Superconductivity

- Introduction
- Thermal properties
- Magnetic properties
- London theory of the Meissner effect
- Microscopic (BCS) theory
- Flux quantization
- Quantum tunneling

Dept of Phys

M.C. Chang

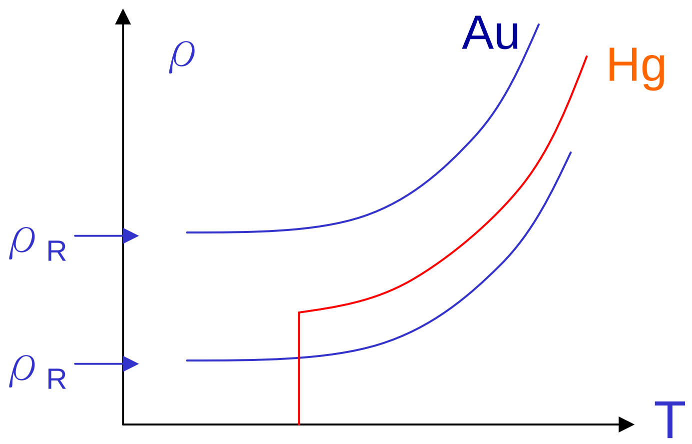
A brief history of low temperature (Ref: 絕對零度的探索)

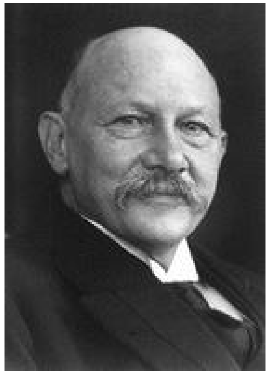
G. Amontons

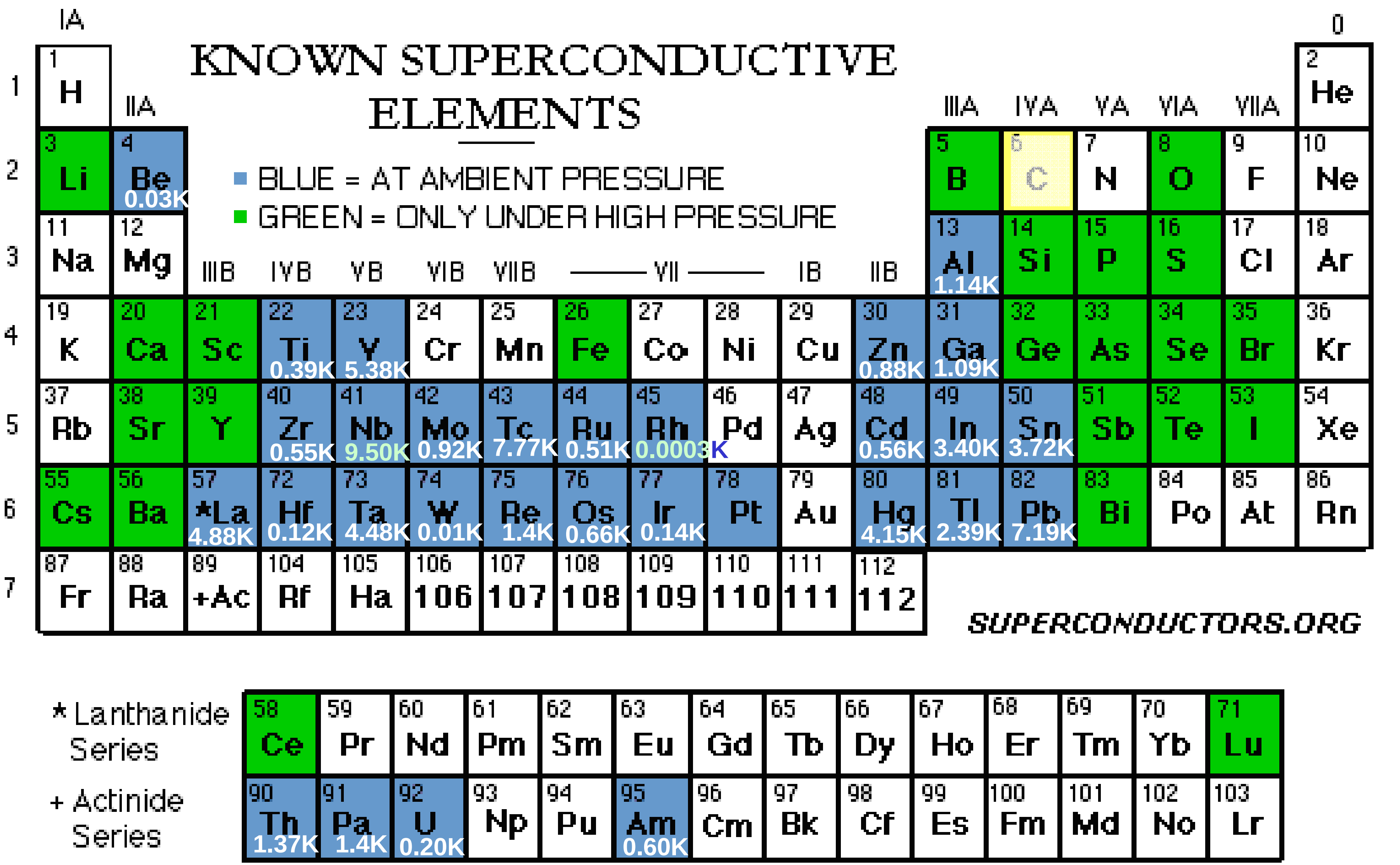
1700

- 1800 Charles and Gay-Lusac (from *P-T* relationship) proposed that the lowest temperature is -273 C (= 0 *K*)
- 1877 Cailletet and Pictet liquified Oxygen (-183 C or 90 *K*)
- soon after, Nitrogen (77 *K*) is liquified
- 1898 Dewar liquified Hydrogen (20 *K*)
- 1908 Onnes liquified Helium (4.2 *K*)

- 1911 Onnes measured the resistance of metal at such a low *T*. To remove residual resistance, he chose mercury. Near 4 *K*, the resistance drops to 0.



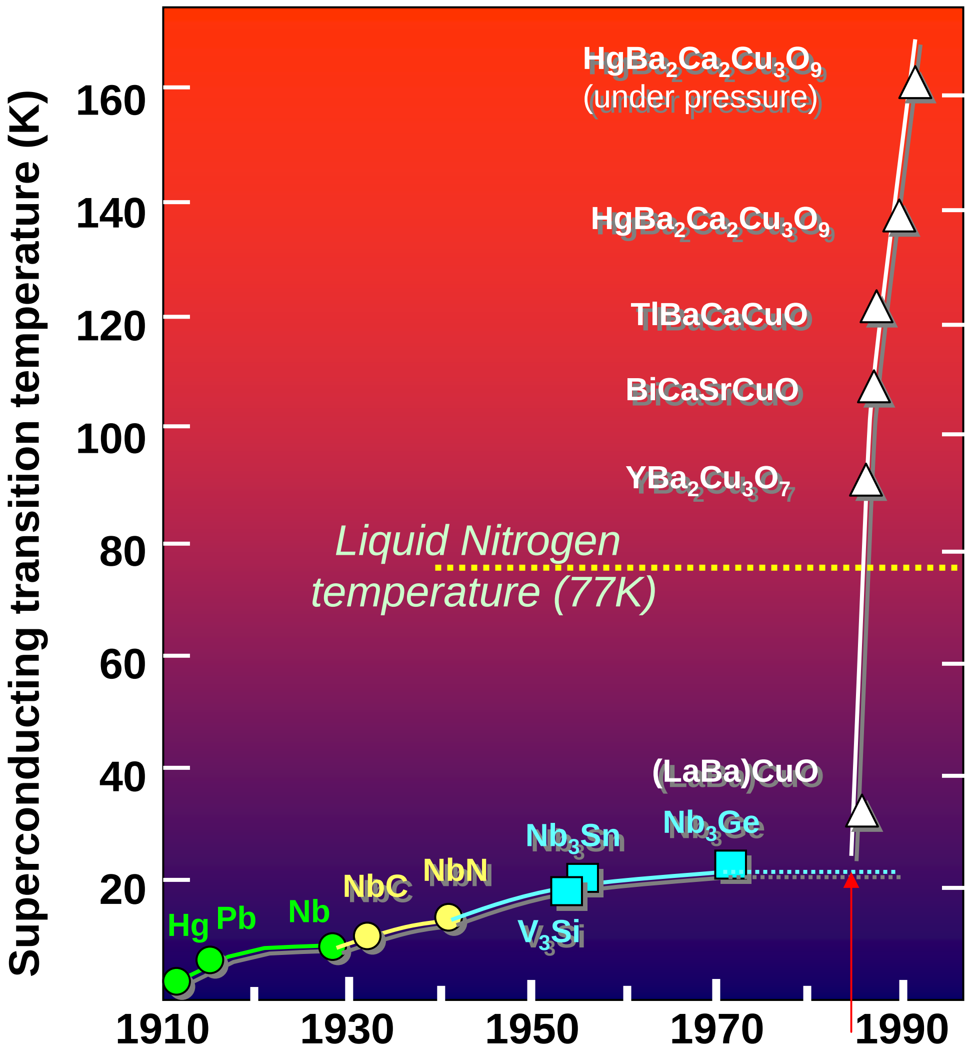




Tc's given are for bulk, except for Palladium, which has been irradiated with He+ ions, Chromium as a thin film, and Platinum as a compacted powder

http://superconductors.org/Type1.htm

Superconductivity in alloys and oxides



Bednorz
Muller
1987

From Cywinski's lecture note

Applications of superconductor

- powerful magnet
    - MRI, LHC...
- magnetic levitation
- SQUID (超導量子干涉儀)
    - detect tiny magnetic field
- quantum bits
- lossless powerline
- …

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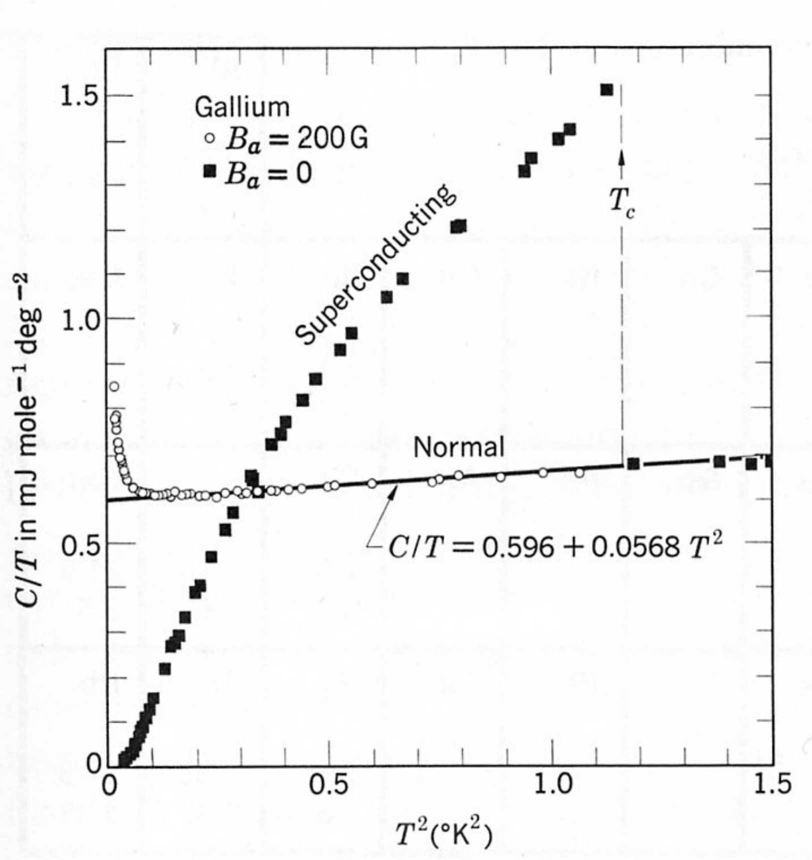
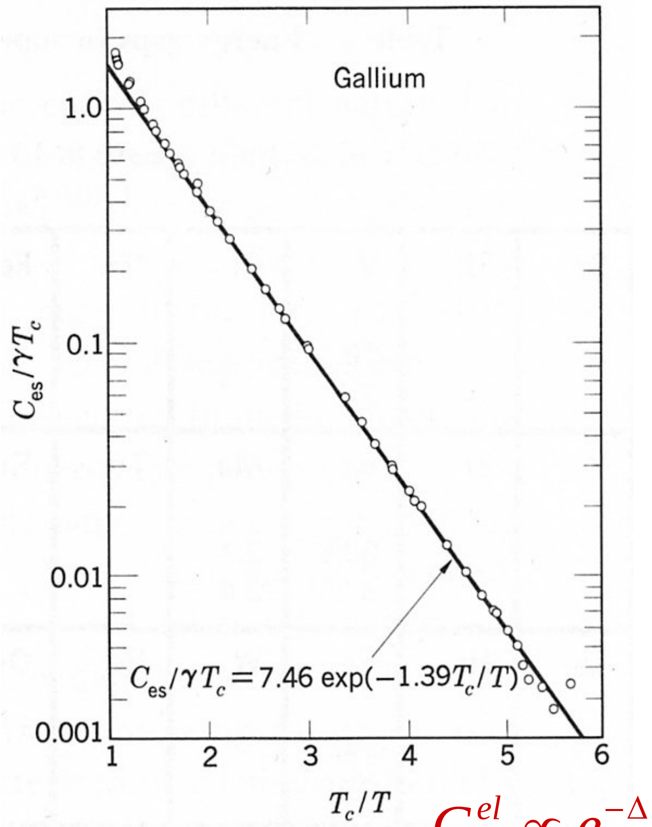
Thermal properties of SC: specific heat





$$C_S^{el} \propto e^{-\Delta(0)/k_B T}$$

For different superconductors,

$$\frac{C_S - C_N}{C_N} \sim 1.43 \text{ at } T_C$$

The exponential dependence with *T* is called "activation" behavior and implies the existence of an energy gap *above Fermi surface.*

