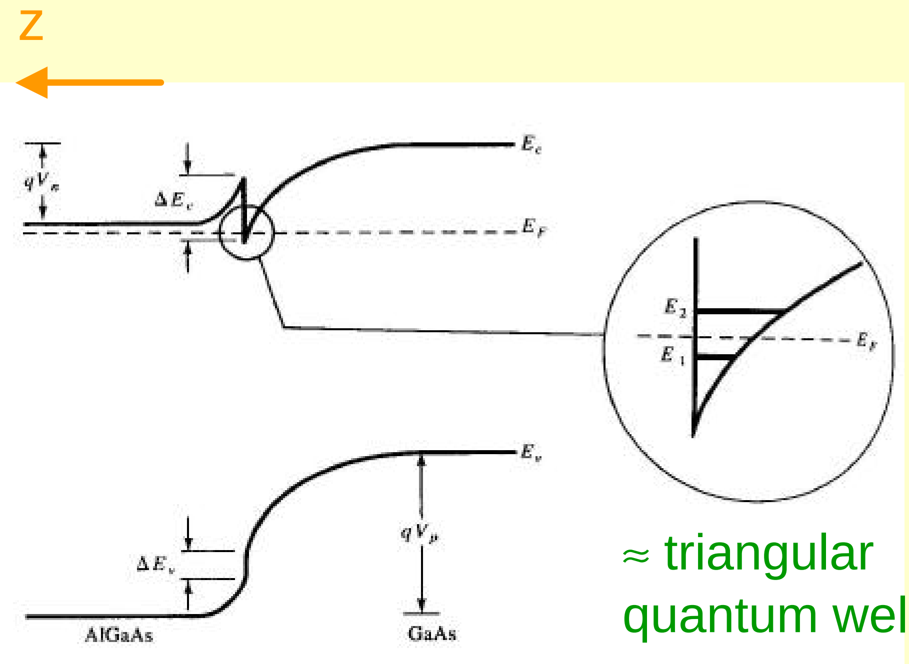
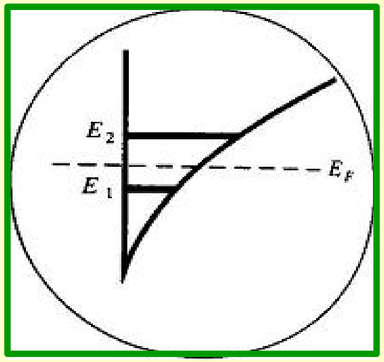


FIG. 3. Typical shape and cross section of a GaAs-$Al_xGa_{1-x}As$ heterostructure used for Hall-effect measurements.



≈ triangular quantum well

QW with structure inversion asymmetry (SIA):



Rashba coupling (Sov. Phys. Solid State, 1960)

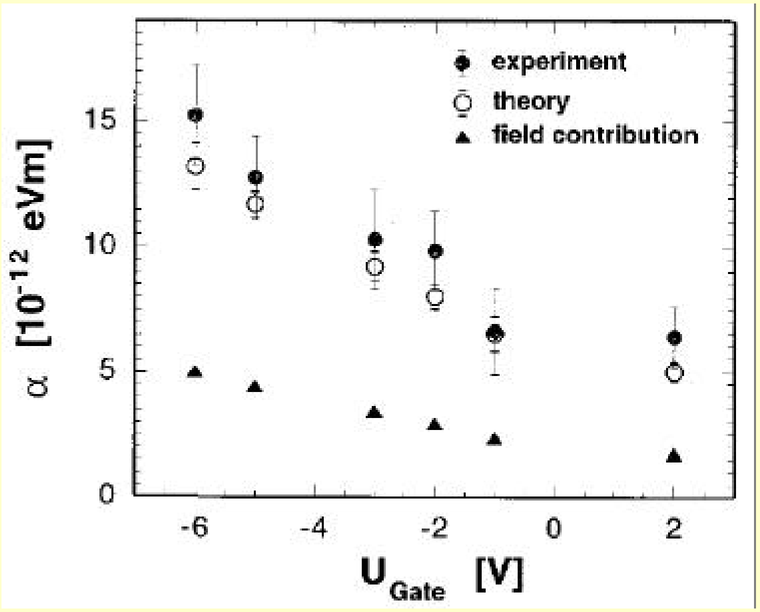
$$H = \frac{p^2}{2m} + \frac{\alpha}{\hbar}\vec{s}\times\vec{p}\cdot\hat{z}$$

- 1974 Ohkawa and Uemura, due to gradient of the confinement potential $\partial V/\partial z$
- 1976 Darr, $\langle \partial V/\partial z \rangle$ for a bound state is actually zero
- 1985 Lassnig, interface/valence band are crucial

Zawadzki's, Semi Sci Tech 2004)

No easy way to calculate α

V_G-dependence of the Bychkov-Rashba parameter



- Can be determined from the beating of dHvA oscillation
- tunable by gate voltage

Engels et al 1997 PRB,

InP/In 0.77 Ga 0.23 As/InP

Intrinsic spin Hall effect in 2DEG

Rashba Hamiltonian (1960)

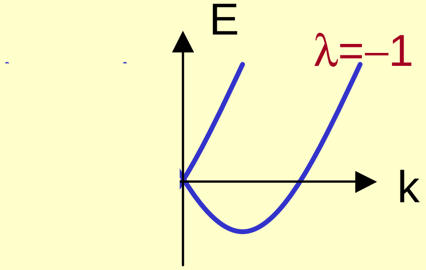
$$H = \frac{p^2}{2m} + \frac{a}{\hbar}\vec{s}\times\vec{p}\cdot\hat{z}$$

