Diamagnetism and paramagnetism

- Langevin diamagnetism
- paramagnetism
  - Hund's rules
  - Lande g-factor
  - Brillouin function
- crystal field splitting
  - quench of orbital angular momentum

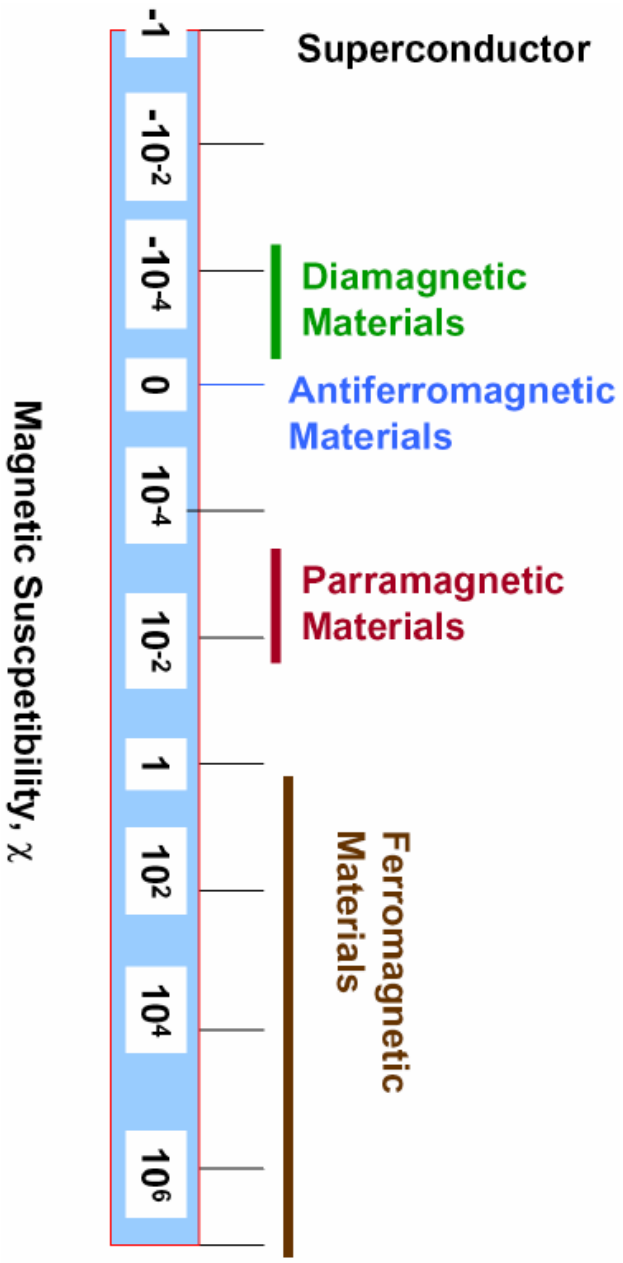
} atom

- nuclear demagnetization

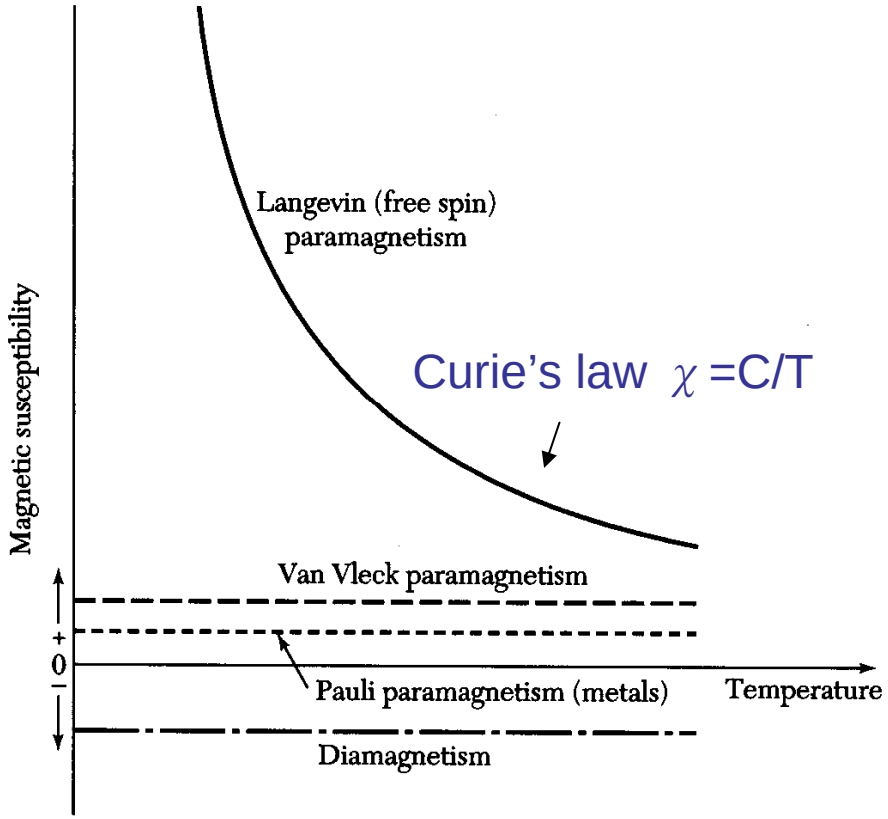
- Pauli paramagnetism and Landau diamagnetism

} free electron gas

Dept of Phys
M.C. Chang


Magnetic Susceptibility, χ
-1
-10-2
-10-4
0
10-4
10-2
1
102
104
106
Superconductor
Diamagnetic Materials
Antiferromagnetic Materials
Parramagnetic Materials
Ferromagnetic Materials

$B=(1+\chi)H$


Magnetic susceptibility
Langevin (free spin) paramagnetism
Curie's law χ =C/T
Van Vleck paramagnetism
+
0
−
Pauli paramagnetism (metals)
Temperature
Diamagnetism

important

Basics

- System energy $E(H)$ $\quad (E \to F = E - TS \text{ if } T \neq 0)$
- magnetization density $M(H) = -\frac{1}{V}\frac{\partial E}{\partial H}$
- susceptibility $\chi \equiv \frac{\partial M}{\partial H} = -\frac{1}{V}\frac{\partial^2 E}{\partial H^2}$

Atomic susceptibility

$$H = \sum_i \left(\frac{p_i^2}{2m} + V_i \right) + \mu_B \left(\vec{L} + g\vec{S} \right) \cdot \vec{H} + \frac{e^2}{2mc} \sum_i A_i^2, \quad \mu_B = \frac{e\hbar}{2mc}$$
$$= H_0 + \Delta H$$