Δ ~ *0.1-1 meV* ($10^{-4\sim-5}$ E_F)

• Connection between energy gap and T_c

MEASURED VALUES[a] OF $2\Delta(0)/k_B T_c$

ELEMENT	$2\Delta(0)/k_B T_c$
Al	3.4
Cd	3.2
Hg (α)	4.6
In	3.6
Nb	3.8
Pb	4.3
Sn	3.5
Ta	3.6
Tl	3.6
V	3.4
Zn	3.2

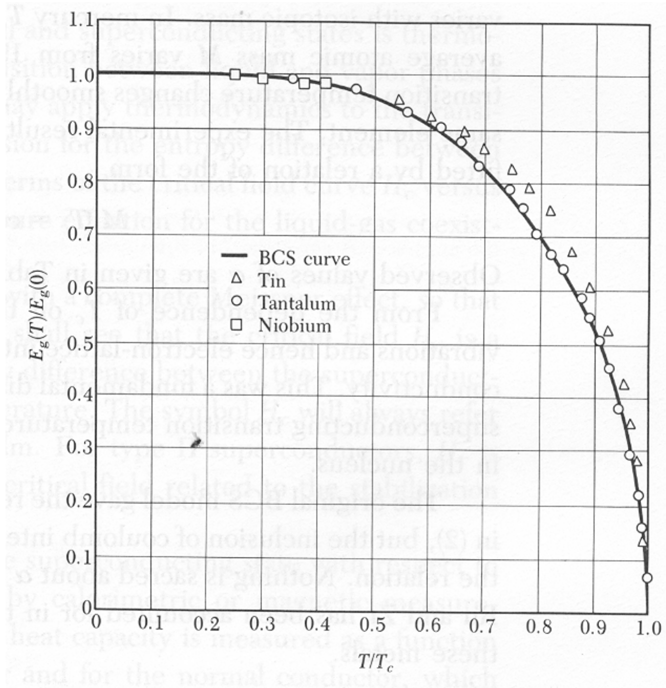
[a] $\Delta(0)$ is taken from tunneling experiments. Note that the BCS value for this ratio is 3.53. Most of the values listed have an uncertainty of ± 0.1.

Δ's scale with different T_c's

$2\Delta(0) \sim 3.5\, k_B T_c$

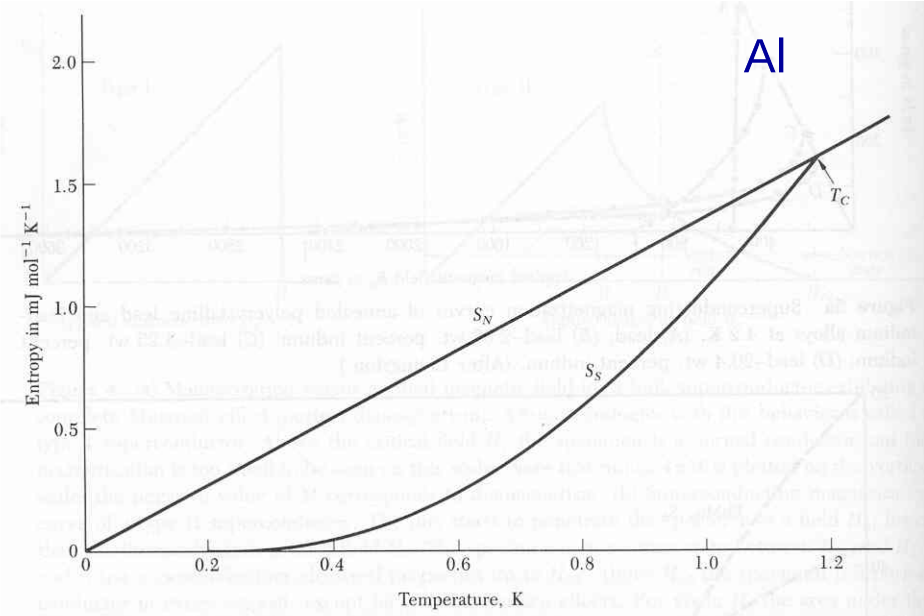
• Temperature dependence of Δ (obtained from Tunneling)

Universal behavior of $\Delta(T)$



$$\frac{\Delta(T)}{\Delta(0)} = 1.74\left(1 - \frac{T}{T_C}\right)^{1/2} \quad \text{for } T \approx T_C$$

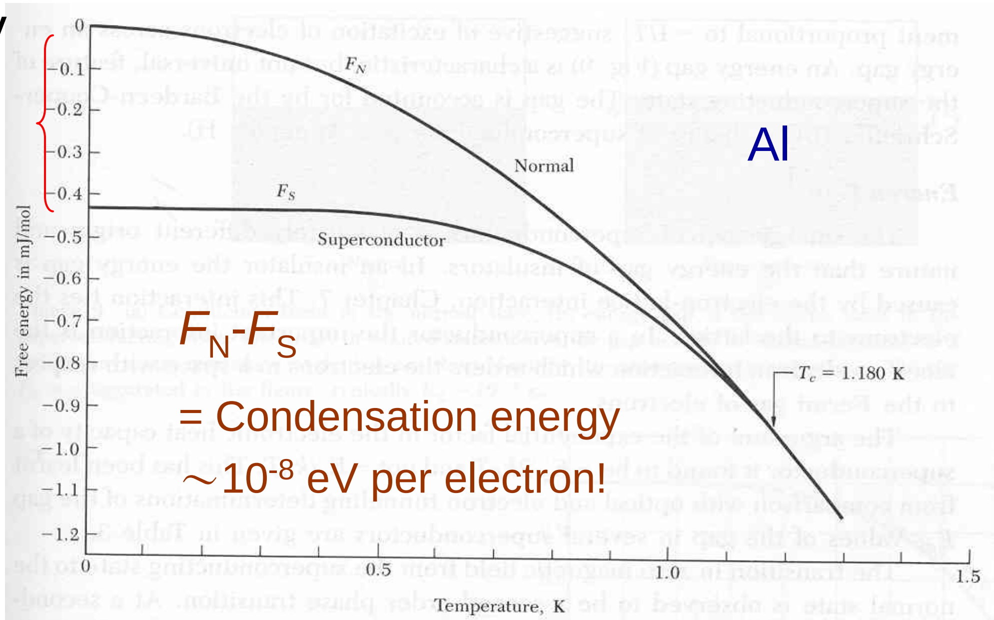
- Entropy



$$C = T\left(\frac{\partial S}{\partial T}\right)_H$$

Less entropy in SC state: more ordering

- free energy



F_N-F_S

= Condensation energy

$\sim 10^{-8}$ eV per electron!

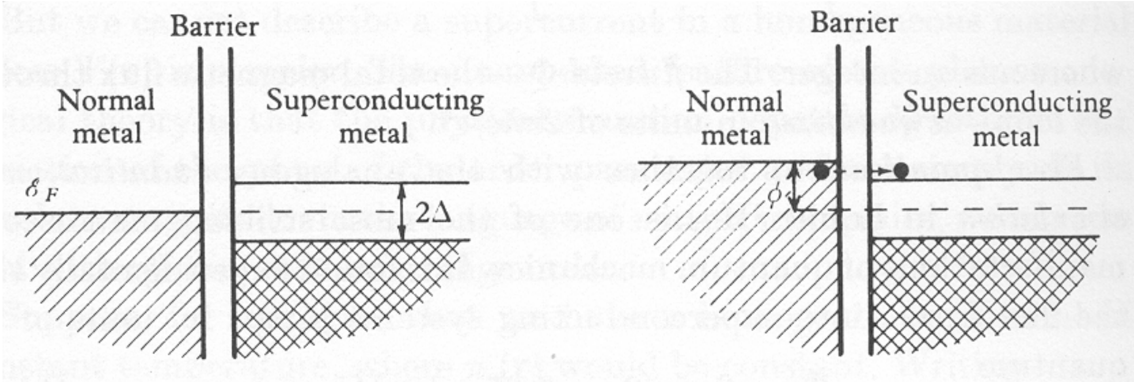
$$S = -\left(\frac{\partial F}{\partial T}\right)_H$$

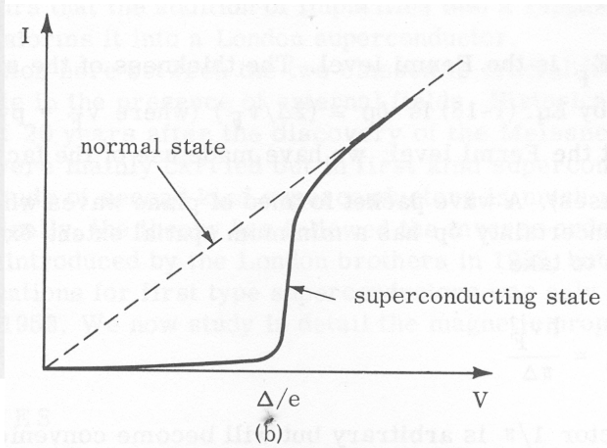
$$\frac{C}{T} = \left(\frac{\partial S}{\partial T}\right)_H = -\left(\frac{\partial^2 F}{\partial T^2}\right)_H$$

2nd order phase transition

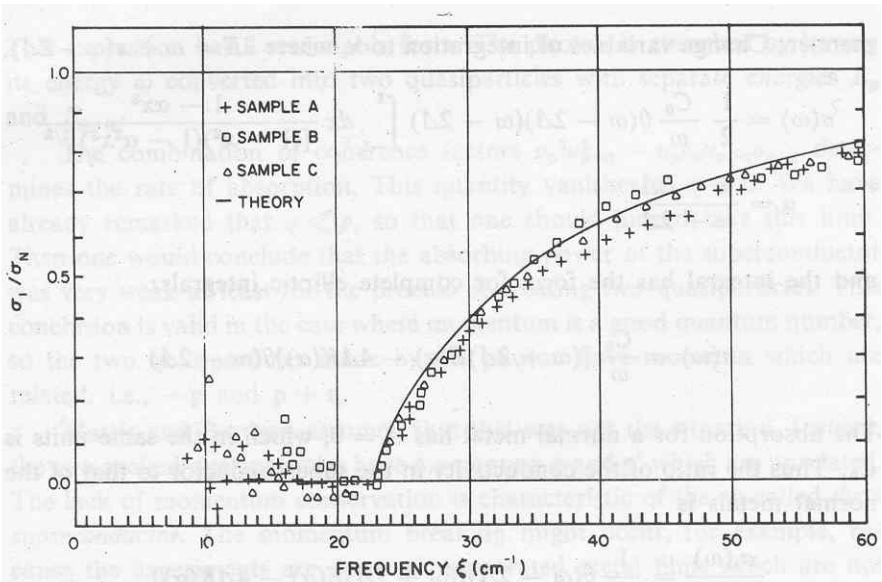
More evidences of energy gap

- Electron tunneling





- EM wave absorption



2Δ suggests excitations created in "e-h" pairs

$$\nu = \frac{2\Delta}{h} = 480\ \text{GHz (microwave)}$$

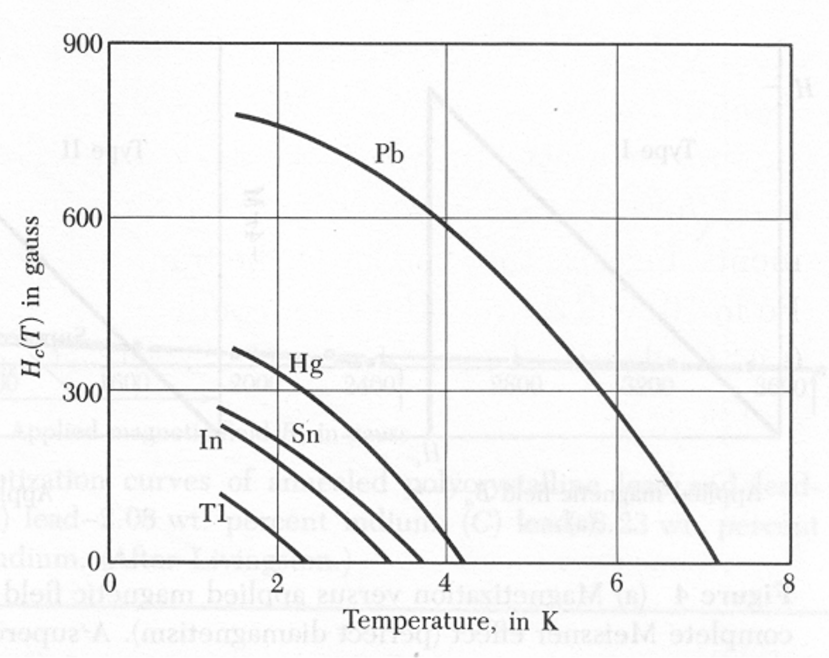
Magnetic property of the superconductor

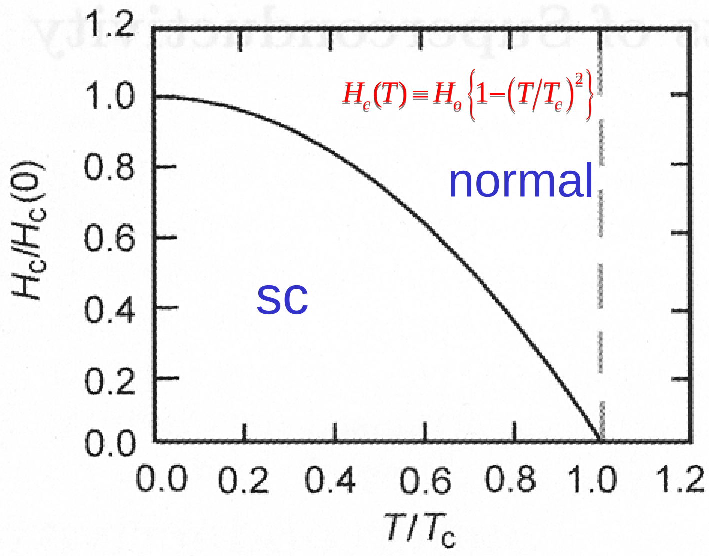
• Superconductivity is destroyed by a strong magnetic field.