$$l = (\vec{s}\times\hat{p})\cdot\hat{z} \quad \text{(helicity)}$$

is a good quantum number

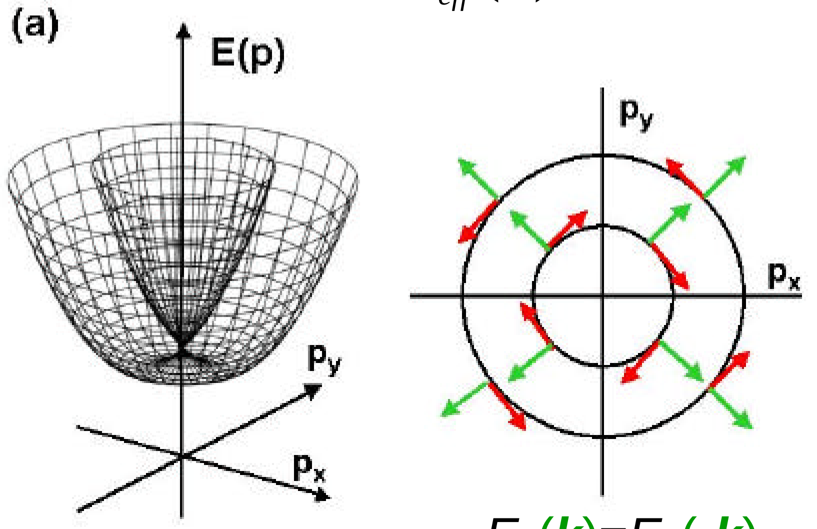
Eigen-energies

$$E_l(\vec{k}) = \frac{\hbar^2 k^2}{2m} + l\,a k, \quad l = \pm 1$$



$$a\vec{s}\times\vec{k}\cdot\hat{z} = -m_B\vec{s}\cdot\vec{B}_{eff}$$

$$\vec{B}_{eff}(\vec{k}) \approx l\,\hat{z}\times\vec{k}$$



$E_\lambda(\boldsymbol{k})=E_\lambda(-\boldsymbol{k})$

Kramer degeneracy

- no space inversion symmetry
- invariant under time reversal

Dynamics of spin under electric perturbation

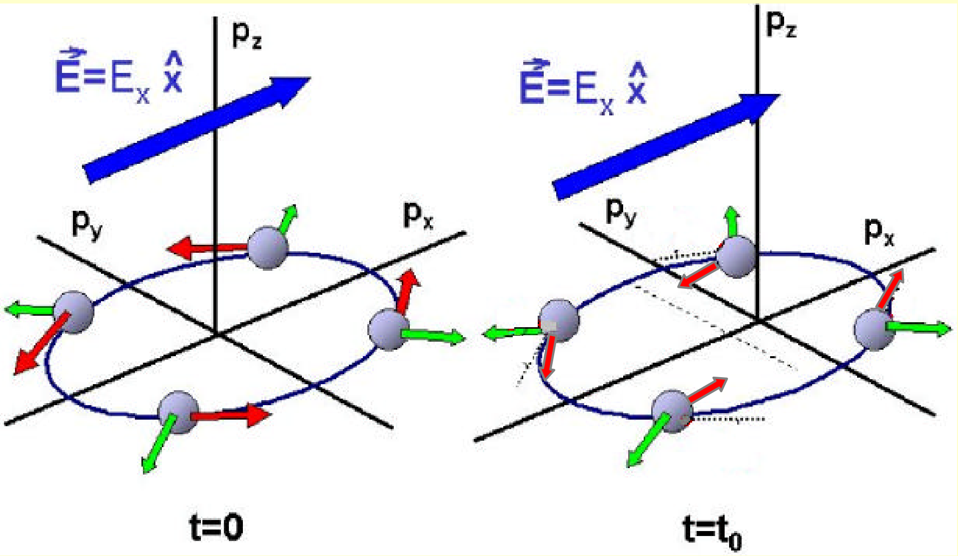
($\lambda=-1$)

$\delta k = -eEt \,//\, -x$

$\delta B_{eff} \approx \lambda z \times \delta k \;\; // -\lambda y$

Landau-Lifshitz eq.

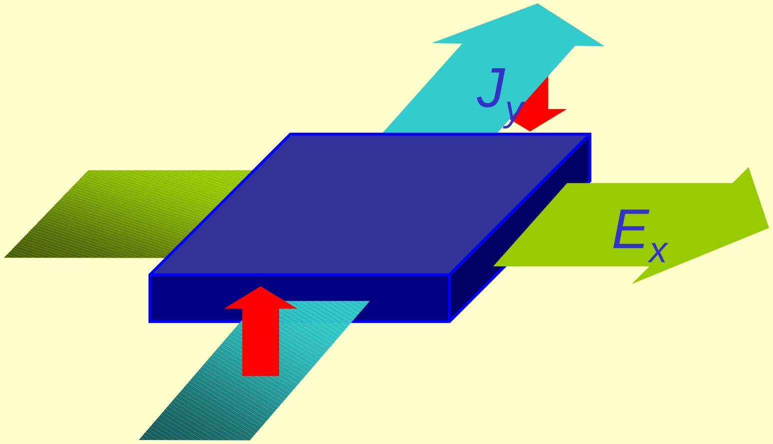
$$\frac{d\vec{S}}{dt} = \vec{S}\times\vec{B}_{eff}(\vec{k}) + g\vec{S}\times\underbrace{\left(\vec{S}\times\vec{B}_{eff}\right)}_{\text{damping}}$$



When both bands are filled,
spin Hall conductivity:

$$|\sigma^{z}_{yx}| = \frac{e}{8\pi} \quad \text{independent of } \alpha$$

- not so for non-parabolic bands
- only for clean system
- not related to Berry curvature



No magnetic field required

Effect of disorder on the intrinsic spin Hall effect (I)

- Rashba system with short-range impurities
  - Inoue et al (2003)
  - Dimitrova (2004)
  - Khaetskii (2004)
  - Raimonde and Schwab (2004)
  - Sheng et al, cond-mat/0504218
  - Nomura et al cond-mat/0506189

$$s_{SH} = s_{SH}^{clean} + s_{SH}^{vertex} = \frac{e}{8p} + \left(-\frac{e}{8p}\right) = 0\,!$$

- Perturbative calculations for other systems
  - If *H*(*k*)=*H*(-*k*), eg. Luttinger model

    then vertex correction is zero (Murakami, PRB 2004)

  - For systems with $H(\vec{k}) = E_0(\vec{k}) + s_x d_y(\vec{k}) - s_y d_x(\vec{k})$