Order of magnitude

- $\mu_B \left(\vec{L} + g\vec{S} \right) \cdot \vec{H} \approx \mu_B H \approx \hbar\omega_c$
  $\approx 10^{-4} eV \quad \text{when } H = 1\,\text{T}$
- $\vec{A}_i = \frac{H}{2}(-y_i, x_i, 0)$

$$\frac{e^2}{2mc} \sum_i A_i^2 \approx \left(\frac{eH}{mc} \right)^2 m a_0^2, \quad a_0 \equiv \frac{\hbar^2}{me^2}$$
$$\approx \frac{(\hbar\omega_c)^2}{e^2 / a_0} \approx 10^{-5} \text{ of the linear term at } H = 1\,\text{T}$$

important

• Perturbation energy (to 2nd order)

$$\Delta E_n = \langle n|\Delta H|n\rangle + \sum_{n'\neq n} \frac{|\langle n|\Delta H|n'\rangle|^2}{E_n - E_{n'}}$$

$$= \mu_B \langle n|\vec{L} + g\vec{S}|n\rangle \cdot \vec{H} + \frac{e^2}{2mc^2}\langle n|\sum_i A_i^2|n\rangle + \sum_{n'}{}' \frac{|\langle n|\mu_B(\vec{L} + g\vec{S})\cdot\vec{H}|n'\rangle|^2}{E_n - E_{n'}}$$

• Filled atomic shell

(applies to noble gas, NaCl-like ions…etc)

Ground state $|0\rangle$:

$$\vec{L}|0\rangle = \vec{S}|0\rangle = 0$$

$$\therefore \quad \Delta E = \frac{e^2}{8mc^2} H^2 \langle 0|\frac{2}{3}\sum_i r_i^2|0\rangle \quad \text{(for spherical charge dist)}$$

For a collection of *N* ions,

$$\chi = -\frac{N}{V}\frac{\partial^2 \Delta E}{\partial H^2} = -\frac{e^2}{6mc^2}\frac{N}{V}\langle 0|\sum_i r_i^2|0\rangle < 0$$

Larmor (or Langevin) diamagnetism

An atom with many electrons

- Without SO coupling

$$H=\sum_i\left(\frac{p_i^2}{2m}+V_i\right)+\frac{1}{2}\sum_{ij}V_{ij}$$

  - Maximally mutually commuting set

$$H, L^2, S^2, L_z, S_z$$

  - Eigenstates (including ground states)

$$|\alpha, L, S, m_L, m_S\rangle$$

- With SO coupling (weak)

$$H=\sum_i\left(\frac{p_i^2}{2m}+V_i\right)+\frac{1}{2}\sum_{ij}V_{ij}+\sum_i\lambda_i\vec{S}_i\cdot\vec{L}_i$$

  - Maximally mutually commuting set

$$H, L^2, S^2, J^2, J_z$$

  - Eigenstates (including ground states)

$$|\alpha, L, S, J, m_J\rangle$$

- single electron ground states

Degeneracy D=d

—— —— … ——

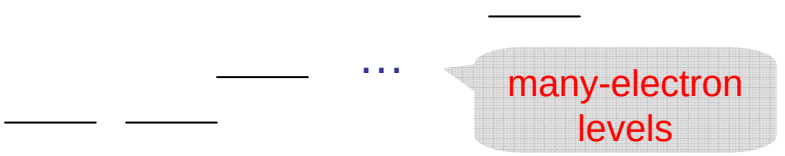
- N-electron ground states

  - Without e-e interaction

Degeneracy D=C^d_N

—— —— … ——

  - With e-e interaction



Ground states

w/o SO: labeled by L,S D=(2L+1)(2S+1)

w/ SO: labeled by L,S,J D=(2J+1)

important

Ground state of an atom with unfilled shell (no *H* field yet!):

non-interacting

- Atomic quantum numbers n, l, m_l, m_s
- Energy of an electron depends on n, l (no m_l, m_s)
- Degeneracy of electron level $\varepsilon_{n,l}$: $2(2l+1)$
- If an atom has *N* (non-interacting) valence electrons, then the degeneracy of the "atomic" ground state (with unfilled $\varepsilon_{n,l}$ shell) is $C_N^{2(2l+1)}$

interacting

e-e interaction will lift this degeneracy partially, and then

- the atom energy is labeled by the conserved quantities *L and S*, each is (2L+1)(2S+1)-fold degenerate
- SO coupling would split these states further, which are labeled by *J* (later).

What's the values of S, L, and J for the atomic ground state?

Use the Hund's rules (1925),

1. Choose the max value of S that is consistent with the exclusion principle
2. Choose the max value of L that is consistent with the exclusion principle and the 1st rule

To reduce Coulomb repulsion, electron spins like to be parallel, electron orbital motion likes to be in high m_l state. Both help disperse the charge distribution.

Example: 2 *e*'s in the *p*-shell ($l_1 = l_2 = 1$, $s_1 = s_2 = 1/2$)

(a) (1,1/2) (b) (0,1/2) (c) (-1,1/2)

(a') (1,-1/2) (b') (0,-1/2) (c') (-1,-1/2)

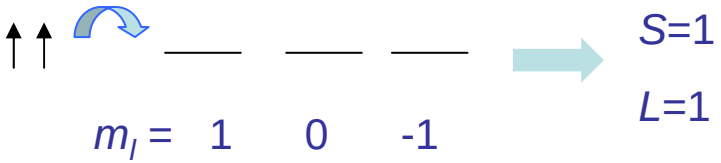
C^6_2 ways to put these 2 electrons in 6 slots

- Spectroscopic notation:

$^{2S+1}X_J \ (X = S, P, D...)$

$^1S_0, {}^3P_{0,1,2}, {}^1D_2$ are o.k.; $^3S_0, {}^1P_1, {}^3D_3$ are not.

(It's complicated. See Eisberg and Resnick App. K for more details)



- Ground state is $^3P_{0,1,2}$,

(2L+1)x(2S+1)=9-fold degenerate

Energy levels of Carbon atom

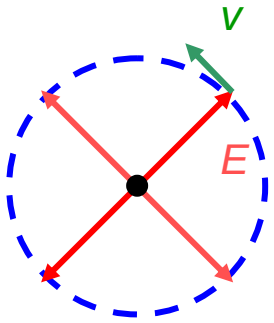
Configuration	Term	J	Level(cm^{-1})
$2s^2 2p^2$	^{3}P	0	0.00000
		1	16.41671
		2	43.41350
$2s^2 2p^2$	^{1}D	2	10192.66
$2s^2 2p^2$	^{1}S	0	21648.02

physics.nist.gov/PhysRefData/Handbook/Tables/carbontable5.htm

- There is also the 3rd Hund's rule related to SO coupling (details below)

important

Review of SO coupling



- An electron moving in a static E field feels an effective B field

$$\vec{B}_{eff} = \vec{E} \times \frac{\vec{v}}{c}$$

- This B field couples with the electron spin

$$H_{SO} = -\vec{\mu} \cdot \vec{B}_{eff}$$

$$= -\left(\frac{q}{mc}\vec{S}\right) \cdot \left(\vec{E} \times \frac{\vec{v}}{c}\right), \quad \vec{E} = -\hat{r}\frac{d\phi}{dr} \text{ for central force, } \phi = +\frac{e}{r}$$

$$= \left(\frac{q}{m^2c^2}\frac{d\phi}{rdr}\right)\vec{S} \cdot \vec{L}$$

(x 1/2 for Thomas precession, 1927)

$$\equiv \lambda \vec{S} \cdot \vec{L}$$

$\lambda > 0$ for less than half-filled (electron-like)
$\lambda < 0$ for more than half-filled (hole-like)

$$= \frac{\lambda}{2}\left(J^2 - L^2 - S^2\right)$$

Quantum states are now labeled by *L, S, J*

(2L+1)x(2S+1) degeneracy is further lifted to become (2J+1)-fold degeneracy

Hund's 3rd rule:

