Collective excitations
in
integer quantum Hall system

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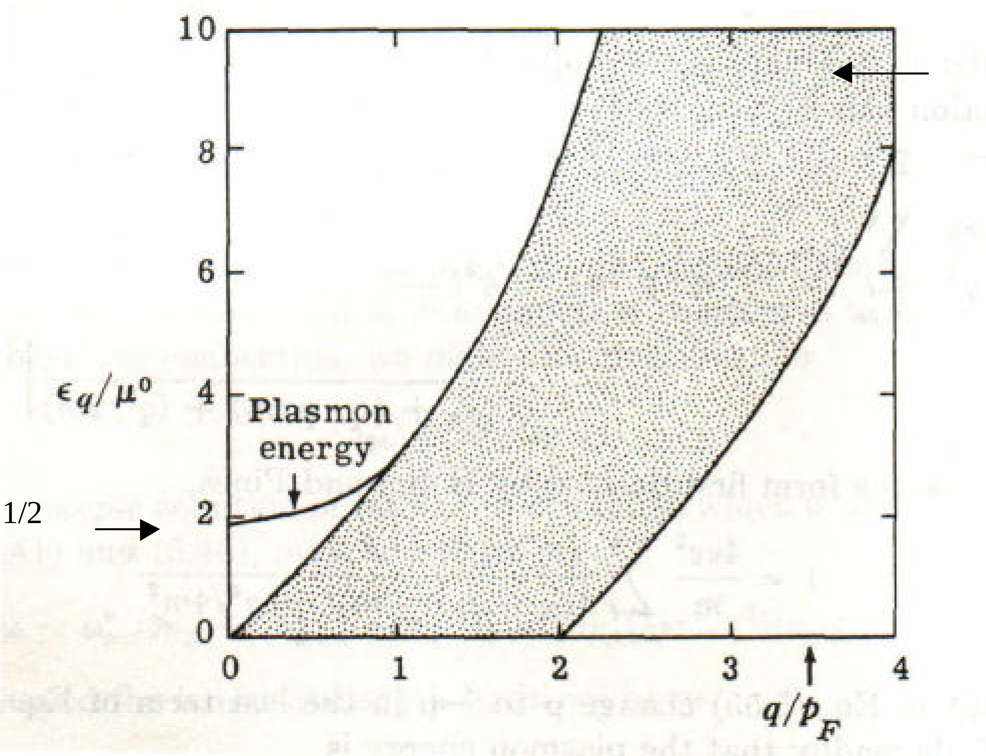
Plasma excitation in 3DEG, NO magnetic field