H_c for metal is of the order of 0.1 Tesla or less.

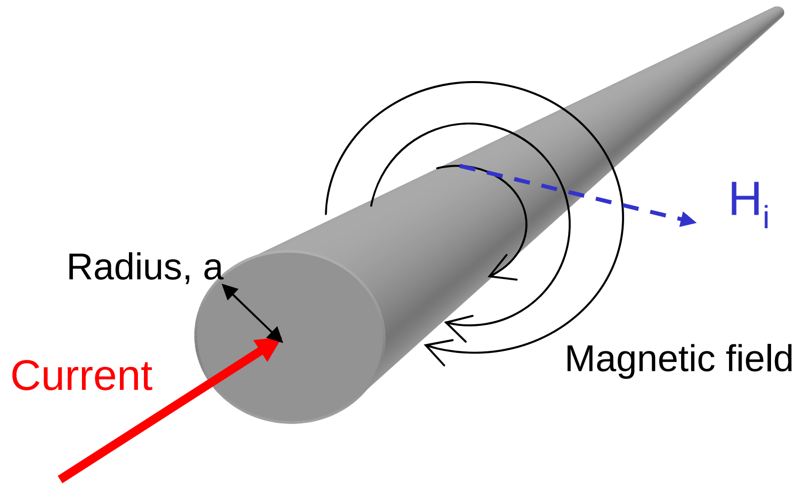
• Temperature dependence of $H_c(T)$

All curves can be collapsed onto a similar curve after re-scaling.





Critical currents (no applied field)



$$\oint \vec{H}\cdot d\vec{\ell} = \frac{4\pi}{c} i \quad \text{so} \quad i_c = \frac{ca}{2} H_c$$

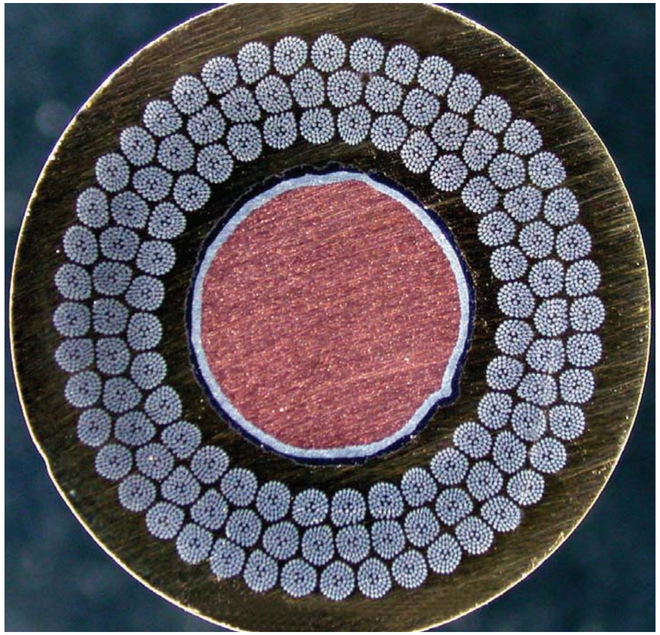
The critical current density of a long thin wire is therefore

$$j_c = \frac{cH_c}{2\pi a}$$ (thinner wire has larger J_c)

$j_c \sim 10^8$ A/cm^2 for H_c=500 Oe, a=500 A

- J_c has a similar temperature dependence as H_c, and T_c is similarly lowered as J increases.

From Cywinski's lecture note

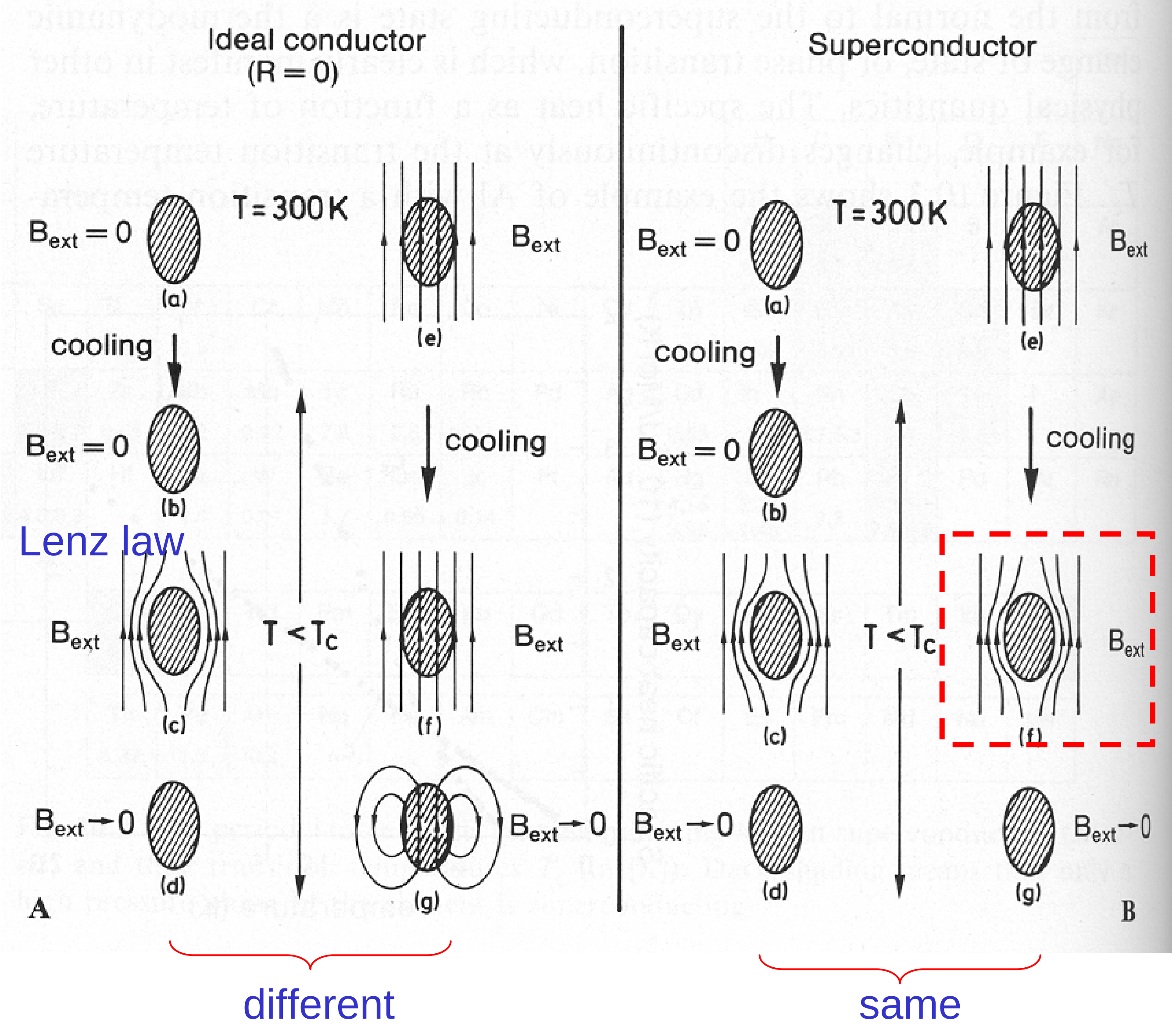


Cross-section through a niobium–tin cable

Phys World, Apr 2011

Meissner effect (Meissner and Ochsenfeld, 1933)

A SC is more than a perfect conductor



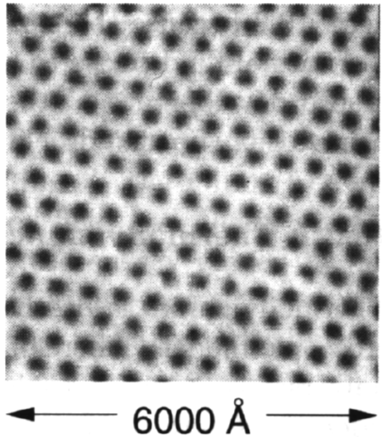
not only $dB/dt=0$
but also $B=0$!

Perfect
diamagnetism

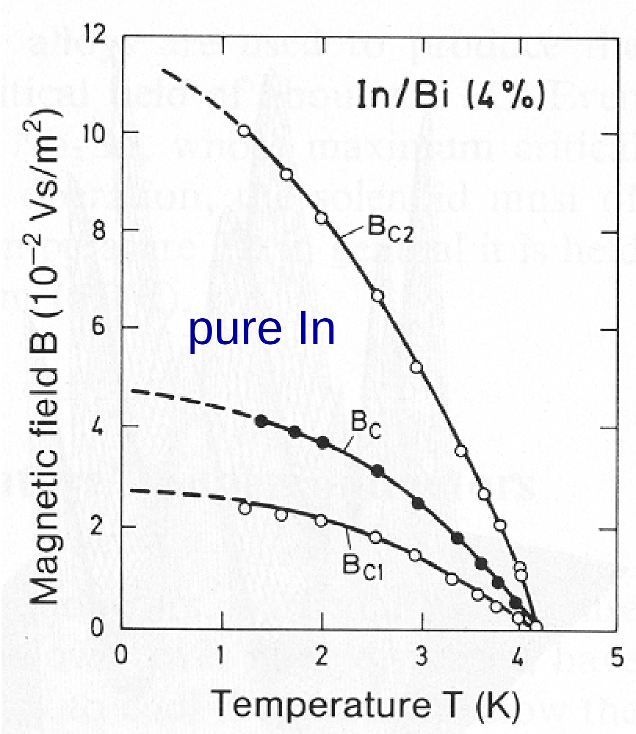
Superconducting alloy: type II SC

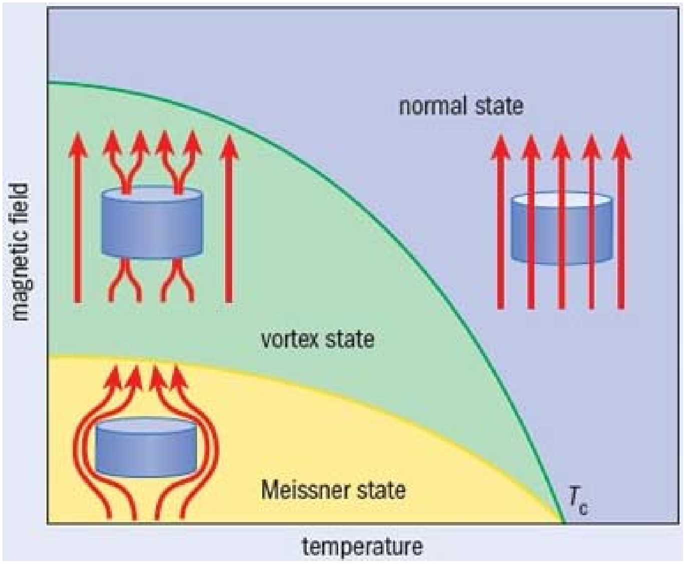
partial exclusion and remains superconducting at high *B* (1935)

(also called intermediate/mixed/vortex/Shubnikov state)



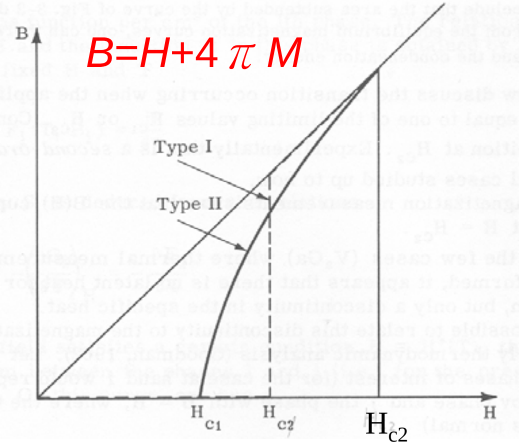
STM image
$NbSe_2$, 1T, 1.8K

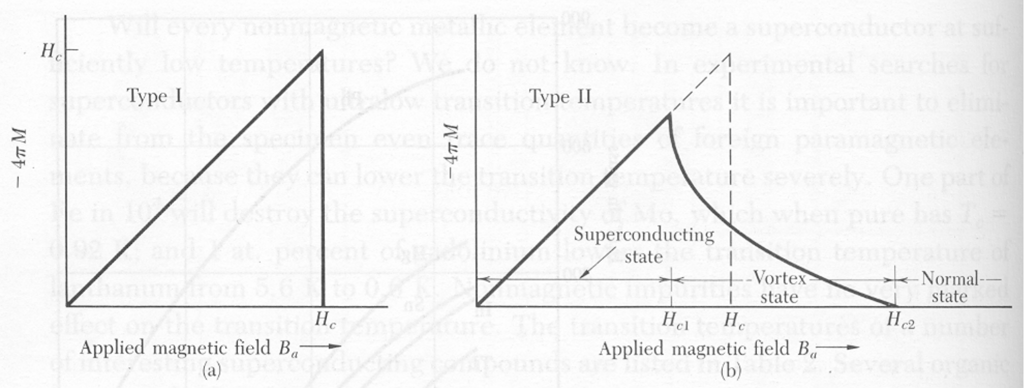


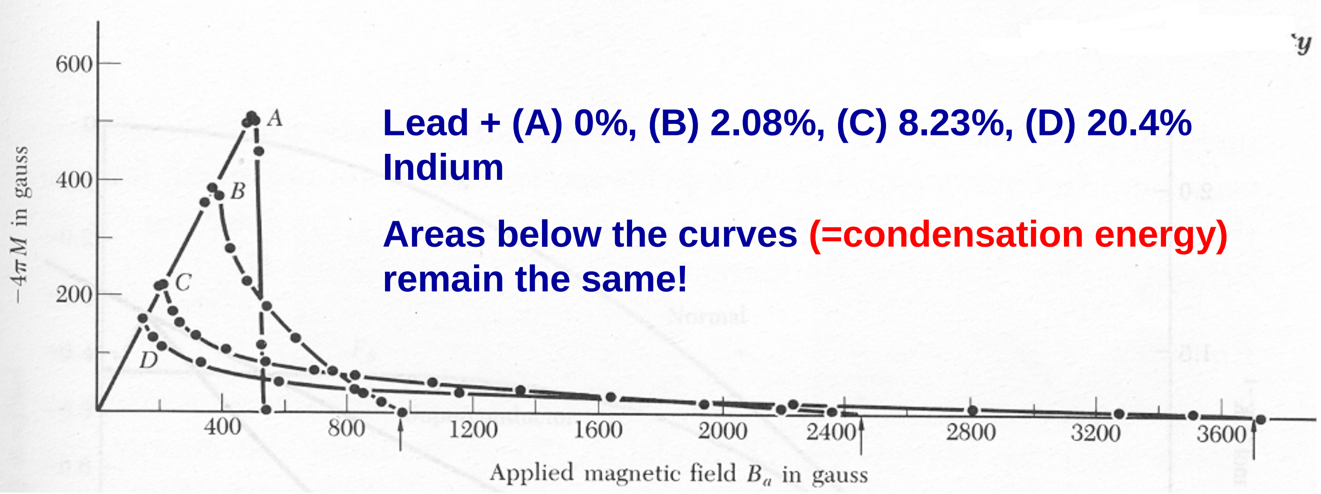


• H_{C2} is of the order of 10~100 Tesla (called hard, or type II, superconductor)

Comparison between type I and type II superconductors







Lead + (A) 0%, (B) 2.08%, (C) 8.23%, (D) 20.4% Indium

Areas below the curves (=condensation energy) remain the same!