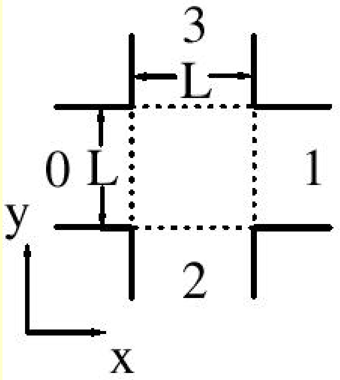
    If $\partial E_0 / \partial \vec{k} \propto \vec{d}$, then perfect cancelation (eg. Rashba)

    otherwise s_s remains finite. (quoted from Murakami's talk)

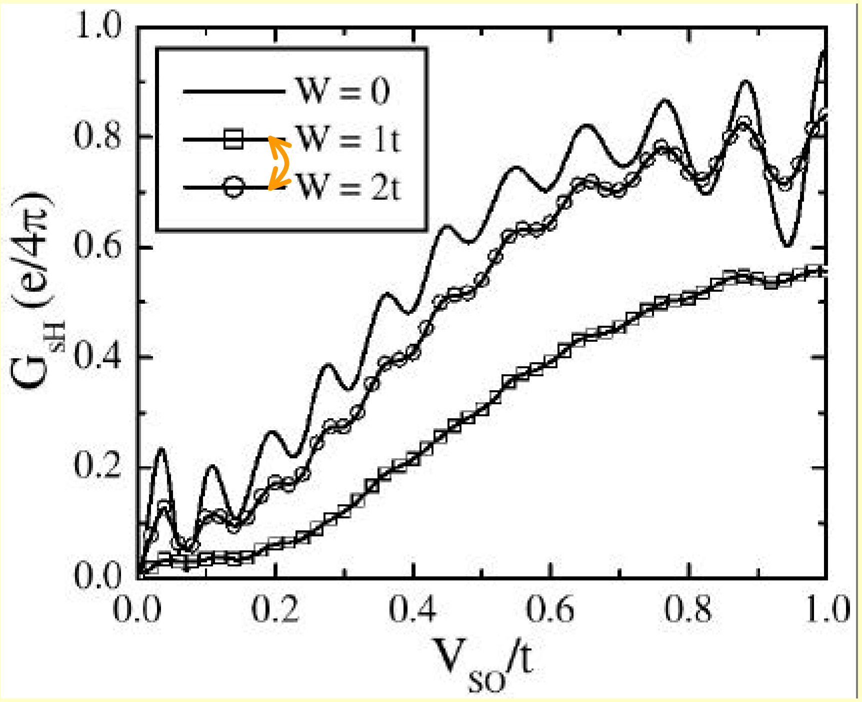
  Spin Hall effect is finite in general

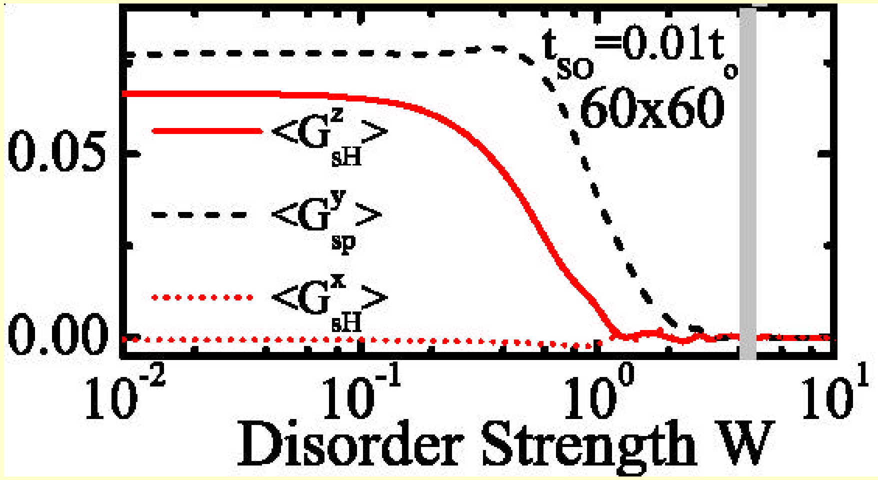
Effect of disorder on the spin Hall effect in Rashba system (II)

- σ_{SH} robust against weak disorder in finite systems



- Nikolic et al, cond-mat/0408693
- Hankiewicz et al, PRB 2004
- Sheng et al, PRL 2005