r_s=4, sodium



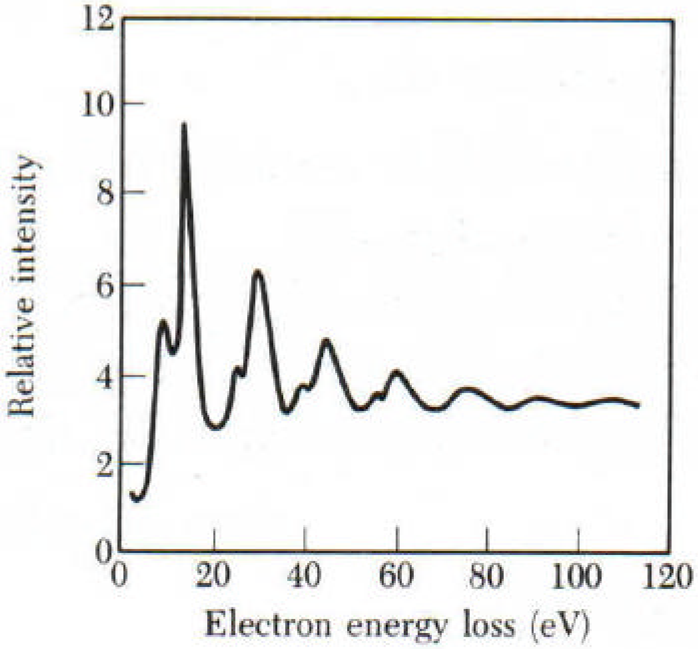
$\omega_P=(4\pi\rho e^2/m)^{1/2}$ →

← Independent electron-hole continuum

2DEG, NO magnetic field ? gapless excitation

Experimental observation

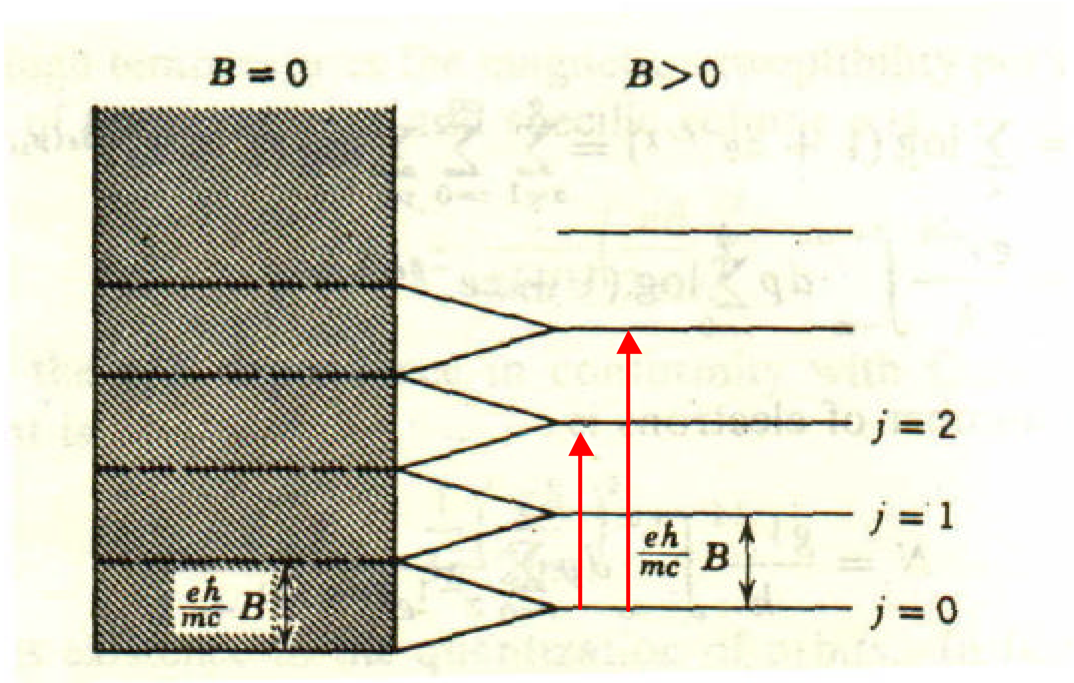
Al: 10.3 eV + 15.3 eV



10.3 eV due to surface plasmons

2DEG in a strong magnetic field

emergence of Landau levels



magnetoplasma modes: inter-LL excitations

can be Integer filling / non-integer filling

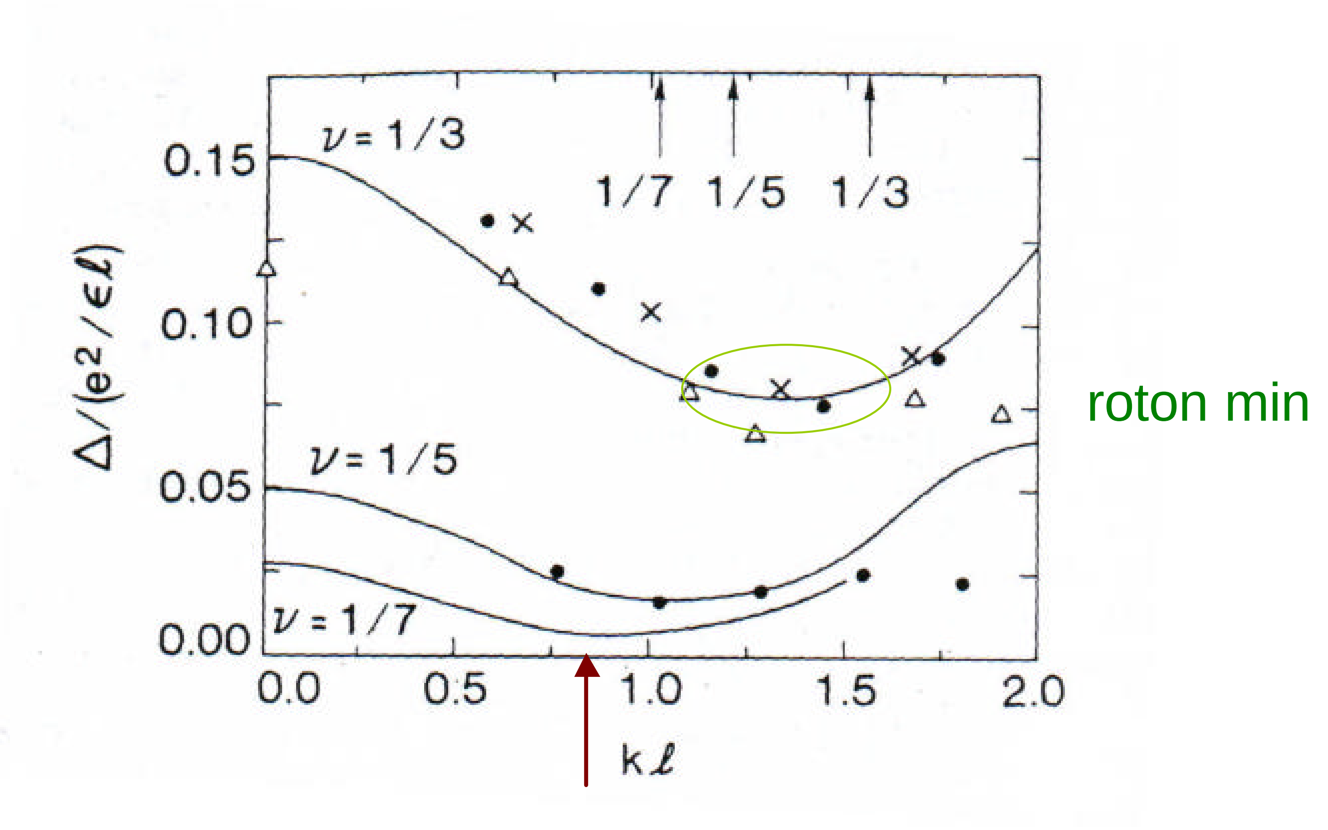