Condensation energy (**for type I**)

$\left(\vec{H} \text{ is } \vec{B}_a \text{ in Kittel}\right)$

$$dF = -\vec{M} \cdot d\vec{H}$$

$$\text{For a SC, } dF_S = \frac{1}{4\pi} H dH$$

$$\rightarrow F_S(H) - F_S(0) = \frac{H^2}{8\pi}$$

$$F_N(H_c) = F_S(H_c)$$

$$F_N(H_c) = F_N(0) \ \text{ for nonmagnetic material}$$

$$\therefore \Delta F = F_N(0) - F_S(0) = \frac{H_c^2}{8\pi}$$

(Magnetic energy density)

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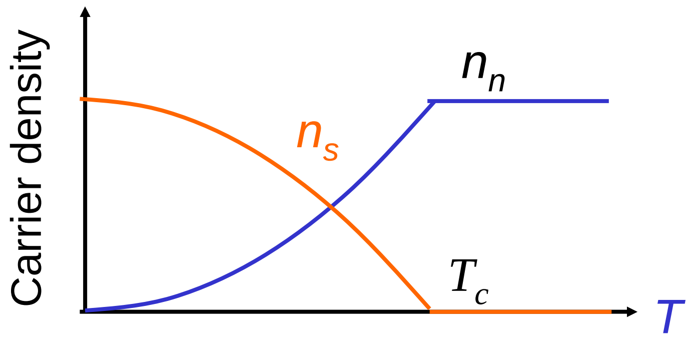
London theory of the Meissner effect (Fritz London and Heinz London, 1934)



Two-fluid model:

$$n_s + n_n = n = \text{constant}$$

- Superfluid density n_s $\sigma = \infty$
- Normal fluid density n_n $\sigma_n = \frac{n_n e^2 \tau}{m}$

Assume

(1) $\frac{d\vec{J}_s}{dt} = \frac{n_s e^2 \vec{E}}{m}$ like free charges

(2) $\vec{J}_n = \sigma_n \vec{E}$

where

$$\vec{J}_s = -en_s\vec{v}_s$$

$$\vec{J}_n = -en_n\vec{v}_n$$

$$\text{Eq.(1)} + \vec{\nabla}\times\vec{E} = -\frac{1}{c}\frac{\partial\vec{B}}{\partial t}$$

$$\Rightarrow \frac{d}{dt}\left(\vec{\nabla}\times\vec{J}_s\right) = -\frac{n_s e^2}{mc}\frac{\partial\vec{B}}{\partial t}$$

London proposed

$$\nabla\times\vec{J}_s = -\frac{n_s e^2}{mc}\vec{B}$$

$$\Rightarrow \vec{J}_s = -\frac{n_s e^2}{mc}\vec{A} + \nabla\phi$$

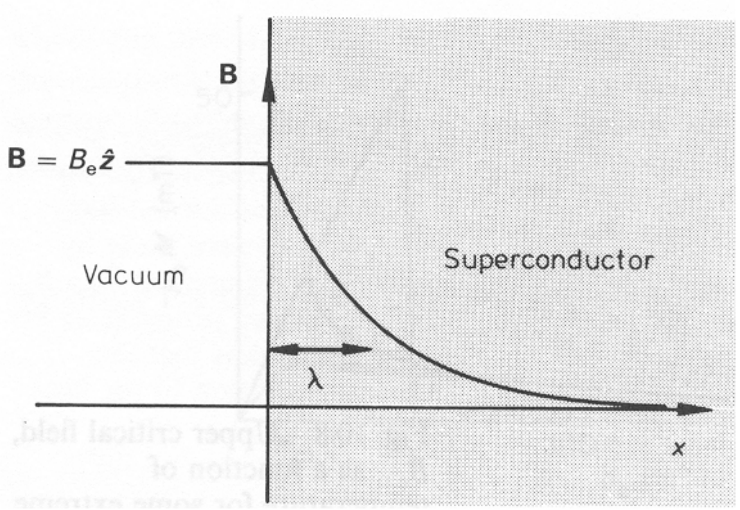
It can be shown that $\nabla\phi$ =0 for simply connected sample (See Schrieffer)

use $\vec{\nabla}\times\vec{B} = \frac{4\pi}{c}\vec{J}_s$ and

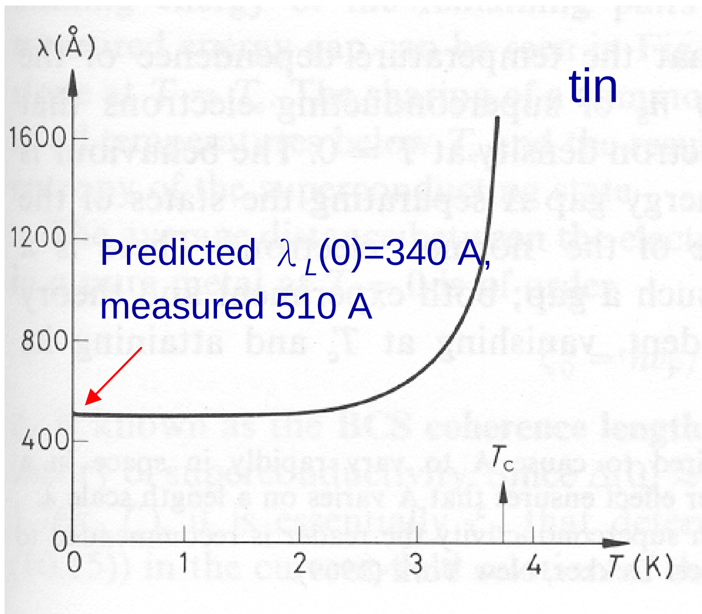
$\nabla\times\left(\nabla\times\vec{v}\right) = \nabla\left(\nabla\cdot\vec{v}\right) - \nabla^2\vec{v}$

$$\Rightarrow \nabla^2\vec{B} = \frac{4\pi\, n_s e^2}{mc^2}\vec{B} \equiv \frac{\vec{B}}{\lambda_L^2}$$

- Penetration length λ_L



- Temperature dependence of λ_L



Outside the SC, ***B***=*B*(*x*) *z*

$$\lambda_L^2 \frac{d^2 B}{dx^2} = B$$

$$\rightarrow B(x) = B_0 e^{-x/\lambda_L}$$

(expulsion of magnetic field)

$$\lambda_L = \sqrt{\frac{mc^2}{4\pi n_S e^2}} \approx 170A \text{ if } n_S = 10^{23}/\text{cm}^3$$

$$\vec{\nabla} \times \vec{B} = \frac{4\pi}{c} \vec{J}_s$$

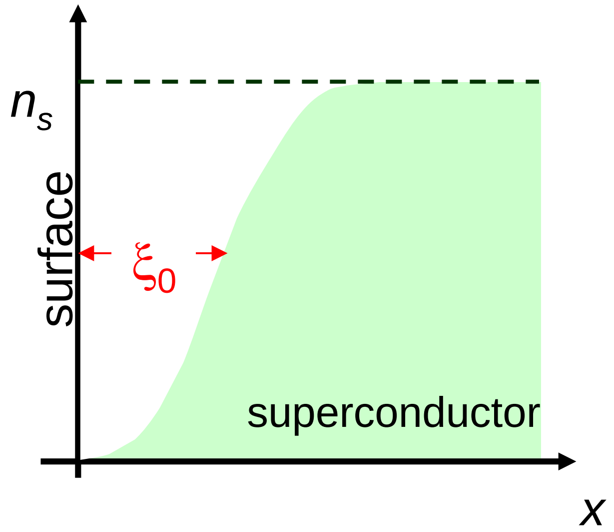
$$\therefore J_{sy} = -\frac{c}{4\pi} \frac{dB}{dx} = \frac{cB_0}{4\pi\lambda_L} e^{-x/\lambda_L}$$

also decays

- Higher T, smaller n_S

$$\lambda(T) = \frac{\lambda(0)}{\left[1 - \left(T/T_C\right)^4\right]^{1/2}}$$

Coherence length ξ_0 (Pippard, 1939)



• In fact, n_s cannot remain uniform near a surface. The length it takes for n_s to drop from full value to *0* is called ξ_0

• Microscopically it's related to the range of the Cooper pair.

• The pair wave function (with range ξ_0) is a superposition of one-electron states with energies within Δ of E_F (A+M, p.742).

Energy uncertainty of a Cooper pair

$$\frac{p\Delta p}{m} \approx \Delta$$

• Therefore, the spatial range of the variation of n_s

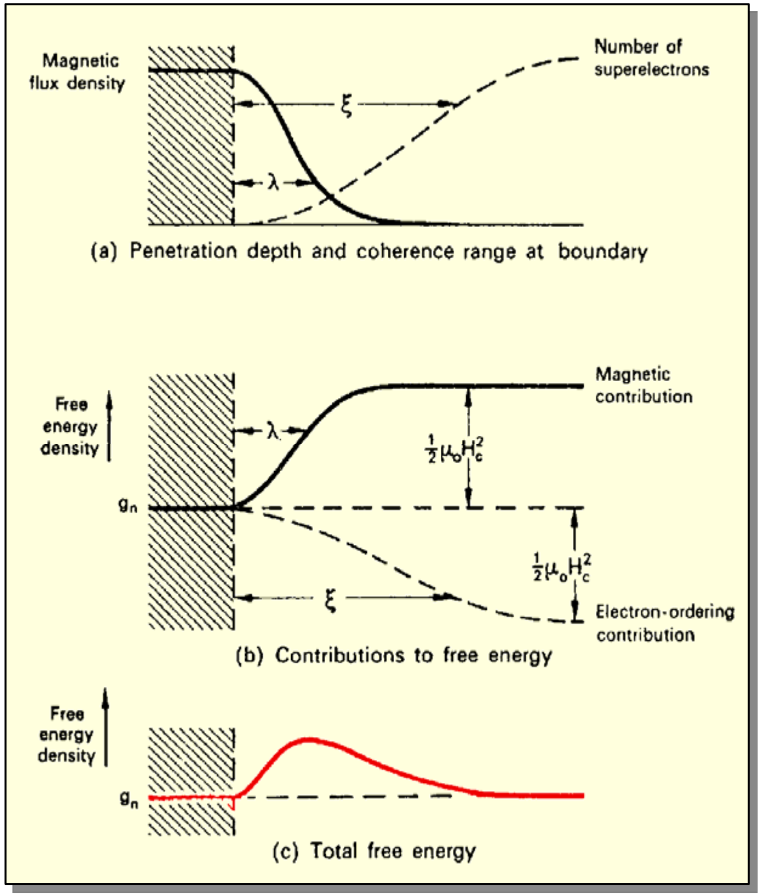
$$\xi_0 \approx \frac{\hbar}{\Delta p} = \frac{\hbar v_F}{\Delta} \quad \leftrightarrow \quad \frac{\hbar v_F}{\pi \Delta} \text{ from BCS theory}$$