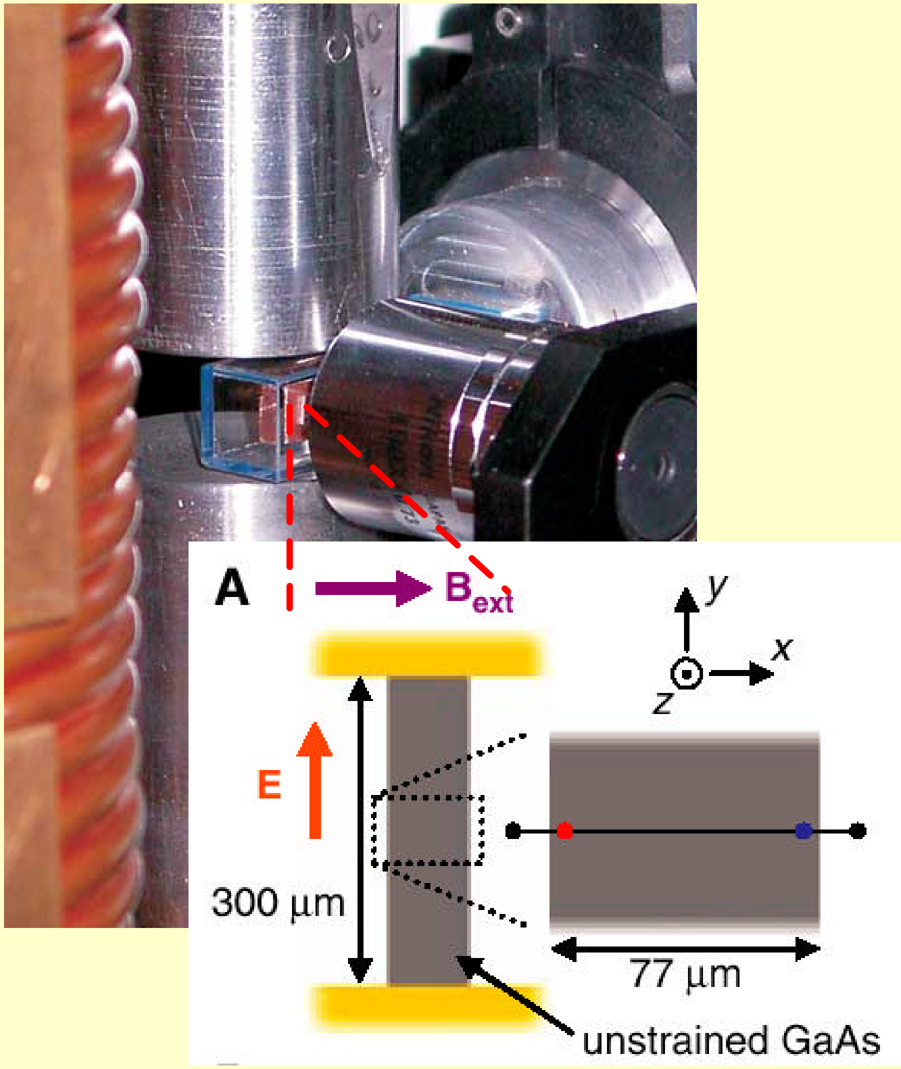


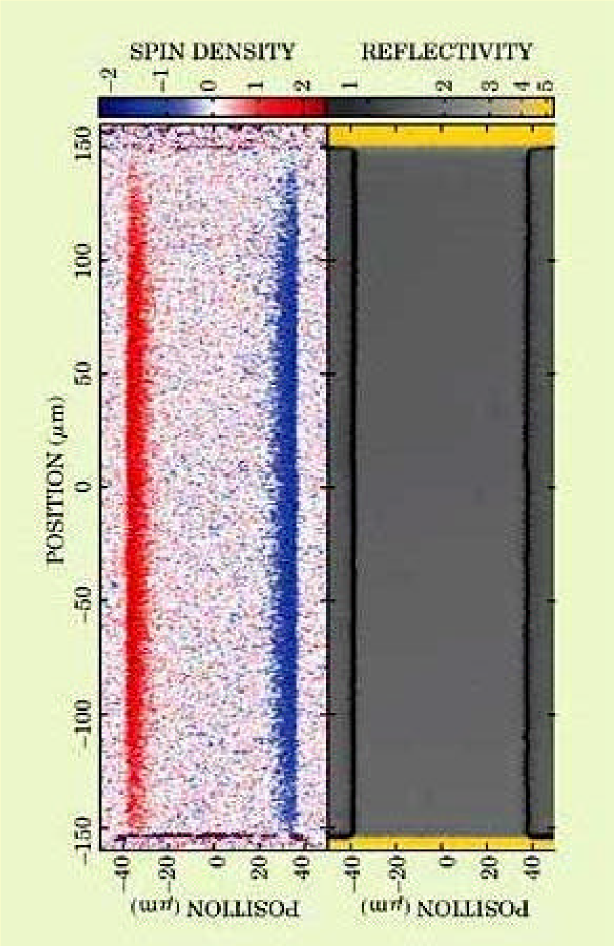


Stronger SO coupling

Spin Hall effect observed (I) (Kato et al, Science 2004)

- Local Kerr effect in strained n-type bulk InGaAs, 0.03% polarization

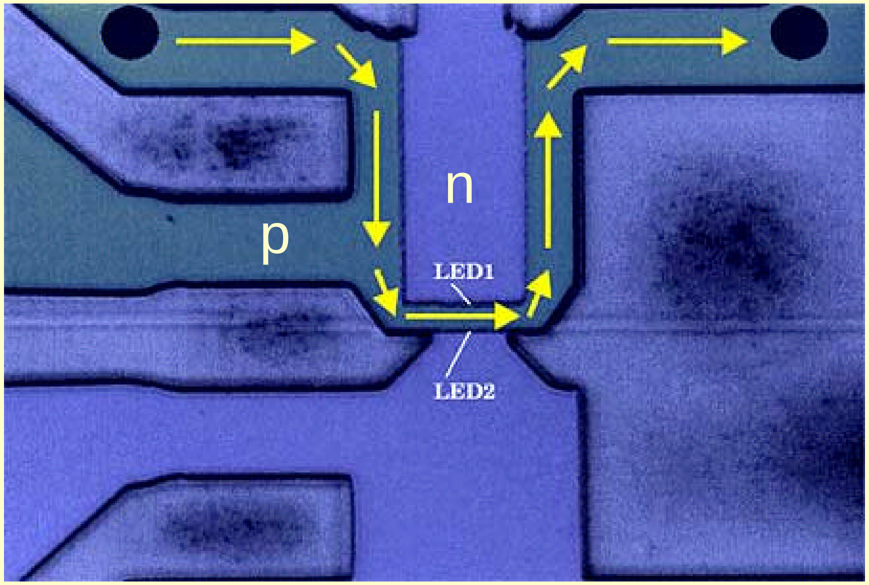


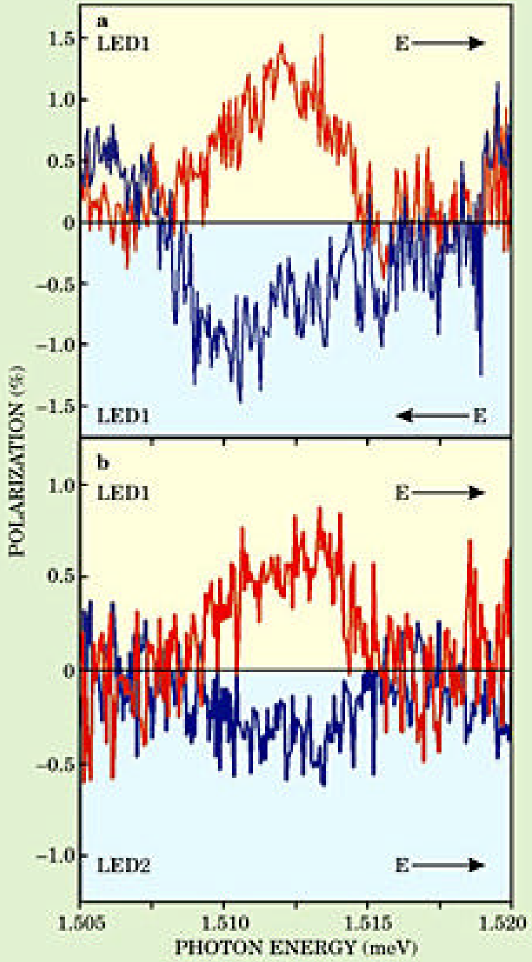


Mostly likely extrinsic.