Inter-LL excitation/intra-LL excitation

Charge excitation / spin excitation

Fractional case, intra-LL charge excitations

Finite energy gap stabilizes the FQHL

Magnetoroton mode similar to rotons in superfluid



indication of Wigner crystal instability?

(Girvin/MacDonald/Platzman 1985 PRL)

Integer case

classification of collective excitations

Intra-LL :

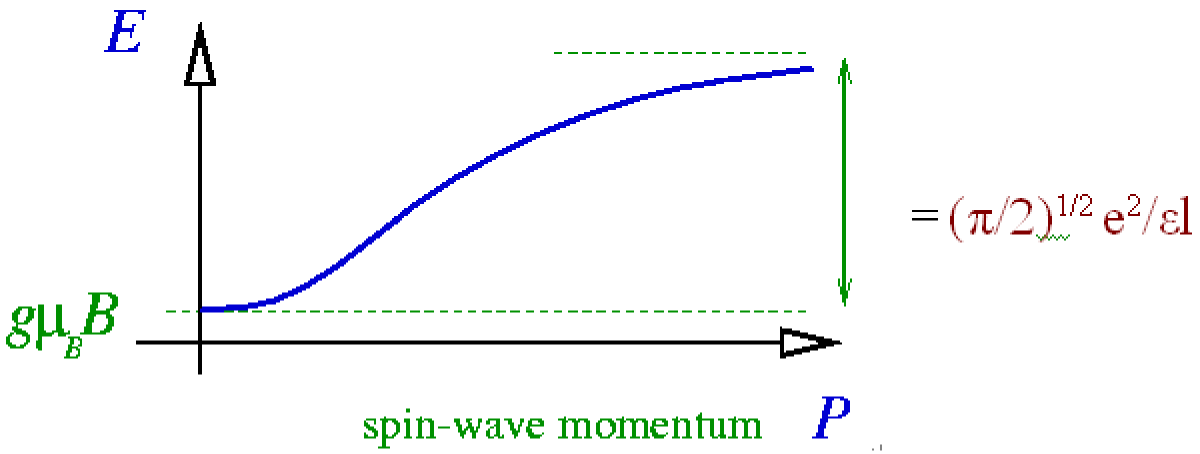
Spin flip excitations only

0?

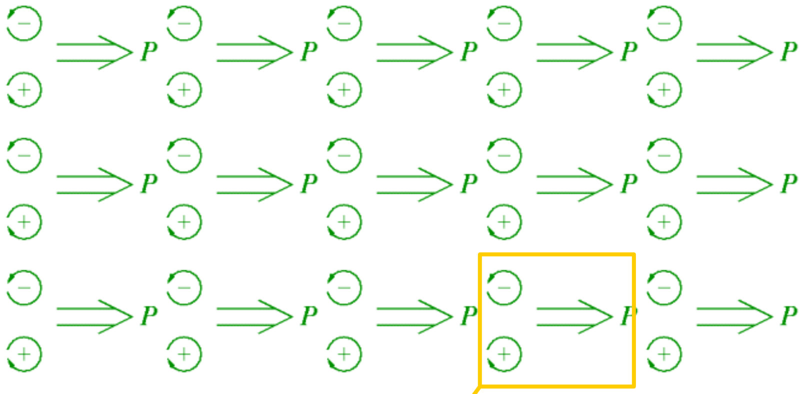
0?