- if less than half-filled, then J=|L-S| has the lowest energy
- if more than half-filled, then J=L+S has the lowest energy

➡ $^3P_0$ is the ground state in previous example

d-shell ($l = 2$)									
n	$l_z = 2,\ 1,\ 0,\ -1,\ -2$	S	$L = \lvert\Sigma l_z\rvert$	J	SYMBOL
1	↓	1/2	2	3/2	$^2D_{3/2}$
2	↓ ↓	1	3	2	$^3F_2$
3	↓ ↓ ↓	3/2	3	3/2	$^4F_{3/2}$
4	↓ ↓ ↓ ↓	2	2	0	$^5D_0$
5	↓ ↓ ↓ ↓ ↓	5/2	0	5/2	$^6S_{5/2}$
6	↑↓ ↑ ↑ ↑ ↑	2	2	4	$^5D_4$
7	↑↓ ↑↓ ↑ ↑ ↑	3/2	3	9/2	$^4F_{9/2}$
8	↑↓ ↑↓ ↑↓ ↑ ↑	1	3	4	$^3F_4$
9	↑↓ ↑↓ ↑↓ ↑↓ ↑	1/2	2	5/2	$^2D_{5/2}$
10	↑↓ ↑↓ ↑↓ ↑↓ ↑↓	0	0	0	$^1S_0$

$J = |L - S|$ (1–4); $J = L + S$ (6–9)

f-shell ($l = 3$)										
n	$l_z = 3,\ 2,\ 1,\ 0, -1, -2, -3$	S	$L = \lvert\Sigma l_z\rvert$	J	
1	↓	1/2	3	5/2	$^2F_{5/2}$
2	↓ ↓	1	5	4	$^3H_4$
3	↓ ↓ ↓	3/2	6	9/2	$^4I_{9/2}$
4	↓ ↓ ↓ ↓	2	6	4	$^5I_4$
5	↓ ↓ ↓ ↓ ↓	5/2	5	5/2	$^6H_{5/2}$
6	↓ ↓ ↓ ↓ ↓ ↓	3	3	0	$^7F_0$
7	↓ ↓ ↓ ↓ ↓ ↓ ↓	7/2	0	7/2	$^8S_{7/2}$
8	↑↓ ↑ ↑ ↑ ↑ ↑ ↑	3	3	6	$^7F_6$
9	↑↓ ↑↓ ↑ ↑ ↑ ↑ ↑	5/2	5	15/2	$^6H_{15/2}$
10	↑↓ ↑↓ ↑↓ ↑ ↑ ↑ ↑	2	6	8	$^5I_8$
11	↑↓ ↑↓ ↑↓ ↑↓ ↑ ↑ ↑	3/2	6	15/2	$^4I_{15/2}$
12	↑↓ ↑↓ ↑↓ ↑↓ ↑↓ ↑ ↑	1	5	6	$^3H_6$
13	↑↓ ↑↓ ↑↓ ↑↓ ↑↓ ↑↓ ↑	1/2	3	7/2	$^2F_{7/2}$
14	↑↓ ↑↓ ↑↓ ↑↓ ↑↓ ↑↓ ↑↓	0	0	0	$^1S_0$

$J = |L - S|$ (1–7); $J = L + S$ (8–13)

*↑ = spin $\frac{1}{2}$; ↓ = spin $-\frac{1}{2}$.

Paramagnetism of an atom with unfilled shell

1) Ground state is nondegenerate (J=0)

$$\Delta E = \mu_B \langle 0 | \vec{L} + g\vec{S} | 0 \rangle \cdot \vec{H} + \frac{e^2}{2mc^2} \langle 0 | \sum_i A_i^2 | 0 \rangle + \sum_n{}' \frac{\left| \langle 0 | \mu_B \left(\vec{L} + g\vec{S} \right) \cdot \vec{H} | n \rangle \right|^2}{E_0 - E_n}$$

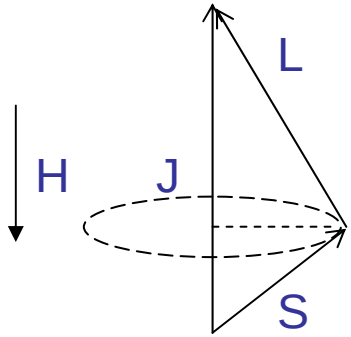
(A+M, Prob 31.4)

Van Vleck PM

2) Ground state is degenerate ($J \neq 0$)

Then the 1st order term almost always >> the 2nd order terms.

$$\vec{m} = -\mu_B \left(\vec{L} + 2\vec{S} \right) = -\mu_B \left(\vec{J} + \vec{S} \right)$$



• Heuristic argument: J is fixed, L and S rotate around J, maintaining the triangle. So the magnetic moment is given by the component of $L+2S$ parallel to J,

$$\vec{S}_{//} = \frac{\vec{J} \cdot \vec{S}}{J^2} \vec{J} = \frac{\vec{J}}{2J^2} \left(J^2 - L^2 + S^2 \right)$$

$$= \frac{\vec{J}}{2J(J+1)} \left[J(J+1) - L(L+1) + S(S+1) \right]$$

$$\therefore \vec{m}_{eff} = -g_J \mu_B \vec{J}$$

Lande g-factor (1921)

$$g_J = 1 + \frac{J(J+1) - L(L+1) + S(S+1)}{2J(J+1)}$$

• $\Delta E(m_J) = g_J \mu_B m_J H$, so $\chi = 0$?

No! these $2J+1$ levels are closely packed ($< kT$), so $F(H)$ is nonlinear (next page).

Langevin paramagnetism

$$Z = \sum_{m_J=-J}^{J} e^{-E(m_J)/k_BT}, \quad \Delta E(m_J) = g_J \mu_B m_J H \left(\sim 1K \text{ at } H = 1\,\mathrm{T}\right)$$

$$F = E - TS = -k_B T \ln Z$$

$$M = -\frac{N}{V}\frac{\partial F}{\partial H} = \frac{N}{V} g_J \mu_B J B_J\left(\frac{g_J \mu_B J H}{k_B T}\right)$$