$\xi_0 \sim 1\ \mu\text{m} >> \lambda$ for type I SC

Penetration depth, correlation length, and surface energy

Type I superconductivity

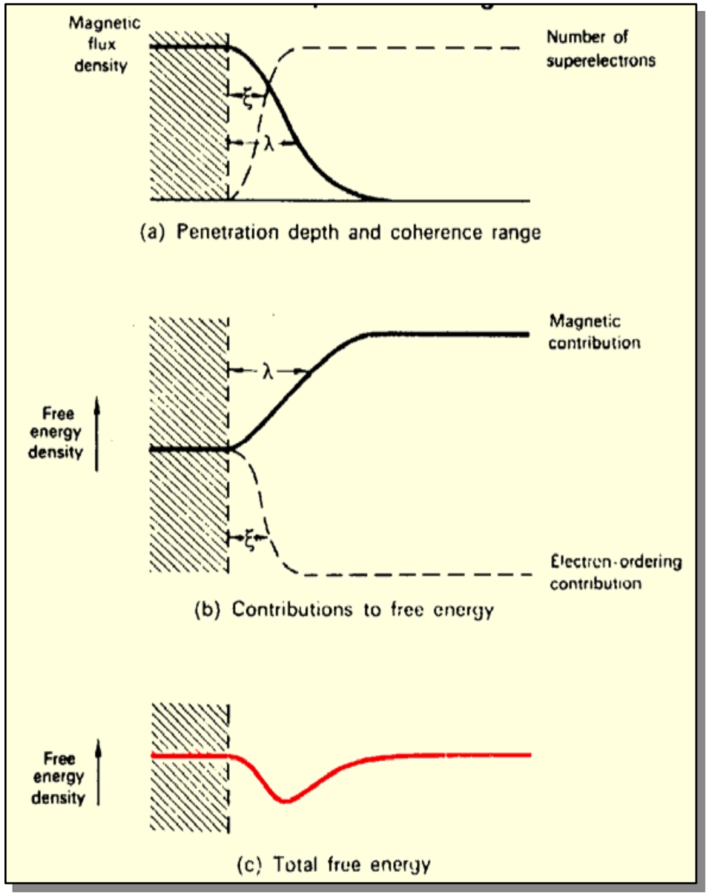
- $\xi_0 > \lambda$, surface energy is positive



- **smaller λ, cost more energy to expel the magnetic field.**
- **When $\xi_0 \gg \lambda$ (type I), there is a net positive surface energy. Difficult to create an interface.**

Type II superconductivity

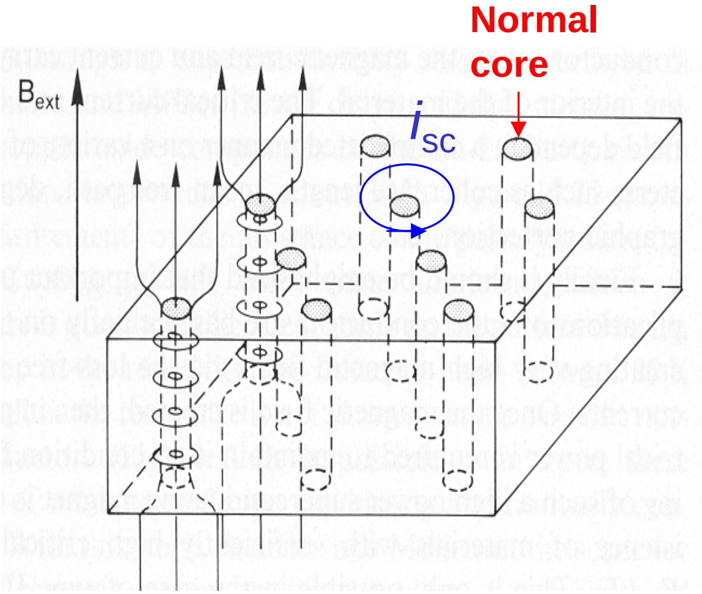
- $\xi_0 < \lambda$, surface energy is negative



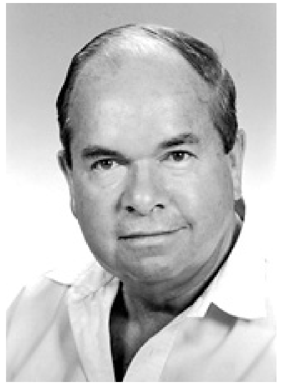
- **smaller ξ_0, get more "negative" condensation energy.**
- **When $\xi_0 \ll \lambda$ (type II), the surface energy is negative. Interface may spontaneously appear.**

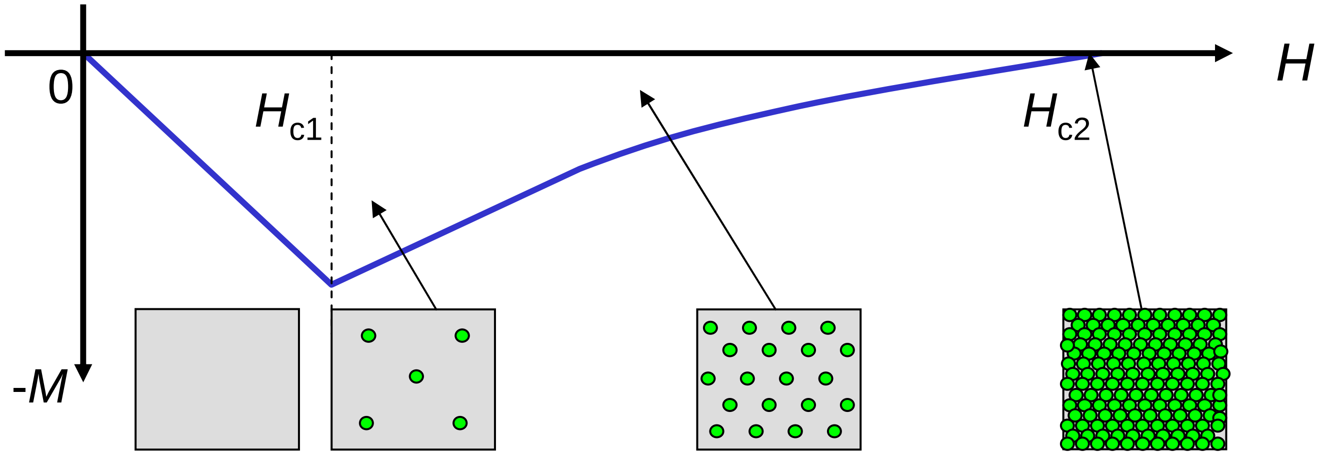
From Cywinski's lecture note

Vortex state of type II superconductor (Abrikosov, 1957)



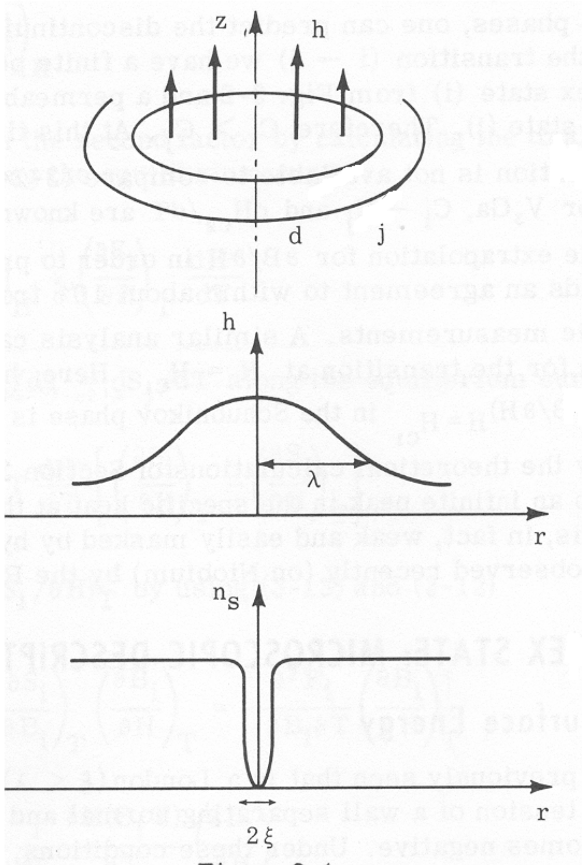
- the magnetic flux ϕ in a vortex is always quantized (discussed later).
- the vortices repel each other slightly.
- the vortices prefer to form a triangular lattice (Abrikosov lattice).
- the vortices can move and dissipate energy (unless pinned by impurity $\leftarrow$ Flux pinning)





From Cywinski's lecture note

Estimation of Hc_1 and Hc_2 (type II)



• Near H_{c1}, there begins with a single vortex with flux quantum ϕ_0, therefore

$$\pi\lambda^2 H_{c1} \approx \phi_0 \rightarrow H_{c1} \approx \frac{\phi_0}{\pi\lambda^2}$$

• Near H_{c2}, vortex are as closely packed as the coherence length allows, therefore

$$N\pi\xi_0^{\ 2} H_{c2} \approx N\phi_0 \rightarrow H_{c2} \approx \frac{\phi_0}{\pi\xi_0^{\ 2}}$$

Therefore, $\dfrac{H_{c2}}{H_{c1}} \approx \left(\dfrac{\lambda}{\xi_0}\right)^2$

Typical values, for Nb_3Sn,

$\xi_0 \sim 34$ A, $\lambda_L \sim 1600$ A

Origin of superconductivity?

• Metal X can (cannot) superconduct because its atoms can (cannot) superconduct?

Neither Au nor Bi is superconductor, but alloy Au_2Bi is!

White tin can, grey tin cannot! (the only difference is lattice structure)

• good normal conductors (Cu, Ag, Au) are bad superconductor;

bad normal conductors are good superconductors, why?

• What leads to the superconducting gap?

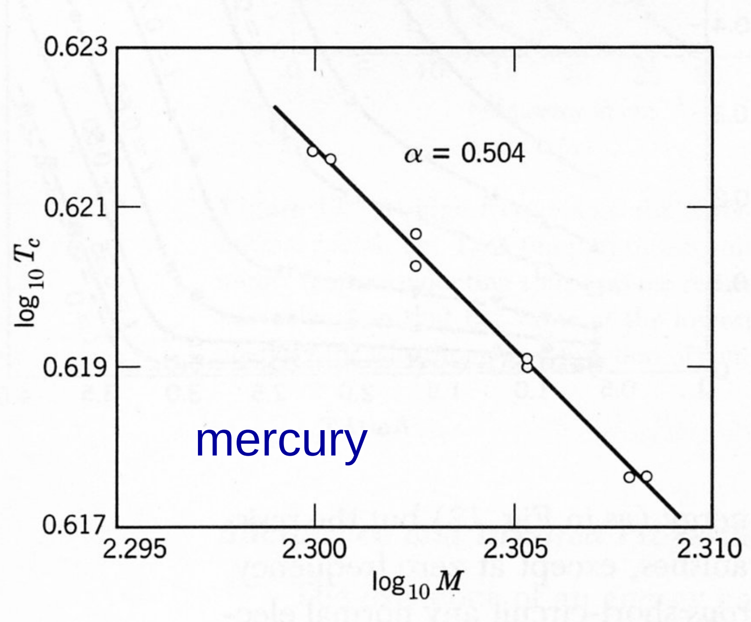
• Failed attempts: polaron, CDW...

• Isotope effect (1950):

It is found that $T_c = \text{const} \times M^{-\alpha}$

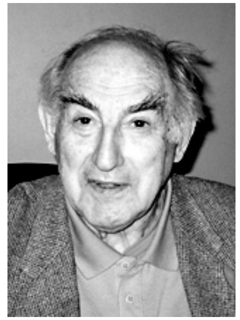
$\alpha \sim$ 1/2 for different materials

$\leftrightarrow$ lattice vibration?



Brief history of the theories of superconductors

• 1935 London: superconductivity is a quantum phenomenon on a macroscopic scale. There is a "rigid" (due to the energy gap) superconducting wave function Ψ.



• 1950

• Frohlich: electron-phonon interaction maybe crucial.

• Reynolds et al, Maxwell: isotope effect

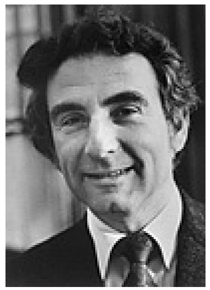
• Ginzburg-Landau theory: ρ_S can be varied in space. Suggested the connection $\rho_S(\vec{r}) = |\psi(\vec{r})|^2$

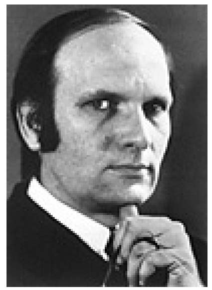
and wrote down the eq. for order parameter Ψ(r) (App. I)

• 1956 Cooper pair: attractive interaction between electrons (with the help of crystal vibrations) near the FS forms a bound state.

• 1957 Bardeen, Cooper, Schrieffer: BCS theory

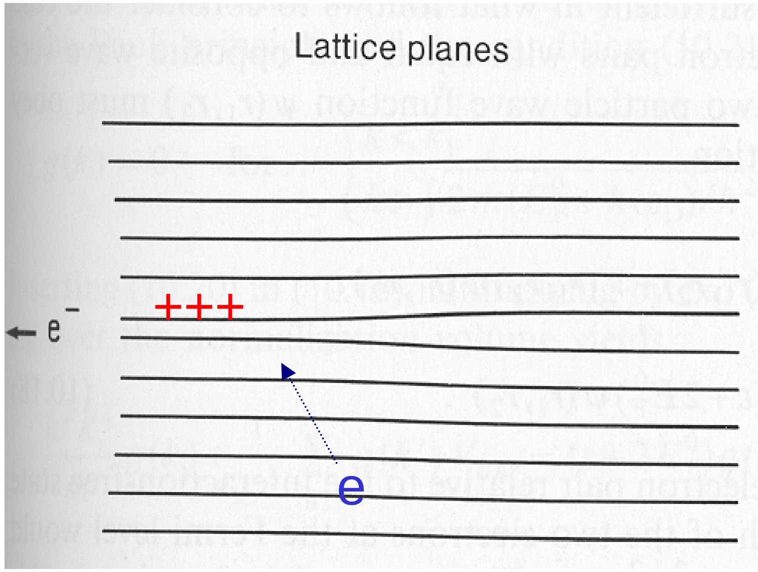
Microscopic wave function for the condensation of Cooper pairs.