Zeeman splitting

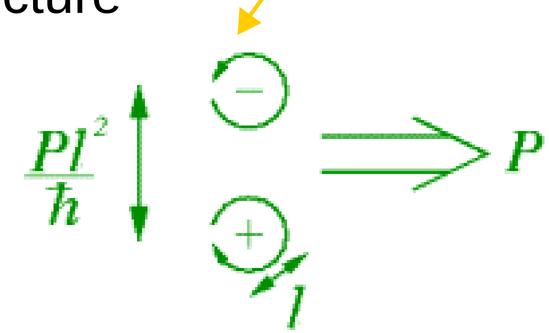
energy dispersion of spin wave



spin wave as coherent superposition of e-h pairs



semiclassical picture



$E_\infty = (\pi/2)^{1/2}\, e^2/\varepsilon l$

= the energy to flip the spin of an electron

and take it far away from the hole

= the exchange energy of an electron in the LLL

Excitation energy

Single particle: cyclotron energy

Zeeman energy

Manybody effect: exchange interaction

polarization and screening

excitonic attraction … etc

Methods of calculation

Equation of motion method

→ Diagrammatic expansion

The pole of response function

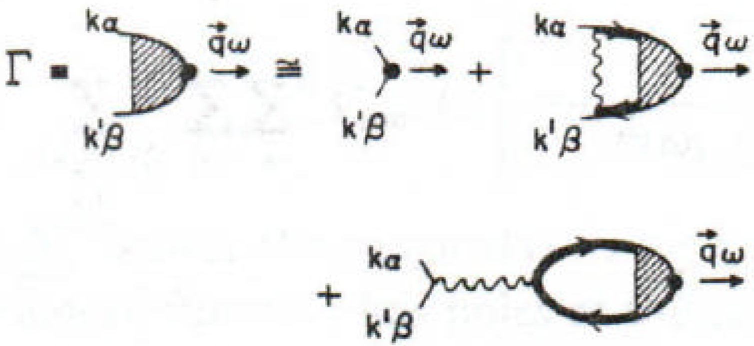
→ energy spectrum of collective excitation

Time dependent HFA

Calculation of response function

Renormalized electron propagator

Bethe-Salpeter equation



ladder

bubble

Response function

$$\chi(k,i\omega) \equiv -\int_0^\infty d\tau e^{i\omega\tau}\langle T_\tau \hat{\rho}(\vec{k},\tau)\hat{\rho}(-\vec{k},0)\rangle.$$

$$= \sum_{n_\alpha n_\beta}\sum_{m_\alpha m_\beta}\langle n_\alpha, m_\alpha|e^{i\vec{k}\cdot\vec{r}}|n_\beta, m_\beta\rangle D_{n_\alpha n_\beta}(i\omega)\Gamma^{m_\alpha m_\beta}_{n_\alpha n_\beta}(k,i\omega)$$

inter/intra-LL indices

free 2-particle propagator

$$D_{n_\alpha' n_\beta'}(i\omega) \equiv \int \frac{d\omega'}{2\pi} G_{n_\alpha'}(i\omega' - i\omega) G_{n_\beta'}(i\omega')$$

BS equation for the vertex

$$\Gamma_{n_\alpha n_\beta}(k,i\omega) = \frac{i^{n_{\alpha\beta}}}{2\pi} e^{-k^2/2}\bar{g}_{n_\alpha n_\beta}(-\bar{\kappa})$$

$$- \sum_{n_\alpha' n_\beta'}\left[V^{n_\alpha' n_\beta'}_{n_\alpha n_\beta}(k) - U^{n_\alpha' n_\beta'}_{n_\alpha n_\beta}(k)\right] D_{n_\alpha' n_\beta'}(i\omega)\Gamma_{n_\alpha' n_\beta'}(k,i\omega).$$

ladder bubble

matrix form of the BS equation

$$\sum_{n_\alpha' n_\beta'} \left[\delta_{n_\alpha,n_\alpha'}\delta_{n_\beta,n_\beta'} D^{-1}_{n_\alpha' n_\beta'}(i\omega) + V^{n_\alpha' n_\beta'}_{n_\alpha n_\beta}(k) - U^{n_\alpha' n_\beta'}_{n_\alpha n_\beta}(k)\right] D_{n_\alpha' n_\beta'}(i\omega)\Gamma_{n_\alpha' n_\beta'}(k, i\omega) = \frac{i^{n_{\alpha\beta}}}{2\pi} e^{-k^2/2}\bar{g}_{n_\alpha n_\beta}(-\bar{\kappa}),$$