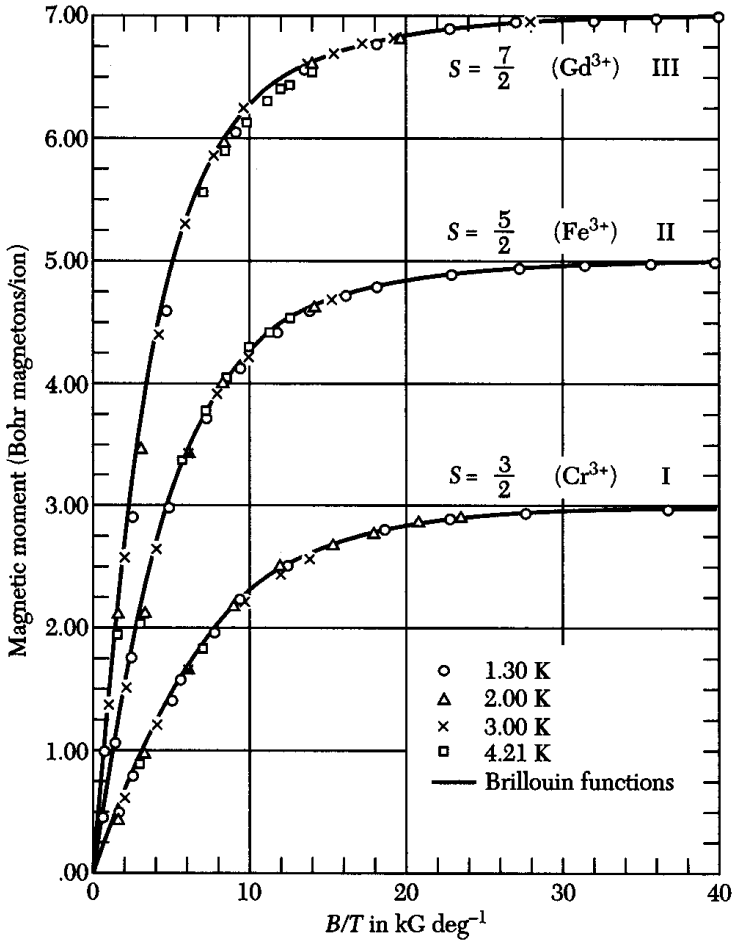
where $B_J(x) \equiv \frac{2J+1}{2J}\coth\left(\frac{2J+1}{2J}x\right) - \frac{1}{2J}\coth\left(\frac{x}{2J}\right)$ **Brillouin function**

- $k_B T << g_J \mu_B J H \quad (x >> 1)$

$$M = \frac{N}{V} g_J \mu_B J, \quad \chi = 0$$

- $k_B T >> g_J \mu_B J H \quad (x << 1)$ $\qquad B_J(x) \sim \frac{J+1}{3J}x$

$$M = \underbrace{\frac{N}{V}\left(g_J \mu_B\right)^2 \frac{J(J+1)}{3k_B T}}_{\chi(T)} H$$



- at room T, χ (para) $\sim$ 500 χ (dia) calculated earlier
- Curie's law $\chi = C/T$ (note: not good for J=0)

$$C = \frac{N}{V}\frac{\left(\mu_B p\right)^2}{3k}, \quad \text{where } p = g_J\sqrt{J(J+1)}$$

effective Bohr magneton number

f-shell (Lanthanides)
鑭系元素

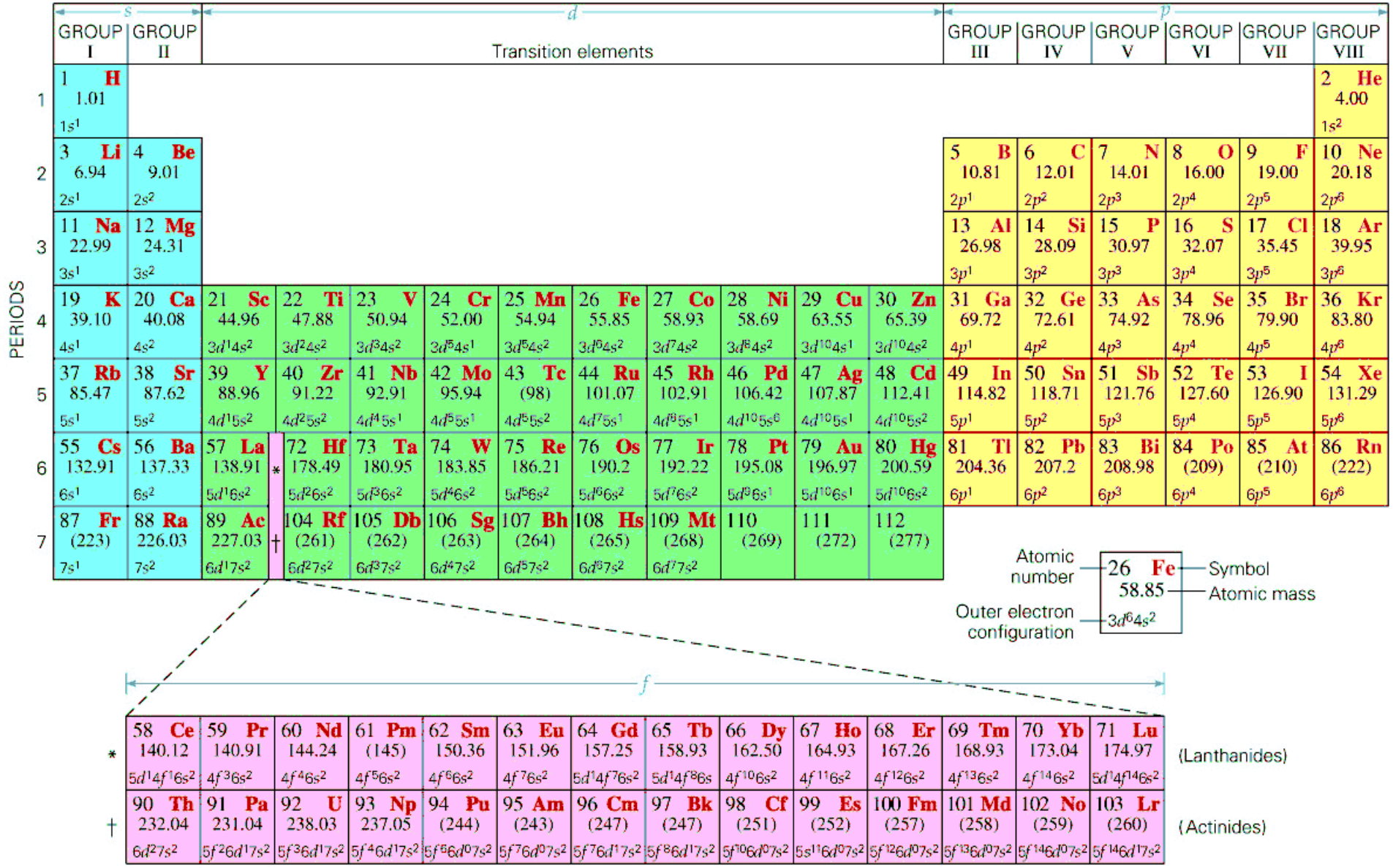
In general (but not always), energy from low to high:
1s 2s 2p 3s 3p 4s 3d 4p 5s 4d 5p 6s 4f 5d …

ELEMENT (TRIPLY IONIZED)	BASIC ELECTRON CONFIGURATION	GROUND-STATE TERM	CALCULATED[b] p	MEASURED[c] p
La	$4f^0$	^1S	0.00	diamagnetic
Ce	$4f^1$	$^2F_{5/2}$	2.54	2.4
Pr	$4f^2$	$^3H_4$	3.58	3.5
Nd	$4f^3$	$^4I_{9/2}$	3.62	3.5
Pm	$4f^4$	$^5I_4$	2.68	—
Sm	$4f^5$	$^6H_{5/2}$	0.84	1.5
Eu	$4f^6$	$^7F_0$	0.00	3.4
Gd	$4f^7$	$^8S_{7/2}$	7.94	8.0
Tb	$4f^8$	$^7F_6$	9.72	9.5
Dy	$4f^9$	$^6H_{15/2}$	10.63	10.6
Ho	$4f^{10}$	$^5I_8$	10.60	10.4
Er	$4f^{11}$	$^4I_{15/2}$	9.59	9.5
Tm	$4f^{12}$	$^3H_6$	7.57	7.3
Yb	$4f^{13}$	$^2F_{7/2}$	4.54	4.5
Lu	$4f^{14}$	^1S	0.00	diamagnetic