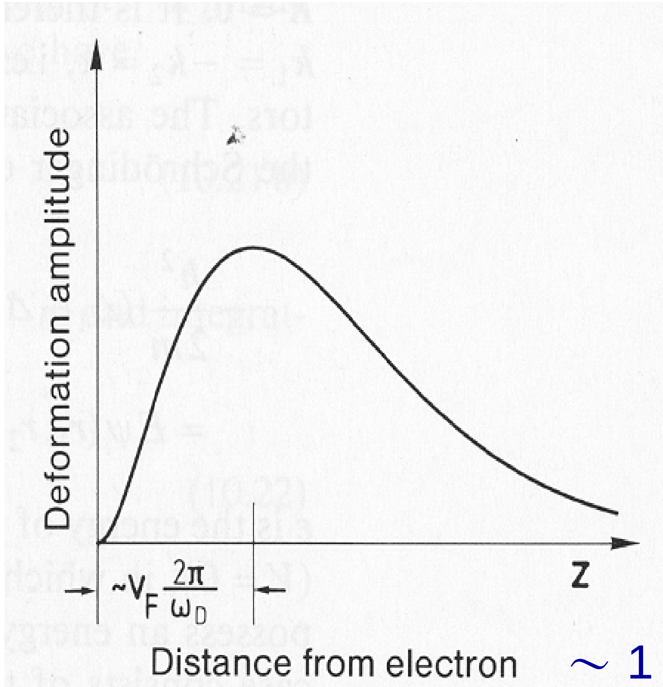




Ref: 1972 Nobel lectures by Bardeen, Cooper, and Schrieffer

Dynamic electron-lattice interaction → Cooper pair





$\sim 1\ \mu$m

Effective attractive interaction
between 2 electrons
(sometimes called overscreening)

(range of a Cooper pair;
coherence length)

Cooper pair, and BCS prediction

- 2 electrons with opposite momenta ($p\uparrow$, $-p\downarrow$) can form a bound state with binding energy (the spin is opposite by Pauli principle)

$$\Delta(0) = 2\hbar\omega_D e^{-\frac{1}{D(E_F)V_{\text{int}}}}, \quad \text{see App. H}$$

- Fraction of electrons involved $\sim kT_c/E_F \sim 10^{-4}$
  - Average spacing between condensate electrons $\sim$ 10 nm

$2\Delta(0) \sim 3.5\, k_B T_c$

- Therefore, within the volume occupied by the Cooper pair, there are approximately (1 μm/10 nm)^3 $\sim 10^6$ other pairs.

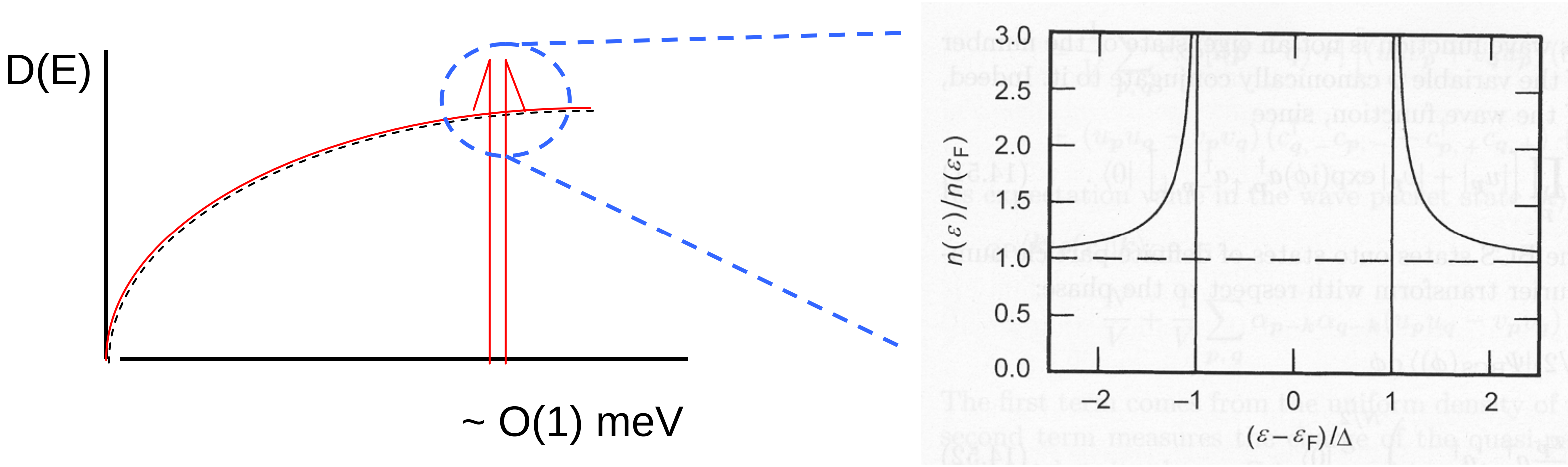
- These pairs (similar to bosons) are highly correlated and form a macroscopic condensate state with (BCS result)

$$k_B T_C = 1.13\hbar\omega_D e^{-\frac{1}{D(E_F)V_{\text{int}}}}$$

$$\hbar\omega_D \le 500\ K,\ \ D(E_F)V_{\text{int}} \le 1/3$$

$$\therefore \quad T_c \le 500 e^{-3} = 25\ K$$ ($\sim$upper limit of T_c)

Energy gap and Density of states

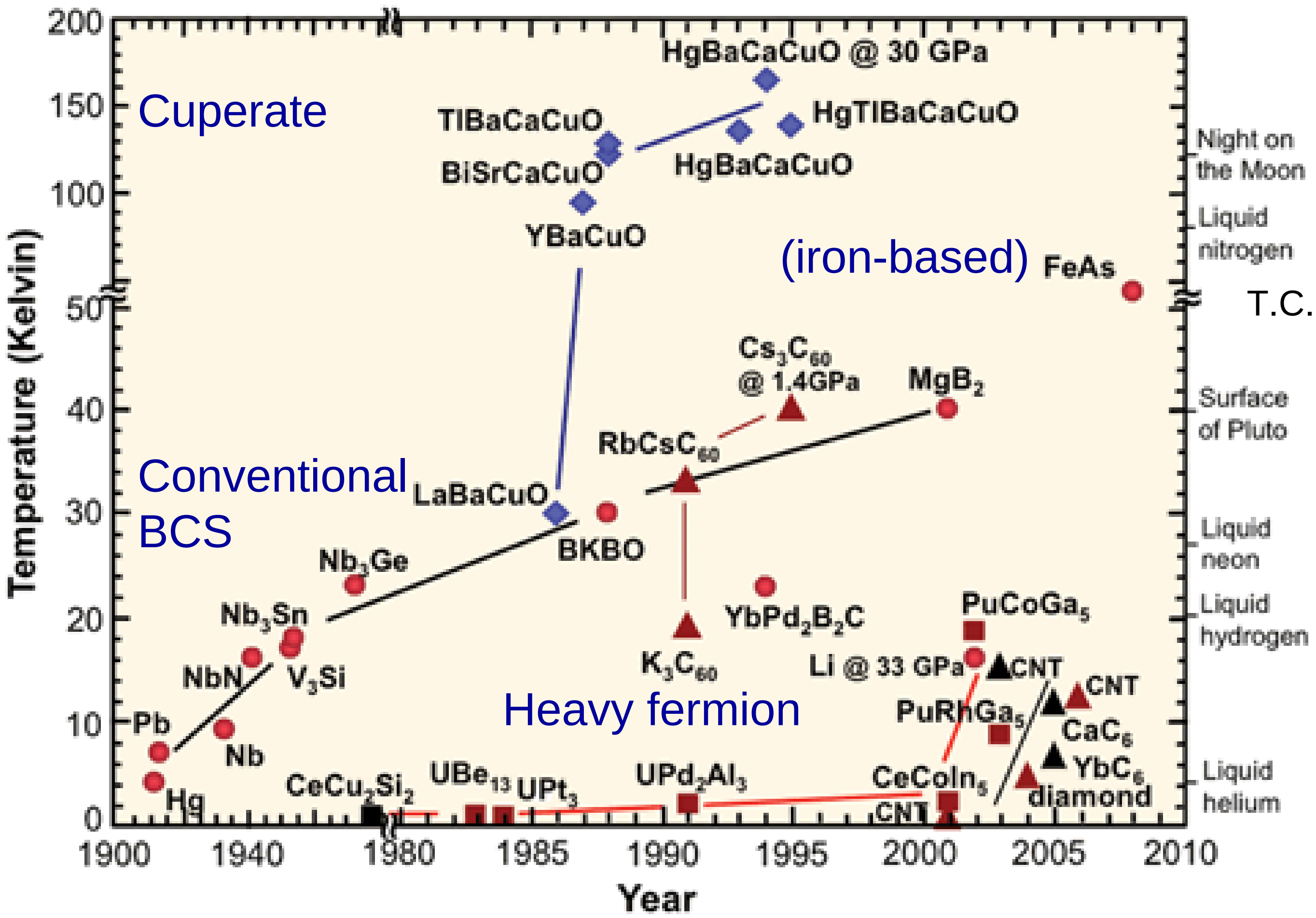


• Electrons within kT_C of the FS have their energy lowered by the order of kT_C during the condensation.

• On the average, energy difference (due to SC transition) per electron is

$$k_B T_C \frac{T_C}{T_F} \simeq 0.1\, meV \times \frac{1}{10^4} \simeq 10^{-8}\ eV$$

Families of superconductors

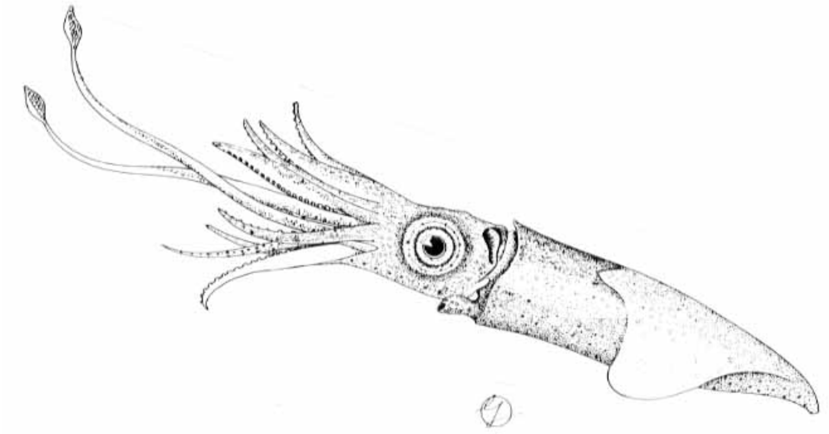


T.C. Ozawa 2008

F. Steglich 1978

wiki

- Introduction
- Thermal properties
- Magnetic properties
- London theory of the Meissner effect
- Microscopic (BCS) theory
- Flux quantization
- Quantum tunneling (Josephson effect, SQUID)



Flux quantization in a superconducting ring

(F. London 1948 with a factor of 2 error, Byers and Yang, also Brenig, 1961)

- Current density operator $\vec{j} = \frac{q}{2m}\left(\psi^* \frac{\hbar}{i}\nabla\psi - \psi\frac{\hbar}{i}\nabla\psi^*\right),\ q=-e$

- SC, in the presence of B $\vec{j} = \frac{q^*}{2m^*}\left[\psi^*\left(\frac{\hbar}{i}\nabla - \frac{q^*}{c}\vec{A}\right)\psi + \psi\left(\frac{\hbar}{i}\nabla - \frac{q^*}{c}\vec{A}\right)^*\psi^*\right]$ $\quad q^* = -2e$, $m^* = 2m$

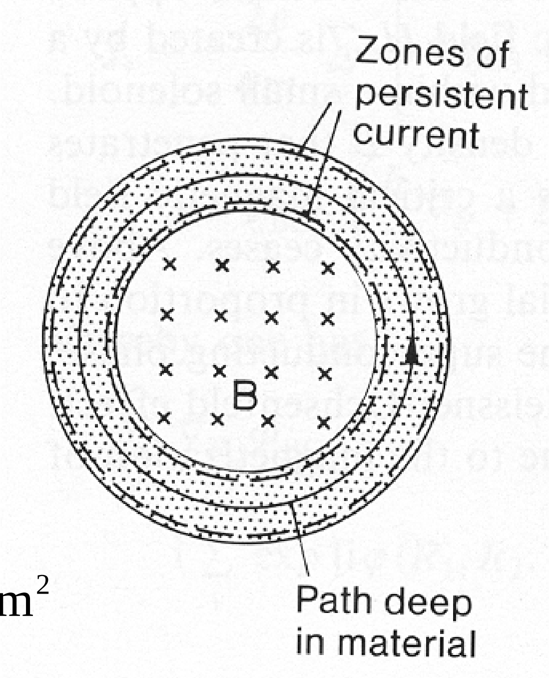
let $\psi = |\psi| e^{i\phi}$ and assume $|\psi|$ vary slowly with $\vec{r}$

then $j = -\left(\frac{e\hbar}{m}\nabla\phi + \frac{2e^2}{mc}\vec{A}\right)|\psi|^2$ ➡ London eq. with $n_s = 2|\psi|^2$

- Inside a ring $\oint \vec{j}\cdot d\vec{\ell} = 0$

$$\Rightarrow \quad \oint \vec{A}\cdot d\vec{\ell} = -\frac{\hbar c}{2e}\oint \nabla\phi\cdot d\vec{\ell} = -\frac{\hbar c}{2e}\Delta\phi$$