Spin Hall effect observed (II) (Wunderlich et al, PRL 2005)

- spin LED in GaAs 2D hole gas,
1% polarization





might be intrinsic?
(Bernevig and Zhang, PRL July 2005)

Spin Hall effect observed (III) (Sih et al, cond-mat/0506704)

- n-type GaAs [110] QW

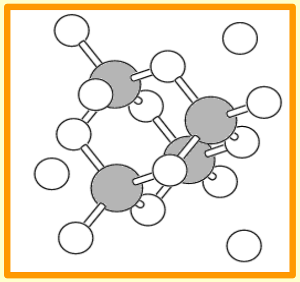
Dresselhaus coupling (PRB 1955):

III-V semiconductor
with bulk inversion asymm (BIA)

$$H(\vec{k}) = \vec{S} \cdot \vec{\Omega}(\vec{k})$$

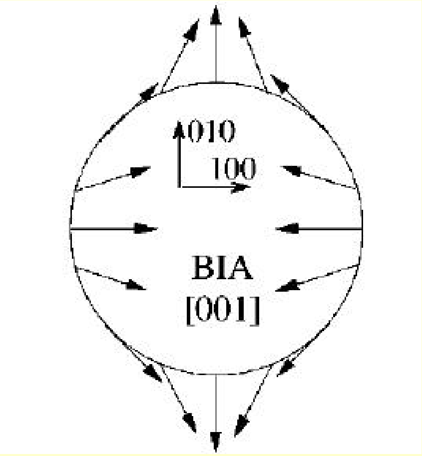
$$\vec{\Omega}(\vec{k}) \approx \left(k_x (k_y^2 - k_z^2), k_y (k_z^2 - k_x^2), k_z (k_x^2 - k_y^2) \right)$$

(BIA)



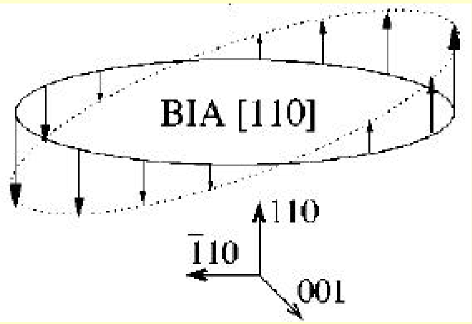
[001] QW, linear

$$\vec{\Omega}(\vec{k}) \approx k_n^2 (-k_x, k_y)$$



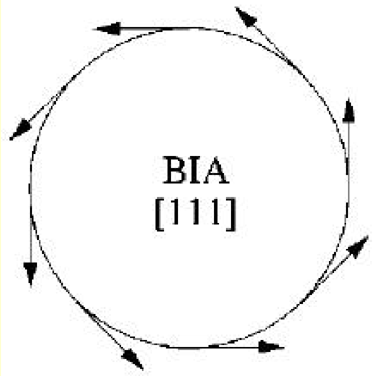
[110] QW

$$\vec{\Omega}(\vec{k}) \approx k_n^2 (-k_x / 2, -k_x / 2)$$



[111] QW

$$\vec{\Omega}(\vec{k}) \approx (2/\sqrt{3}) k_n^2 (k_{y,} - k_x)$$



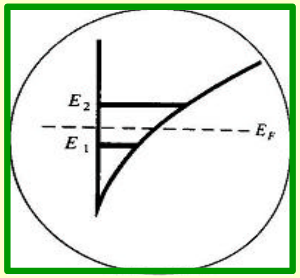
Rashba and Dresselhaus,
[001] quantum well:

Rashba

$$H = \frac{p^2}{2m^*} + \boxed{\frac{a}{\hbar}\left(s_x p_y - s_y p_x \right)}$$
$$+ \boxed{\frac{b}{\hbar}\left(s_x p_x - s_y p_y \right)}$$

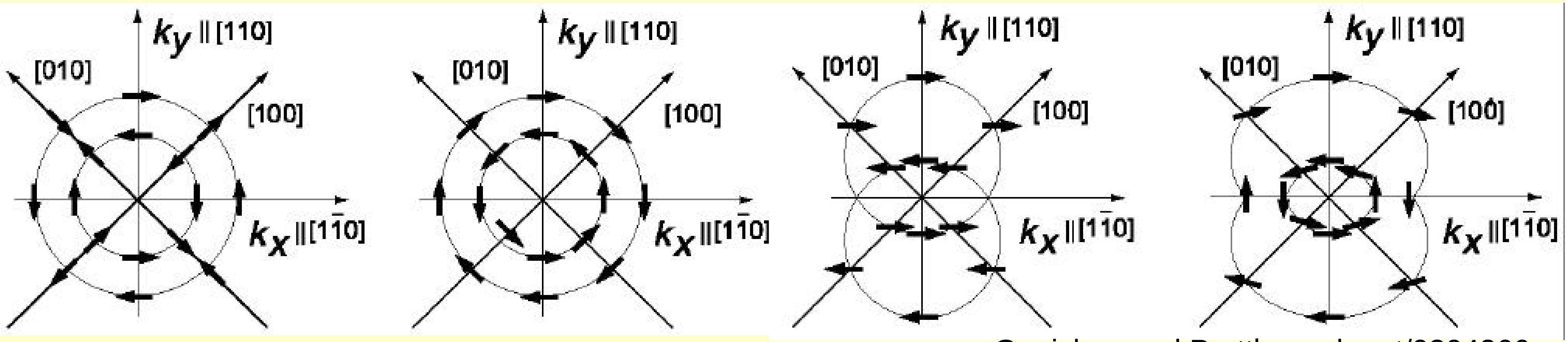
Dresselhaus

(SIA)