$J=0$

Due to low-lying J-multiplets (see A+M, p.657)

• Before ionization, La: $5p^6\ 6s^2\ 5d^1$; Ce: $5p^6\ 6s^2\ 4f^2$ …


s
d
p
GROUP I
GROUP II
Transition elements
GROUP III
GROUP IV
GROUP V
GROUP VI
GROUP VII
GROUP VIII
PERIODS
1 H 1.01 1s1
2 He 4.00 1s2
3 Li 6.94 2s1
4 Be 9.01 2s2
5 B 10.81 2p1
6 C 12.01 2p2
7 N 14.01 2p3
8 O 16.00 2p4
9 F 19.00 2p5
10 Ne 20.18 2p6
11 Na 22.99 3s1
12 Mg 24.31 3s2
13 Al 26.98 3p1
14 Si 28.09 3p2
15 P 30.97 3p3
16 S 32.07 3p4
17 Cl 35.45 3p5
18 Ar 39.95 3p6
19 K 39.10 4s1
20 Ca 40.08 4s2
21 Sc 44.96 3d14s2
22 Ti 47.88 3d24s2
23 V 50.94 3d34s2
24 Cr 52.00 3d54s1
25 Mn 54.94 3d54s2
26 Fe 55.85 3d64s2
27 Co 58.93 3d74s2
28 Ni 58.69 3d84s2
29 Cu 63.55 3d104s1
30 Zn 65.39 3d104s2
31 Ga 69.72 4p1
32 Ge 72.61 4p2
33 As 74.92 4p3
34 Se 78.96 4p4
35 Br 79.90 4p5
36 Kr 83.80 4p6
37 Rb 85.47 5s1
38 Sr 87.62 5s2
39 Y 88.96 4d15s2
40 Zr 91.22 4d25s2
41 Nb 92.91 4d45s1
42 Mo 95.94 4d55s1
43 Tc (98) 4d55s2
44 Ru 101.07 4d75s1
45 Rh 102.91 4d85s1
46 Pd 106.42 4d105s0
47 Ag 107.87 4d105s1
48 Cd 112.41 4d105s2
49 In 114.82 5p1
50 Sn 118.71 5p2
51 Sb 121.76 5p3
52 Te 127.60 5p4
53 I 126.90 5p5
54 Xe 131.29 5p6
55 Cs 132.91 6s1
56 Ba 137.33 6s2
57 La 138.91 5d16s2
72 Hf 178.49 5d26s2
73 Ta 180.95 5d36s2
74 W 183.85 5d46s2
75 Re 186.21 5d56s2
76 Os 190.2 5d66s2
77 Ir 192.22 5d76s2
78 Pt 195.08 5d96s1
79 Au 196.97 5d106s1
80 Hg 200.59 5d106s2
81 Tl 204.36 6p1
82 Pb 207.2 6p2
83 Bi 208.98 6p3
84 Po (209) 6p4
85 At (210) 6p5
86 Rn (222) 6p6
87 Fr (223) 7s1
88 Ra 226.03 7s2
89 Ac 227.03 6d17s2
104 Rf (261) 6d27s2
105 Db (262) 6d37s2
106 Sg (263) 6d47s2
107 Bh (264) 6d57s2
108 Hs (265) 6d67s2
109 Mt (268) 6d77s2
110 (269)
111 (272)
112 (277)
Atomic number
26 Fe
58.85
Symbol
Atomic mass
Outer electron configuration
3d64s2
f
58 Ce 140.12 5d14f16s2
59 Pr 140.91 4f36s2
60 Nd 144.24 4f46s2
61 Pm (145) 4f56s2
62 Sm 150.36 4f66s2
63 Eu 151.96 4f76s2
64 Gd 157.25 5d14f76s2
65 Tb 158.93 5d14f86s
66 Dy 162.50 4f106s2
67 Ho 164.93 4f116s2
68 Er 167.26 4f126s2
69 Tm 168.93 4f136s2
70 Yb 173.04 4f146s2
71 Lu 174.97 5d14f146s2
(Lanthanides)
90 Th 232.04 6d27s2
91 Pa 231.04 5f26d17s2
92 U 238.03 5f36d17s2
93 Np 237.05 5f46d17s2
94 Pu (244) 5f66d07s2
95 Am (243) 5f76d07s2
96 Cm (247) 5f76d17s2
97 Bk (247) 5f86d17s2
98 Cf (251) 5f106d07s2
99 Es (252) 5s116d07s2
100 Fm (257) 5f126d07s2
101 Md (258) 5f136d07s2
102 No (259) 5f146d07s2
103 Lr (260) 5f146d17s2
(Actinides)

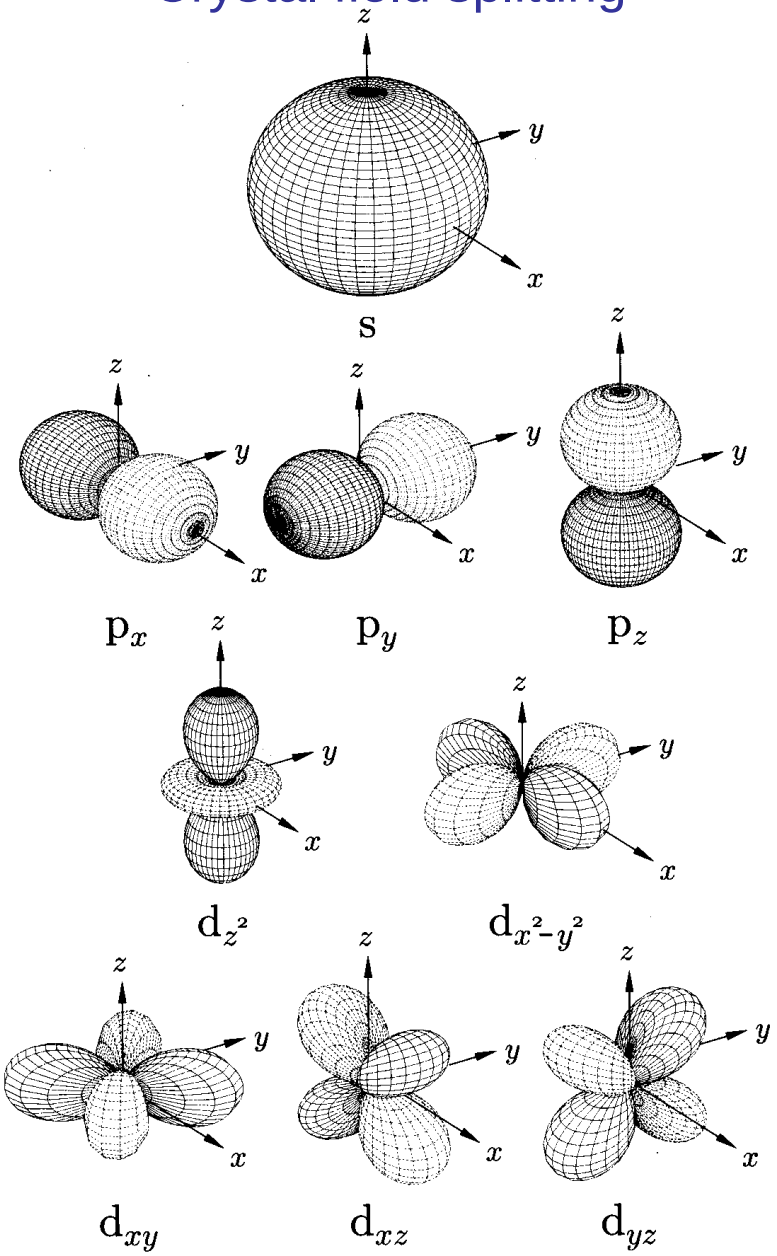
3d-shell (transition metal ions)