$$D^{-1}_{n_\alpha' n_\beta'}(i\omega) = \left(i\omega - (n_\alpha' - n_\beta')\omega_C - (\Sigma^{n_0}_{n_\alpha'} - \Sigma^{n_0}_{n_\beta'})\right)$$

self-energy

bubble (Coulomb local field)

$$U^{n_\alpha' n_\beta'}_{n_\alpha n_\beta}(k) \equiv i^{n_{\alpha\beta} - n_{\alpha\beta}'} e^{-k^2/2}\bar{g}_{n_\alpha n_\beta}(-\bar{\kappa}) \frac{\tilde{v}(k)}{2\pi} g_{n_\alpha' n_\beta'}(-\bar{\kappa})$$

ladder (exchange local field)

$$V^{n_\alpha' n_\beta'}_{n_\alpha n_\beta}(k) \equiv 2\pi \int d^2\vec{r_1} \int d^2\vec{r_2}\, \Phi^{\kappa}_{n_\alpha, n_\beta}(\vec{r_1}, \vec{r_2}) v(\vec{r_1} - \vec{r_2}) \bar{\Phi}^{\kappa}_{n_\alpha', n_\beta'}(\vec{r_1}, \vec{r_2})$$

exciton wave function

poles of the response function

= eigenvalues of the matrix eq.

consider ?n=1, neglect inter-mode coupling

one of the diagonal element

$$M_{2,1}^{2,1}(k,\omega) = \omega - \omega_C - \frac{e^2}{\epsilon l_0}(\tilde{\Sigma}^1{}_{n=2} - \tilde{\Sigma}^1{}_{n=1}) - \frac{e^2}{\epsilon l_0}\tilde{U}_{2,1}^{2,1}(k) + \frac{e^2}{\epsilon l_0}\tilde{V}_{2,1}^{2,1}(k) = 0.$$

transition energy =

cyclotron energy + self energy correction (k independent)

+ Coulomb local field correction

+ exchange local field correction

General behavior of the manybody corrections

k → 0

- bubble correction → 0
- ladder correction → self-energy correction

Kohn theorem: the cyclotron energy gap is NOT renormalized by many-body correction

k → ∞

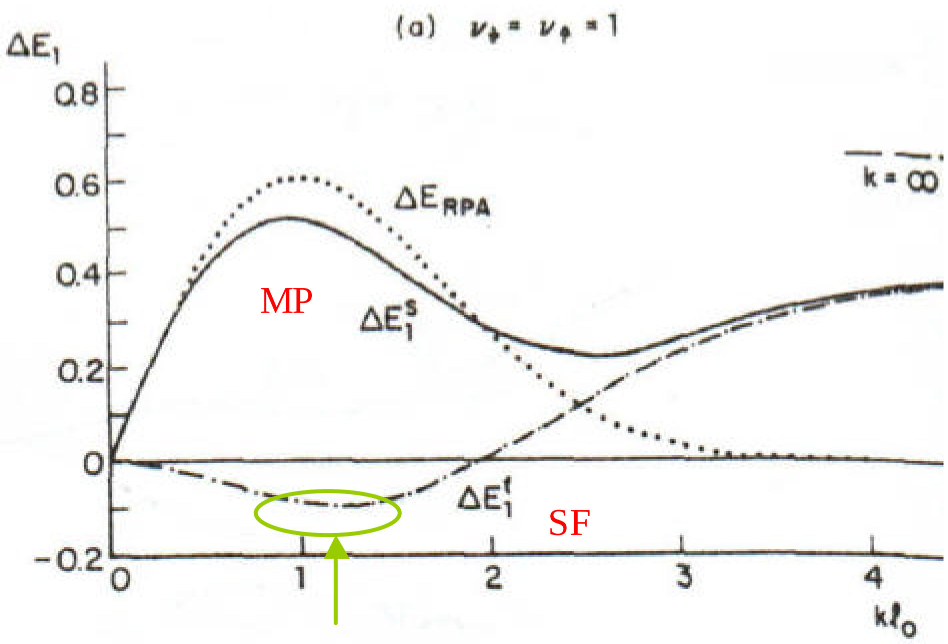
- bubble correction → 0 exponentially
- ladder correction → 0 algebraically

Coulomb interaction prohibits spin-flip

→ no bubble correction for spin-flip excitaions

energy dispersion determined solely by ladder correction

$\nu = 2$



roton feature (spin triplet roton)