Effective magnetic field:

BIA SIA BIA=SIA BIA≠SIA



Ganichev and Prettl, cond-mat/0304266

$$\sigma^{z}_{xy} = \frac{e}{8\pi} N$$

For 2D electron systems, with Rashba and Dresselhaus coupling,

N=1 if Rashba > Dresselhuas

N=-1 if Dresselhaus > Rashba (Shen, PRB 2004)

For 2D hole system with (cubic) Rashba, N=9

(Schliemann and Loss, PRB 2005)

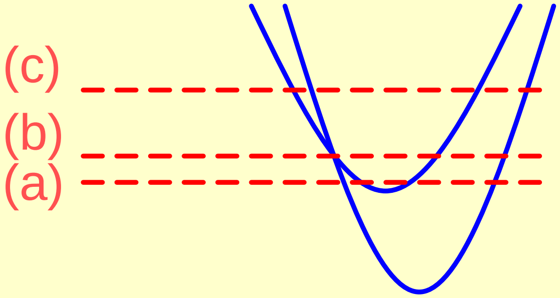
Rashba-Dresselhaus system in an in-plane magnetic field

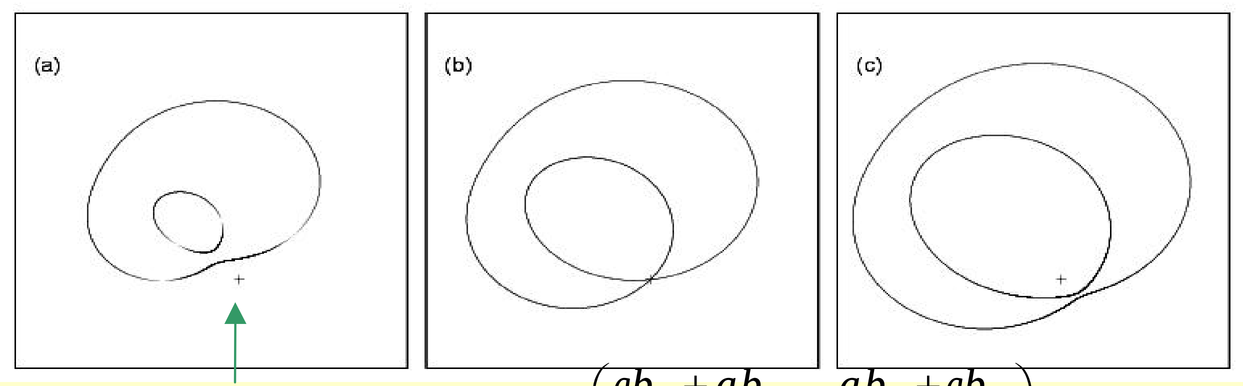
$$H=\frac{p^2}{2m^*}+\frac{\alpha}{\hbar}\left(\sigma_x p_y-\sigma_y p_x\right)+\frac{\gamma}{\hbar}\left(\sigma_x p_x-\sigma_y p_y\right)+\beta_x\sigma_x+\beta_y\sigma_y$$

Eigen-energies:

$$E_\lambda(\vec{k})=E_0(\vec{k})+\lambda\sqrt{\left(\gamma k_x+\alpha k_y+\beta_x\right)^2+\left(\alpha k_x+\gamma k_y-\beta_y\right)^2},\qquad \lambda=\pm$$

Distorted Fermi surfaces (generic cases):





Point of degeneracy $\vec{k}_0=\left(\frac{\gamma\beta_x+\alpha\beta_y}{\alpha^2-\gamma^2},-\frac{\alpha\beta_x+\gamma\beta_y}{\alpha^2-\gamma^2}\right)$

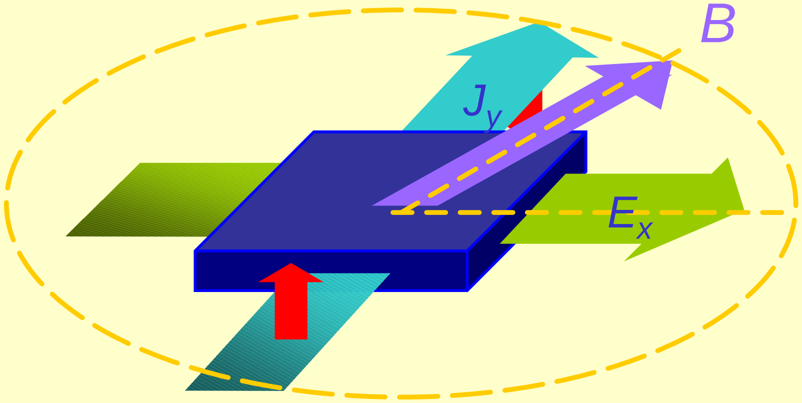
Parameters: $\alpha\approx 1\ \mathrm{eV}\cdot\mathrm{Å}$ (tunable by gate voltage)

γ of the same order

$\beta=(\mathrm{g}^*/2)\mu_B B,\quad \mu_B\approx 0.06\ \ \mathrm{meV/T}$

$k_F=\sqrt{2\pi n}\approx 10^2/\mathrm{Å}\ \ \text{for } n\approx 10^{11}/\mathrm{cm}^2$