ELEMENT (AND IONIZATION)	BASIC ELECTRON CONFIGURATION	GROUND-STATE TERM	CALCULATED[b] p ($J = S$)	CALCULATED[b] p ($J = \lvert L \pm S \rvert$)	MEASURED[c] p
Ti^{3+}	$3d^1$	$^2D_{3/2}$		1.55	—
V^{4+}	$3d^1$	$^2D_{3/2}$		1.55	1.8
V^{3+}	$3d^2$	$^3F_2$		1.63	2.8
V^{2+}	$3d^3$	$^4F_{3/2}$		0.77	3.8
Cr^{3+}	$3d^3$	$^4F_{3/2}$		0.77	3.7
Mn^{4+}	$3d^3$	$^4F_{3/2}$		0.77	4.0
Cr^{2+}	$3d^4$	$^5D_0$		0	4.8
Mn^{3+}	$3d^4$	$^5D_0$		0	5.0
Mn^{2+}	$3d^5$	$^6S_{5/2}$		5.92	5.9
Fe^{3+}	$3d^5$	$^6S_{5/2}$		5.92	5.9
Fe^{2+}	$3d^6$	$^5D_4$		6.70	5.4
Co^{2+}	$3d^7$	$^4F_{9/2}$		6.54	4.8
Ni^{2+}	$3d^8$	$^3F_4$		5.59	3.2
Cu^{2+}	$3d^9$	$^2D_{5/2}$		3.55	1.9

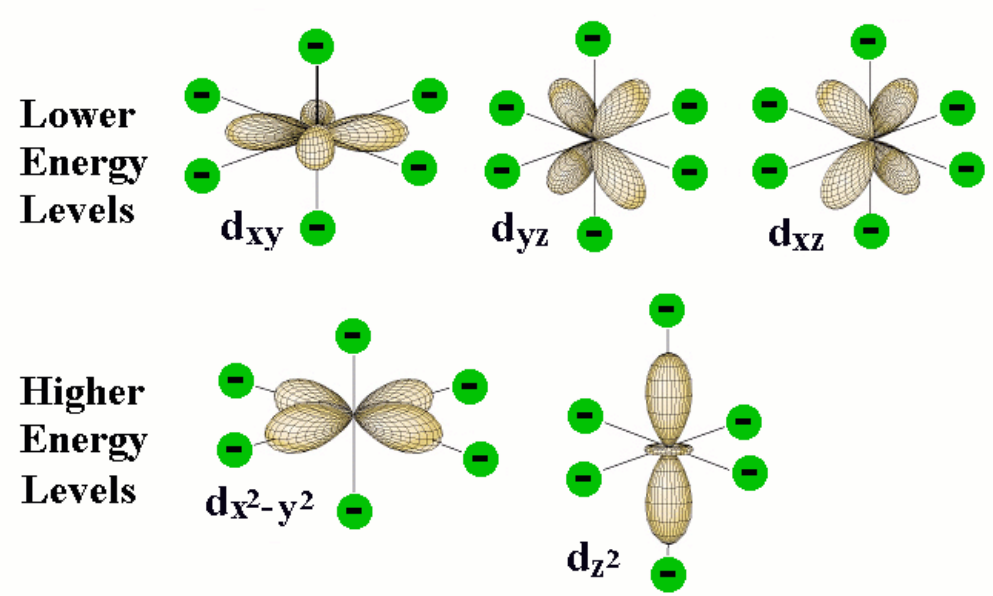
?

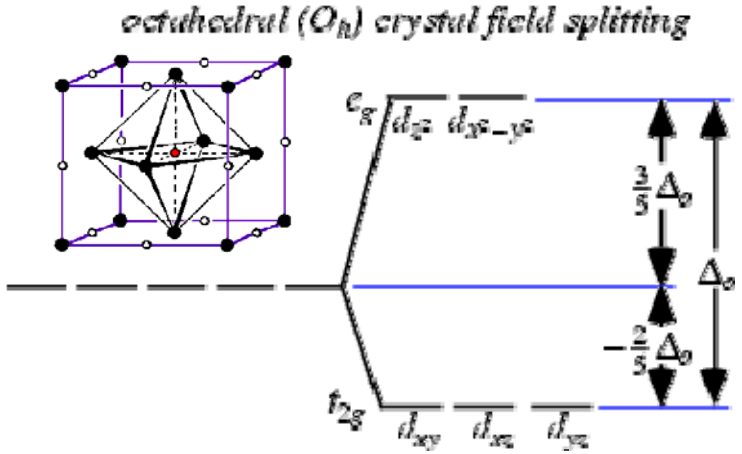
- Curie's law is still good, but *p* is mostly wrong
- Much better improvement if we let *J*=*S*

Crystal field splitting



In a crystal, crystal field may be more important than the LS coupling





- Different symmetries would have different splitting patterns.

淬滅

Quench of orbital angular momentum

- Due to crystal field, energy levels are now labeled by *L* (not *J*)
- Orbital degeneracy not lifted by crystal field may be lifted by 1) LS coupling, or 2) Jahn-Teller effect, or both.

Spontaneous lattice distortion

- The stationary state ϕ of a non-degenerate level can be chosen as real

when $t \to -t$,

$\psi \to \psi^*$ ($= \psi$ if nondegenerate)

- $\langle \psi | \vec{L} | \psi \rangle = \langle \psi | \vec{r} \times \frac{\hbar}{i} \nabla | \psi \rangle$ is purely imaginary

but $\langle \psi | \vec{L} | \psi \rangle$ has to be real also

$\therefore \langle \psi | \vec{L} | \psi \rangle = 0$

$\left(\langle \psi | L^2 | \psi \rangle \right.$ can still be non-zero$\left. \right)$

- for 3d ions, crystal field > SO interaction
- for 4f ions, SO interaction > crystal field (because 4f is hidden inside 5p and 6s shells)
- for 4d and 5d ions that have stronger SO interaction, the 2 energies maybe comparable and it's more complicated.