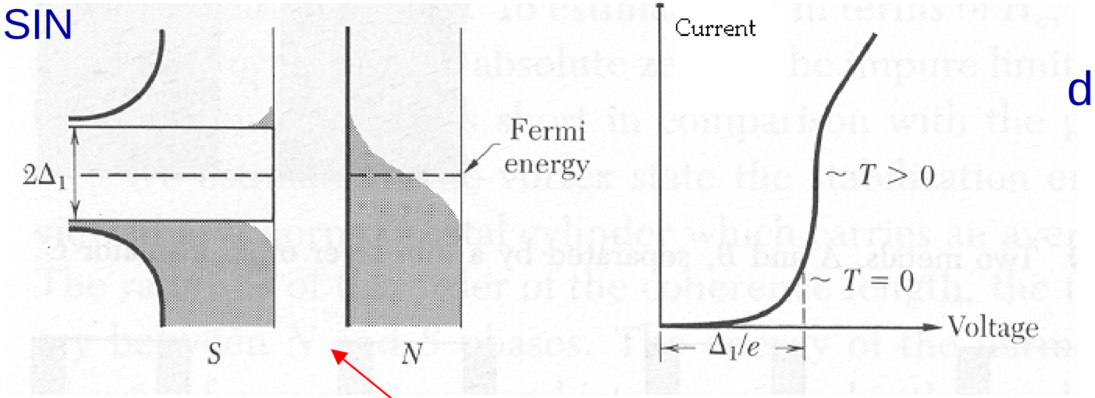
$$\therefore \quad \text{flux} \quad |\Phi| = n\frac{hc}{2e} = n\phi_0, \qquad \phi_0 \equiv \frac{hc}{2e} = 2\times10^{-7}\ \text{gauss-cm}^2$$



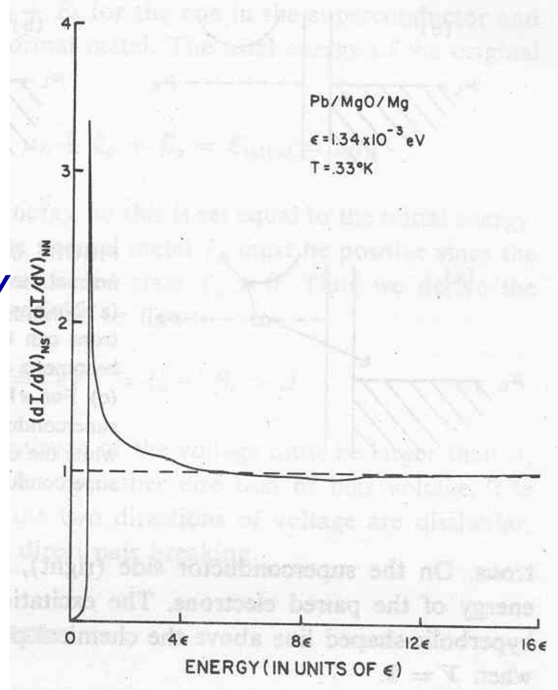
- $\phi_0 \sim$ the flux of the Earth's magnetic field through a human red blood cell (~ 7 microns)

Single particle tunneling (Giaever, 1960)

- SIN

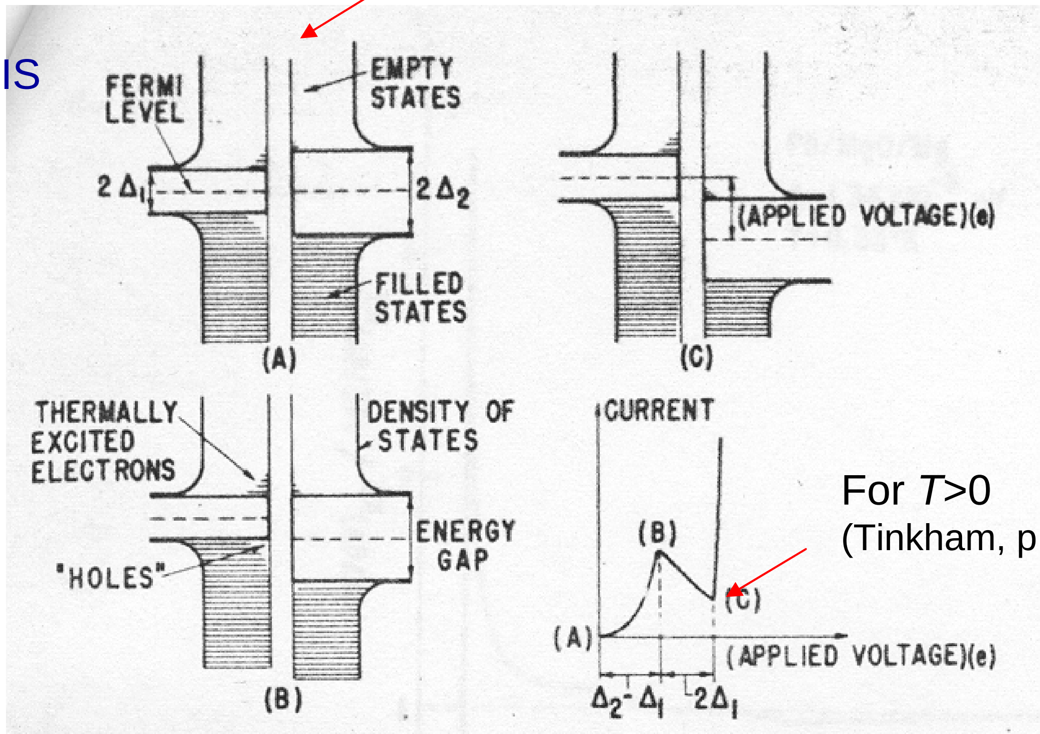


dI/dV



20-30 A thick

- SIS



For T>0
(Tinkham, p.77)

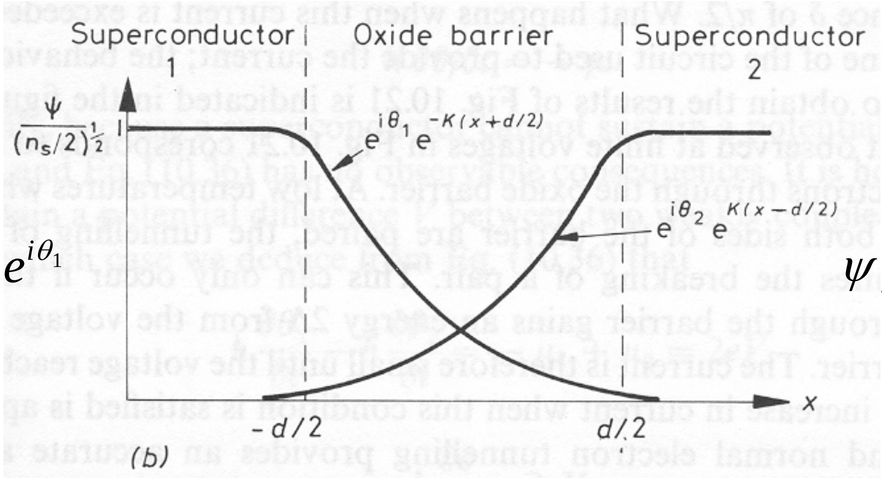
Ref: Giaever's 1973 Nobel prize lecture

Josephson effect (Cooper pair tunneling) Josephson, 1962

1) DC effect:

There is a DC current through SIS in the absence of voltage.

$$\psi_1 = |\psi_1| e^{i\theta_1}$$



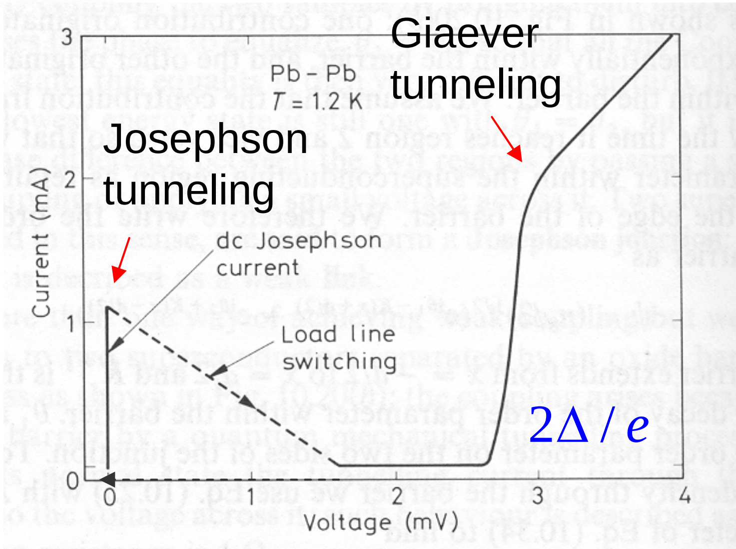
$$\psi_2 = |\psi_2| e^{i\theta_2}$$

$$\psi = \sqrt{n_S / 2} \left(e^{i\theta_1 - K(x+d/2)} + e^{i\theta_2 + K(x-d/2)} \right)$$

$$j = \frac{ie\hbar n_S}{2m} K e^{-Kd} \left(-e^{i(\theta_1 - \theta_2)} + e^{i(\theta_2 - \theta_1)} \right)$$

$$= j_0 \sin \delta_0$$

$$j_0 \equiv e\hbar n_S K e^{-Kd} / m, \; \delta_0 \equiv \theta_1 - \theta_2$$



2) AC Josephson effect

Apply a DC voltage, then there is a rf current oscillation.

$$\psi = \langle N-1 | \hat{\psi} | N \rangle \propto e^{-i(E_N - E_{N-1})t/\hbar} = e^{-i\mu t/\hbar}$$

$$\rightarrow \theta_i(t) = -\mu_i t/\hbar + \theta_i \quad (i=1,2)$$

$$\mu_1 - \mu_2 = -2eV$$

$$\therefore \delta = \frac{2eV}{\hbar} t + \delta_0 \qquad \Rightarrow \qquad j = j_0 \sin\left(\frac{2eV}{\hbar} t + \delta_0\right)$$

(see Kittel, p.290 for an alternative derivation)

- An AC supercurrent of Cooper pairs with freq. ν =2eV/h, a weak microwave is generated.
- ν can be measured very accurately, so tiny ΔV as small as 10^{-15} V can be detected.
- Also, since V can be measured with accuracy about 1 part in 10^{10}, so 2e/h can be measured accurately.
- JJ-based voltage standard (1990):

1 V $\equiv$ the voltage that produces ν =483,597.9 GHz (exact)

- advantage: independent of material, lab, time (similar to the quantum Hall standard).

3) DC+AC: Apply a DC+ rf voltage, then there is a DC current

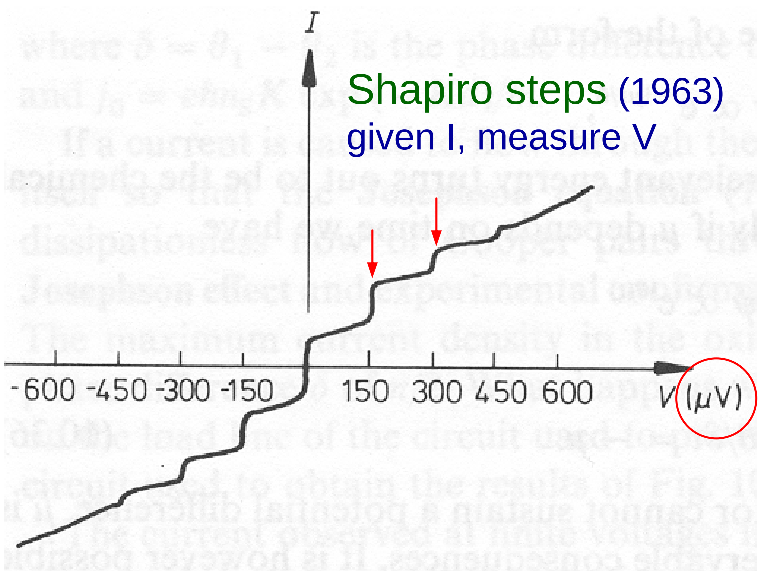
$$V = V_0 + \upsilon \cos \omega t$$

$$j = j_0 \sin\left[\frac{2e}{\hbar}\left(V_0 t + \frac{\upsilon}{\omega}\sin \omega t\right) + \delta_0\right]$$

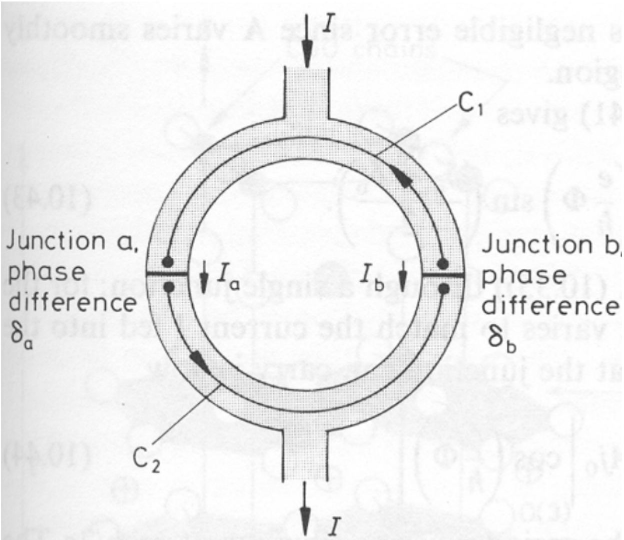
$$= j_0 \sum_n (-1)^n J_n\left(\frac{2e\upsilon}{\hbar\omega}\right)\sin\left(\frac{2eV_0}{\hbar}t - n\omega t + \delta_0\right)$$

$$\Rightarrow \quad \text{there is DC current at } V_0 = n\frac{\hbar\omega}{2e}$$

• Another way of providing a voltage standard



SQUID (Superconducting QUantum Interference Device)



$$j = j_0 \sin\delta_a + j_0 \sin\delta_b$$

$$= 2 j_0 \cos\left(\frac{\delta_a - \delta_b}{2}\right)\sin\left(\frac{\delta_a + \delta_b}{2}\right)$$

Similar to $\oint \vec{A}\cdot d\vec{\ell} = -\frac{\hbar c}{2e}\oint \nabla\theta\cdot d\vec{\ell}$