Effect of in-plane magnetic field on spin Hall conductivity

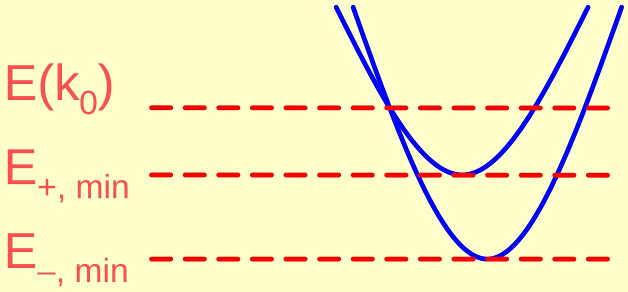


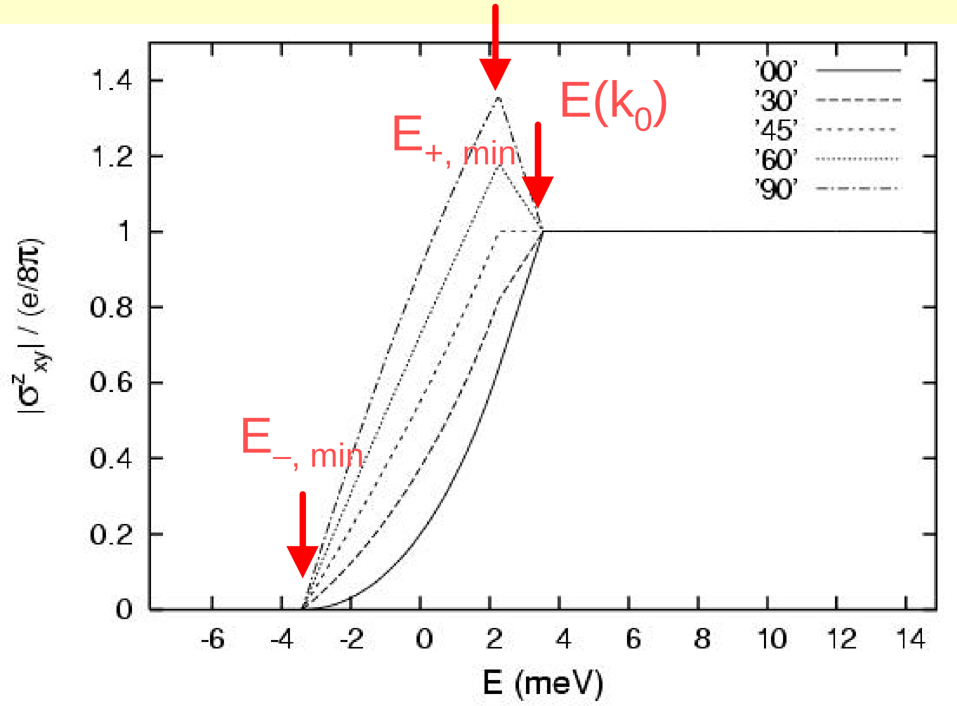
σ^h_{mn} **Kubo formula**

$$= \frac{1}{i\hbar}\sum_{\substack{k,l,l'\\(l\neq l')}} \frac{f_{\vec{k},l}-f_{\vec{k},l'}}{\omega^2_{ll'}(\vec{k})}\left\langle \vec{k},l\left|j^h_m\right|\vec{k},l'\right\rangle\left\langle \vec{k},l'\left|j_n\right|\vec{k},l\right\rangle,$$

$$j^h_m = \frac{\hbar}{4}\left(v_m\sigma^h + \sigma^h v_m\right); \qquad j_n = -ev_n$$

For γ = 0 (pure Rashba)

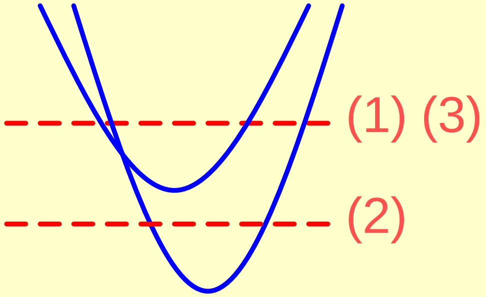




$\sigma^z_{xy}(\vec{B})$ **could be changed by 100% simply by rotating the magnetic field**

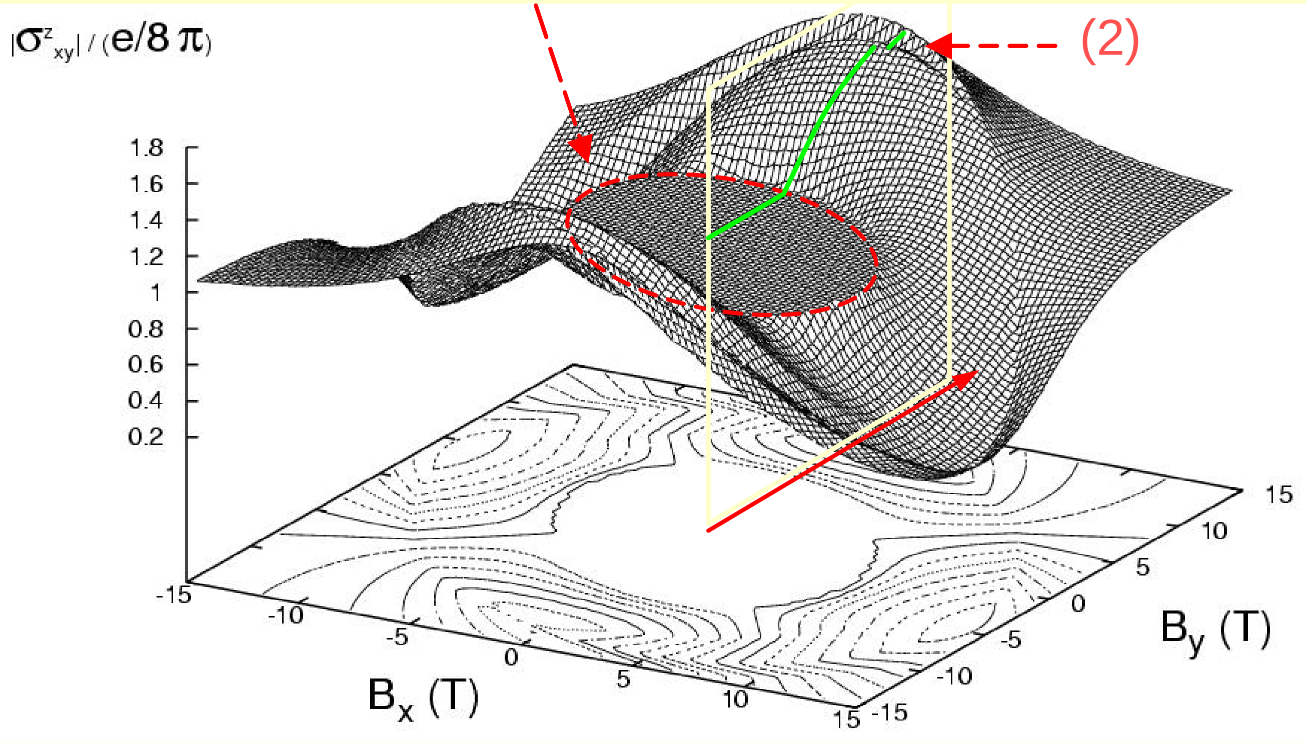
Spin Hall conductivity (electron density fixed)