- Langevin diamagnetism
- paramagnetism
    - Hund’s rules
    - Lande g-factor
    - Brillouin function
- crystal field splitting
    - quench of orbital angular momentum
- nuclear demagnetization
- Pauli paramagnetism and Landau diamagnetism

Adiabatic demagnetization (proposed by Debye, 1926)

- The first method to reach below 1K

$$Z = \sum_{m_J=-J}^{J} e^{-E(m_J)/kT}, \quad \text{assume } E(m_J) \propto H$$

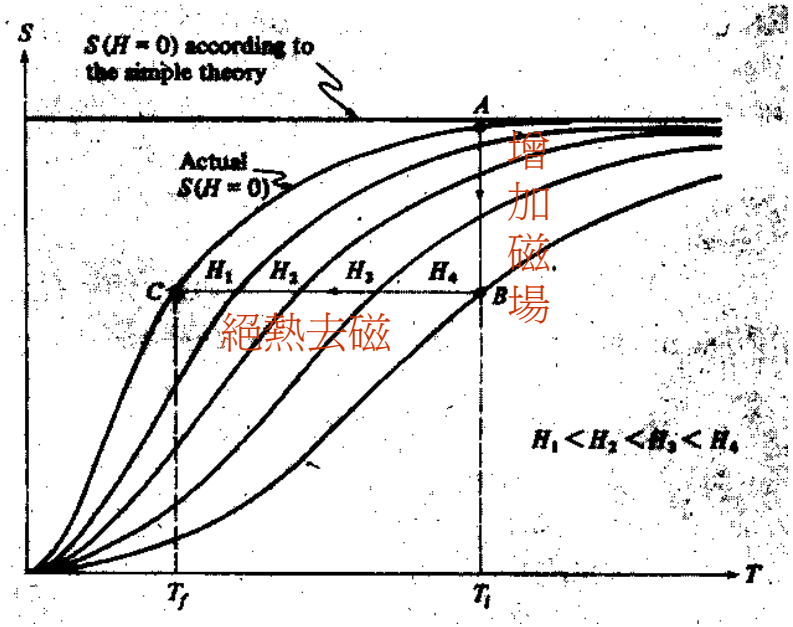
$$F = -kT \ln Z\left(\frac{H}{kT}\right)$$

$$S = -\frac{\partial F}{\partial T} = S\left(\frac{H}{kT}\right)$$

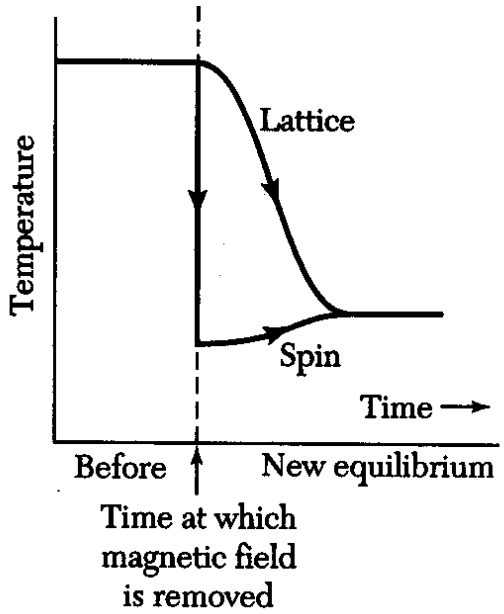
- If S=constant, then $kT \sim H$ $\quad T_f = T_i \frac{H_f}{H_i}$

$\therefore$ We can reduce H to reduce T

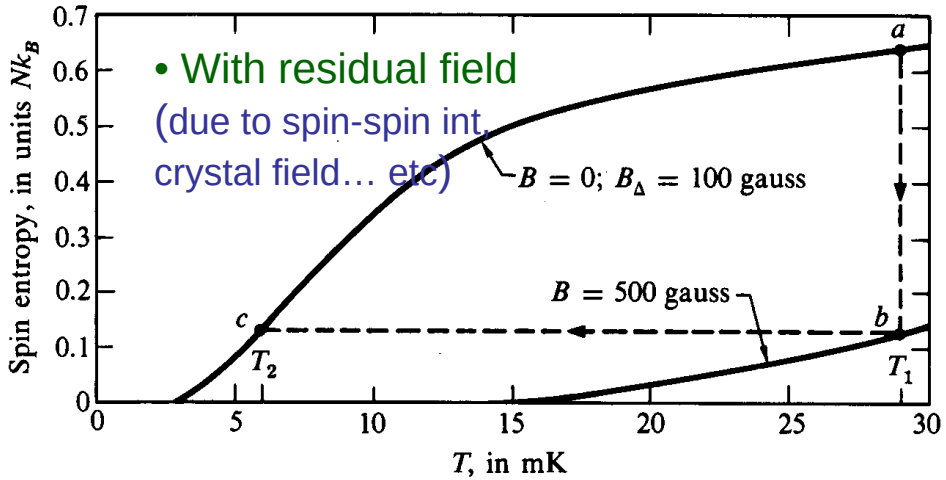
- Without residual field



Freezing is effective only if spin specific heat is dominant (usually need $T \ll T_D$)

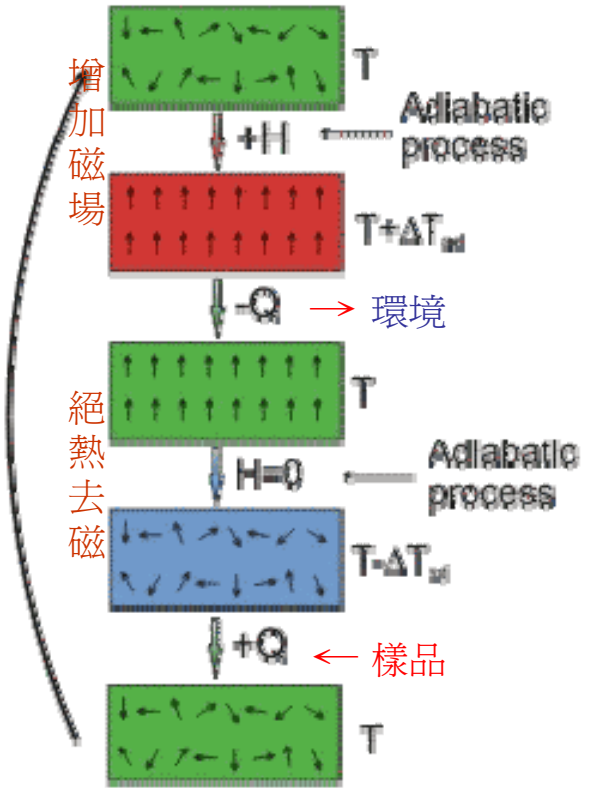


- With residual field (due to spin-spin int, crystal field… etc)

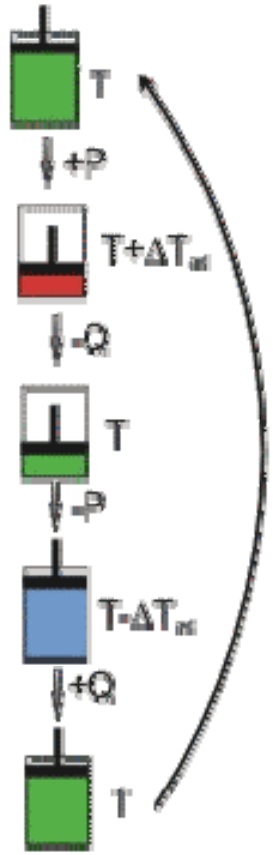


Can reach 10^{-6} K (dilution refrig only 10^{-3} K)