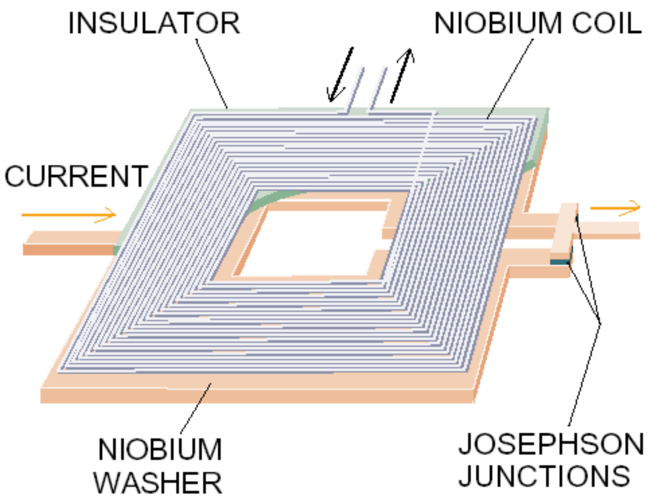
We now have $\frac{2\mathrm{e}}{\hbar \mathrm{c}}\int_{C_1} \vec{A}\cdot d\vec{\ell} = \theta_{b1} - \theta_{a1}$

$$\frac{2\mathrm{e}}{\hbar \mathrm{c}}\int_{C_2} \vec{A}\cdot d\vec{\ell} = \theta_{a2} - \theta_{b2}$$

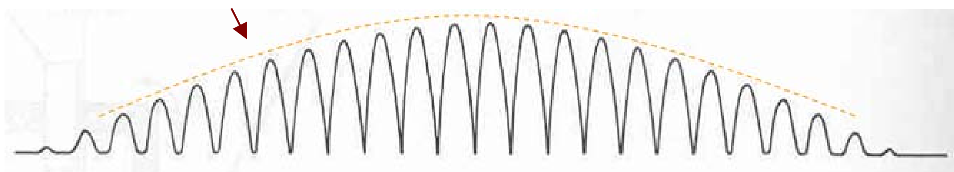
$$\Rightarrow \delta_a - \delta_b = \frac{2e}{\hbar c}\oint_C \vec{A}\cdot d\vec{\ell} = 2\pi\frac{\phi}{\phi_0}$$

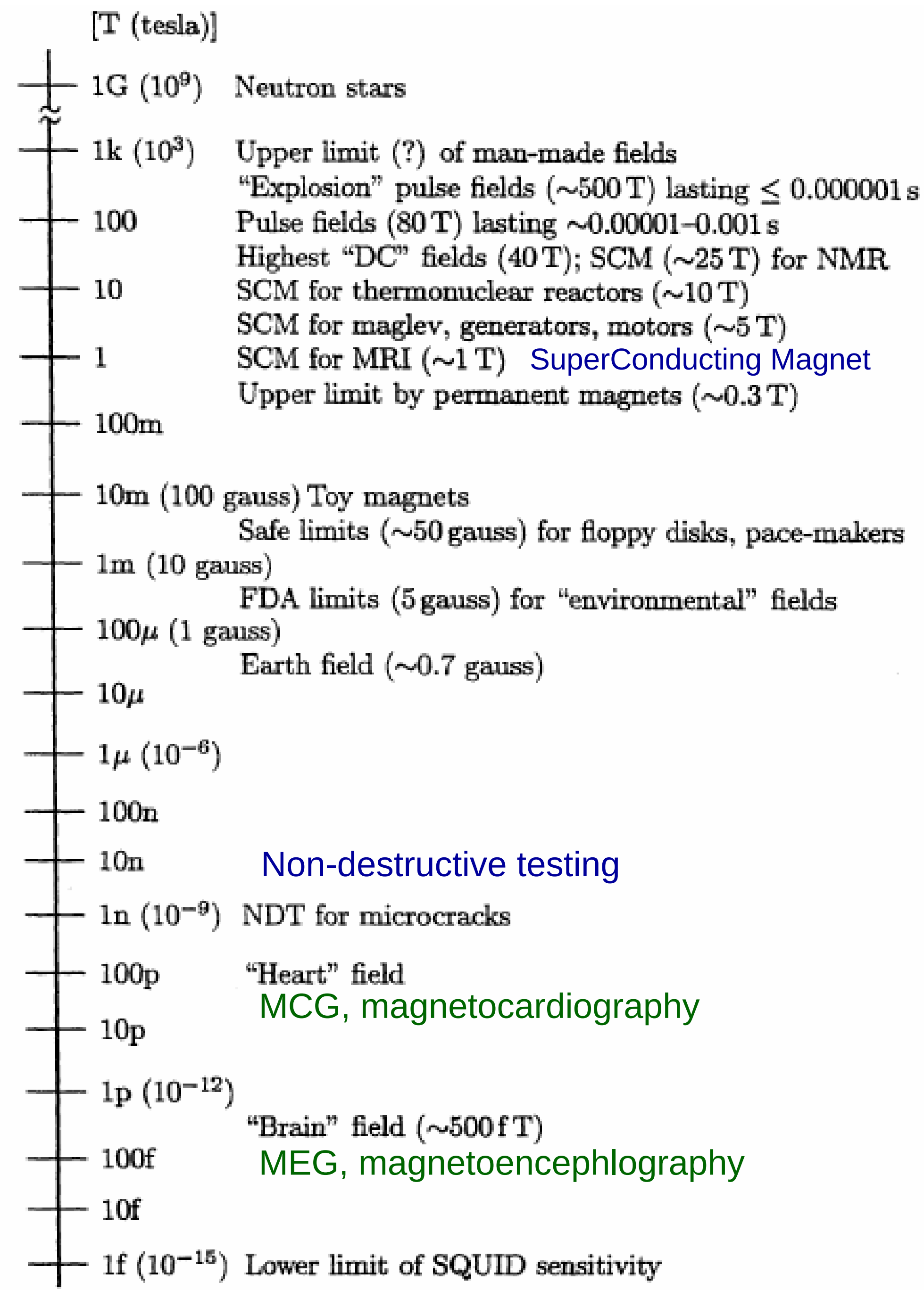
$$\therefore j_{\max} = 2 j_0 \left|\cos\left(\frac{2\pi}{2}\frac{\phi}{\phi_0}\right)\right|$$

The current of a SQUID with area 1 cm² could change from max to min by a tiny ΔH=10^{-7} gauss!

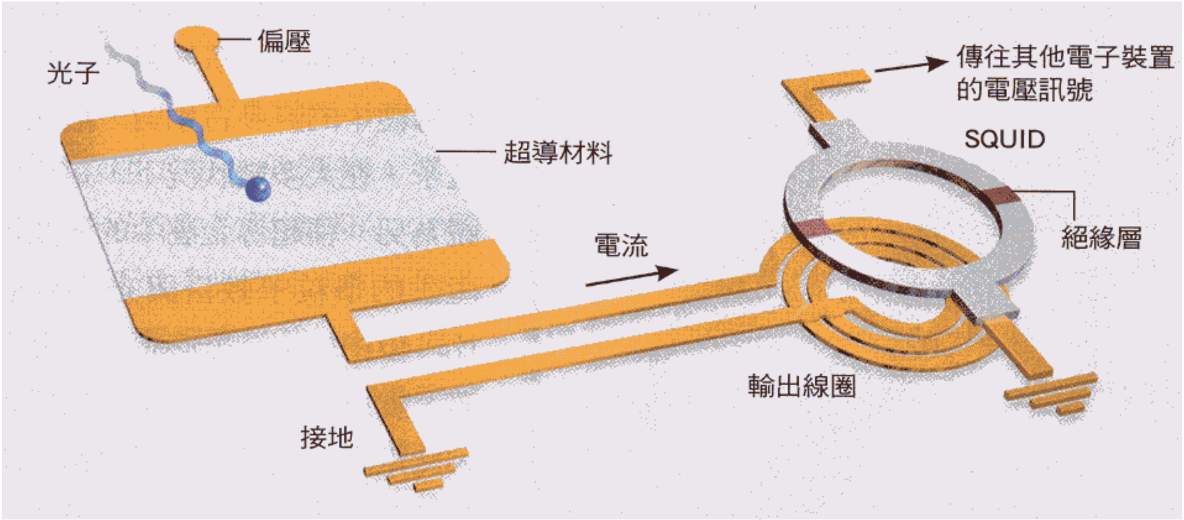


For junction with finite thickness

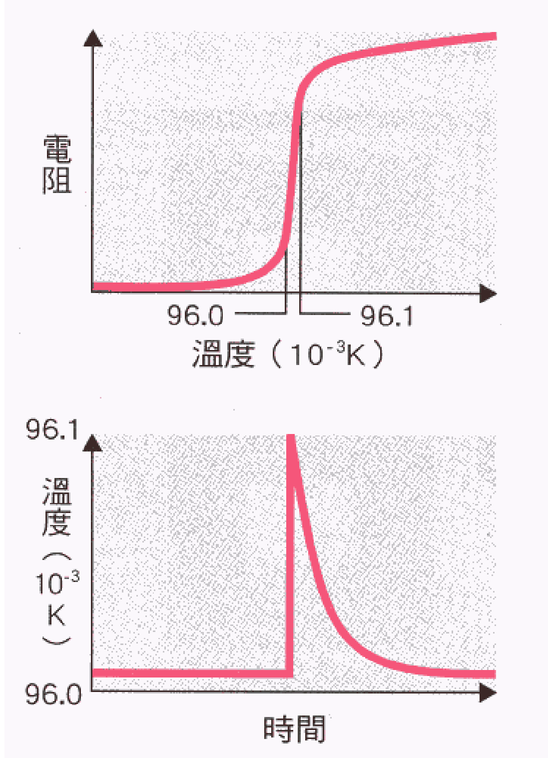


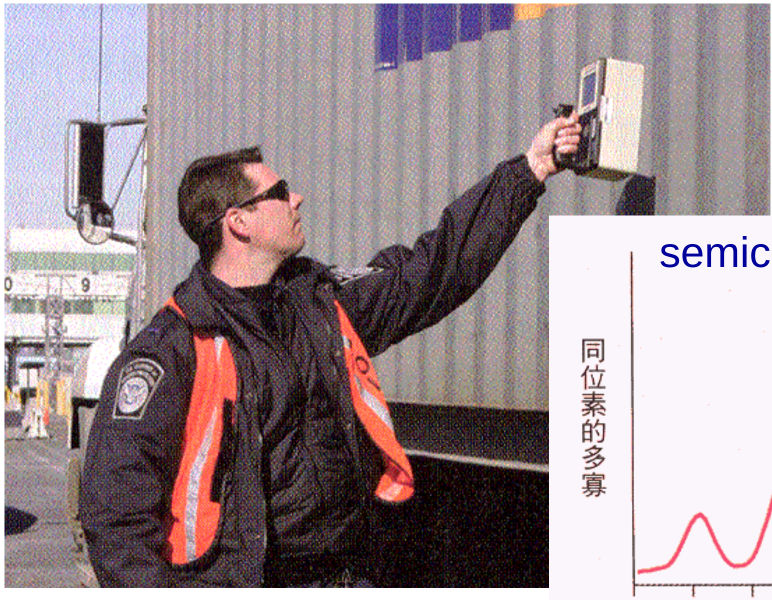

[T (tesla)]
1G (10^9) Neutron stars
1k (10^3) Upper limit (?) of man-made fields
"Explosion" pulse fields (~500 T) lasting ≤ 0.000001 s
100 Pulse fields (80 T) lasting ~0.00001–0.001 s
Highest "DC" fields (40 T); SCM (~25 T) for NMR
10 SCM for thermonuclear reactors (~10 T)
SCM for maglev, generators, motors (~5 T)
1 SCM for MRI (~1 T)
SuperConducting Magnet
Upper limit by permanent magnets (~0.3 T)
100m
10m (100 gauss) Toy magnets
Safe limits (~50 gauss) for floppy disks, pace-makers
1m (10 gauss)
FDA limits (5 gauss) for "environmental" fields
100μ (1 gauss)
Earth field (~0.7 gauss)
10μ
1μ (10^{-6})
100n
10n
Non-destructive testing
1n (10^{-9}) NDT for microcracks
100p "Heart" field
MCG, magnetocardiography
10p
1p (10^{-12})
"Brain" field (~500 f T)
100f
MEG, magnetoencephlography
10f
1f (10^{-15}) Lower limit of SQUID sensitivity

Super-sentitive photon detector



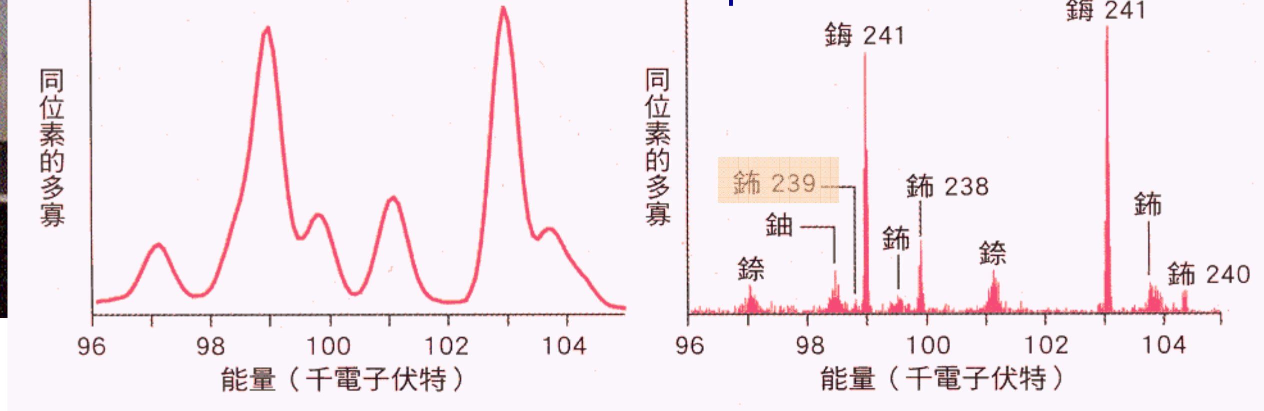
Transition edge sensor





semiconductor detector

superconductor detector



科學人,2006年12月