$$s^{x}_{xy} = s^{y}_{xy} = 0$$



Boundary of plateau *E(k₀)* = μ

$$b_x^2 + 4agb_x b_y + b_y^2 = z\frac{\left(a^2 - g^2\right)^2}{a^2 + g^2}$$



M.C. Chang, PRB 2005

Acknowledgement: M.F. Yang

Existence of charge Hall effect?

Thouless formula (PRL 1982)

$$s^{\,l}_{xy} = \frac{e^2}{\hbar}\sum_{\vec{k}\ filled}\Omega_l(\vec{k}),$$

$$|\vec{k},+\rangle = \frac{1}{\sqrt{2}}\begin{pmatrix}1\\ -ie^{iq}\end{pmatrix},\quad |\vec{k},-\rangle = \frac{1}{\sqrt{2}}\begin{pmatrix}-ie^{iq}\\ 1\end{pmatrix},$$

$$\tan q = \frac{gk_x + ak_y + b_x}{ak_x + gk_y - b_y}$$

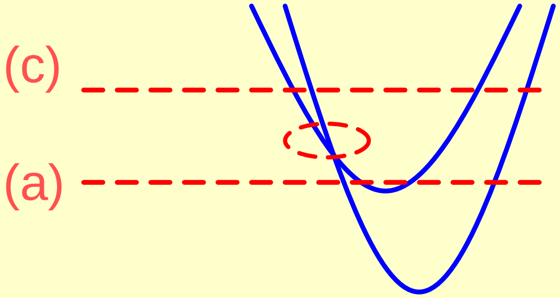
Berry curvature

$$\Omega_l(\vec{k}) = i\sum_{l'\neq l}\frac{\langle \vec{k},l|v_x|\vec{k},l'\rangle\langle\vec{k},l'|v_y|\vec{k},l\rangle - \langle\vec{k},l|v_y|\vec{k},l'\rangle\langle\vec{k},l'|v_x|\vec{k},l\rangle}{w^2_{ll'}(\vec{k})} = 0$$

at every *k*, except at degenerate point k_0

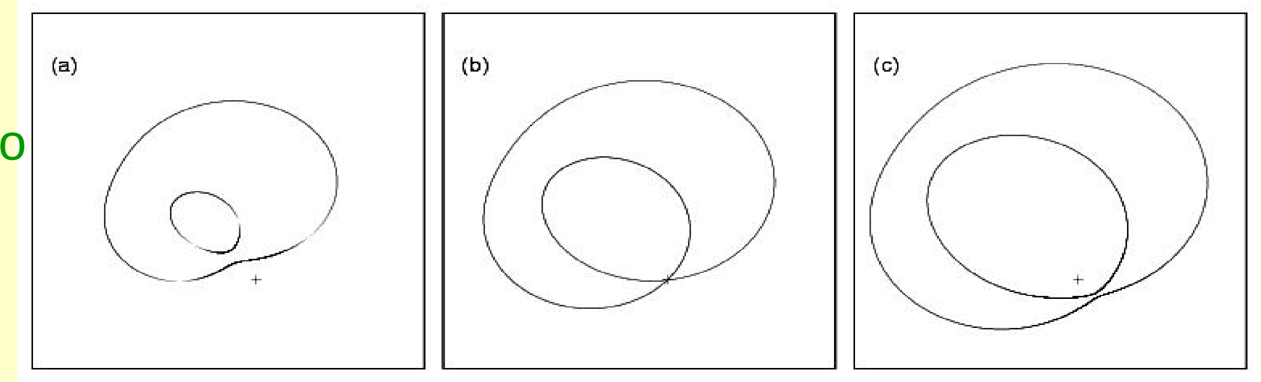
Berry phase

$$\Gamma_l = \oint d\vec{k}\cdot\langle\vec{k},l|i\frac{\partial}{\partial\vec{k}}|\vec{k},l\rangle = \begin{cases}-lp & \text{for } a^2>g^2\\ 0 & \text{for } a^2=g^2\\ +lp & \text{for } a^2<g^2\end{cases}$$



$$\Rightarrow \quad \Omega_l(\vec{k}) = -\mathrm{sgn}(a^2-g^2)\,lpd(\vec{k}-\vec{k}_0)$$

Hall conductivity is zero wherever the chemical potential is



0+0=0

(−π)+π =0

Issues on the spin current in SO coupled systems

(Rashba, cond-mat/0408119)

- spin current is not well defined (total spin not conserved)

$$\frac{\partial}{\partial t} S^a + \nabla \cdot \vec{J}^a = \mathrm{Re}\, y^{+} \dot{s}^a y \,,$$ Spin torque

$$\text{where } S^a \equiv y^{+} s^a y \,, \quad \vec{J}^a \equiv (1/2)\, \mathrm{Re}\, y^{+} \left(s^a \vec{v} + \vec{v} s^a \right) y$$ Spin flux

- existence of background spin current Rashba, PRB 2003

  (which produces no spin accumulation)

$$\vec{J}^x(\vec{k}, l) = a/2\hat{y}; \quad \vec{J}^y(\vec{k}, l) = -a/2\hat{x}$$

- no experimental procedure to measure it directly

  (accumulation? Induced electric field?) Meier and Loss, PRL 2003

- connection with Maxwell eqs?

  (Bernevig and Zhang, PRL Aug 2005)