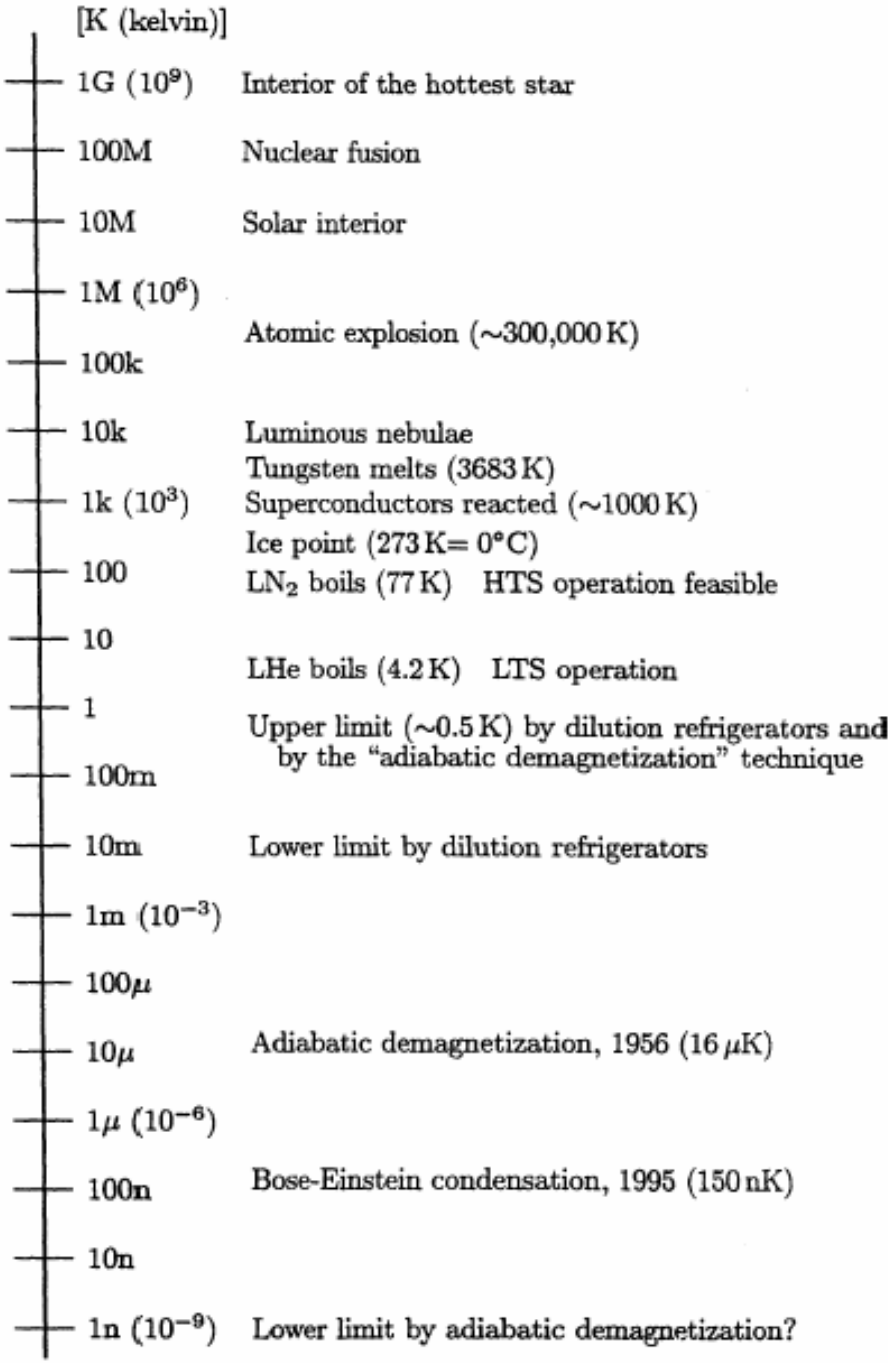
analogy



Magnetic refrigeration



Vapor cycle refrigeration



wikipedia

Pauli paramagnetism for free electron gas (1925)

- Orbital response to *H* neglected, consider only spin response
- One of the earliest application of the exclusion principle

$$N = N_\uparrow + N_\downarrow$$

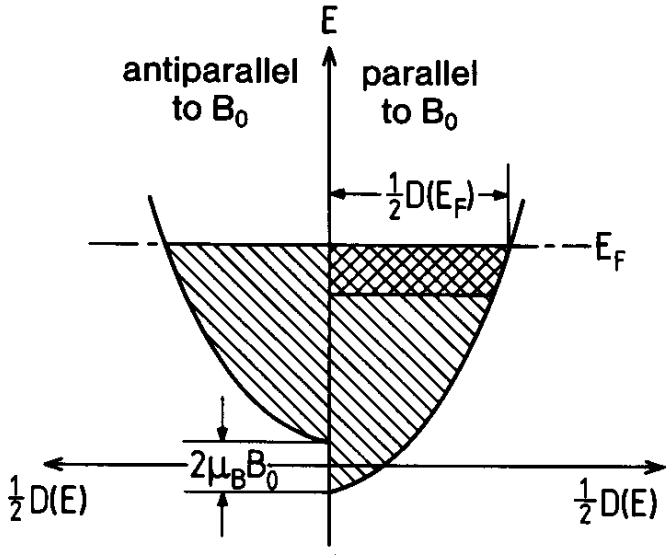
$$M = \frac{1}{V}\left(N_\uparrow - N_\downarrow\right)\mu_B$$

For $T << T_F$,

$$n_\uparrow - n_\downarrow \cong g(\varepsilon_F)\mu_B H, \quad g = g_\uparrow + g_\downarrow$$

$$\therefore M = g(\varepsilon_F)\mu_B^2 H$$

$$\Rightarrow \quad \chi_{Pauli} = g(\varepsilon_F)\mu_B^2 = \left(\frac{\alpha}{2\pi}\right)^2 \left(k_F a_0\right)^{1} \sim 10^{-6}$$



$$g(\varepsilon) = \frac{m}{\pi^2\hbar^3}\sqrt{2m\varepsilon} \qquad a_0 = \frac{\hbar^2}{me^2}$$

$$\mu_B = \frac{e\hbar}{2mc} \qquad \alpha = \frac{e^2}{\hbar c}$$

- unlike the PM of magnetic ions, here the magnitude $\sim$ DM's (supressed by Pauli exclusion principle)

Landau diamagnetism for free electron gas (1930)

Landau began his paper with the remark, "It has until now been more or less quietly assumed that the magnetic properties of electrons, other than spin, originate exclusively from the binding of electrons in atoms."

- The orbital response neglected earlier gives slight DM
- The calculation is not trivial. For free electron gas,

$$\chi_{Landau} = -\frac{e^2 k_F}{12\pi^2 mc^2}$$
$$= -\frac{1}{3}\chi_{Pauli}$$

- So far we have learned PM and DM for a free electron gas.

How do we separate these contributions in experiment?

X-ray magnetic circular dichroism (XMCD)

Hoddeson, Our of the crystal maze, p.126