Sec 5.10 Quasicrystal

"perfectly ordered materials that never repeat themselves"

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Metallic Phase with Long-Range Orientational Order and No Translational Symmetry

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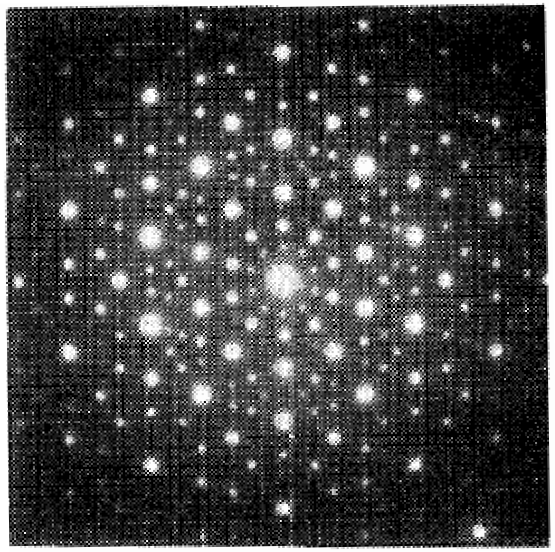
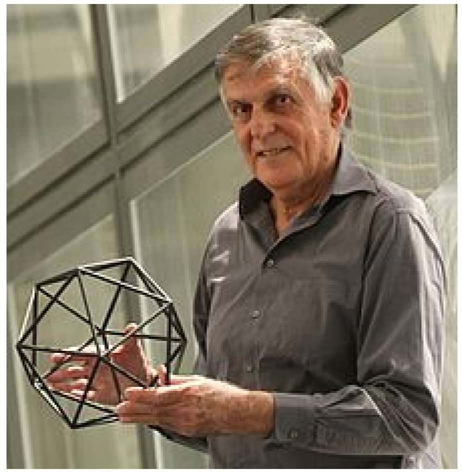


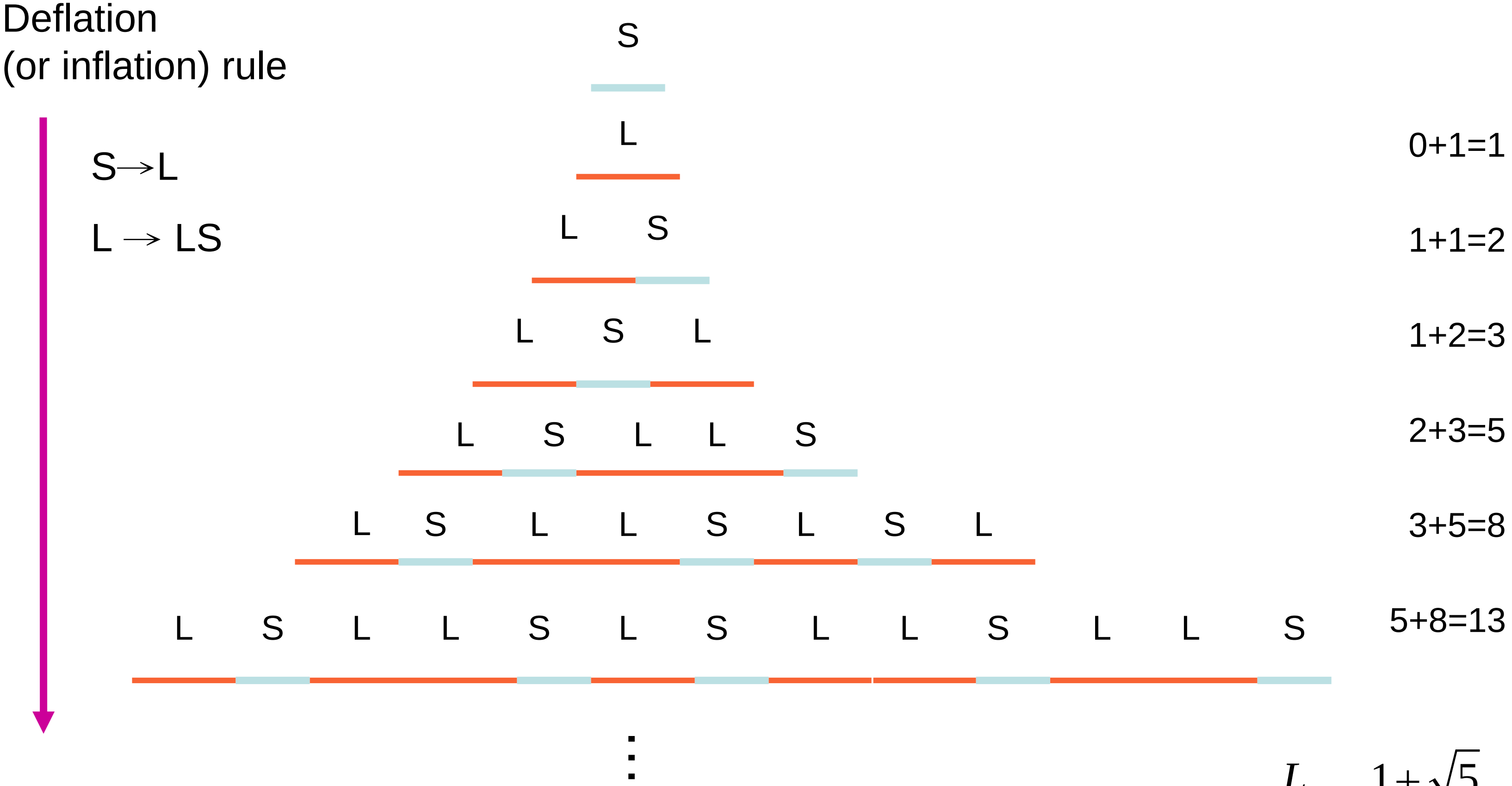
Electron diffraction pattern of Al-Mn alloy (cooling rate 10^6 k/s).



Dept of Phys
M.C. Chang

An example of 1D quasicrystal

Deflation
(or inflation) rule



Fibonacci sequence (1202)

$$\tau = \lim_{n\to\infty} \frac{L_n}{S_n} = \frac{1+\sqrt{5}}{2}$$

• No periodicity, but with perfect order
(i.e. locations of S and L are predictable)

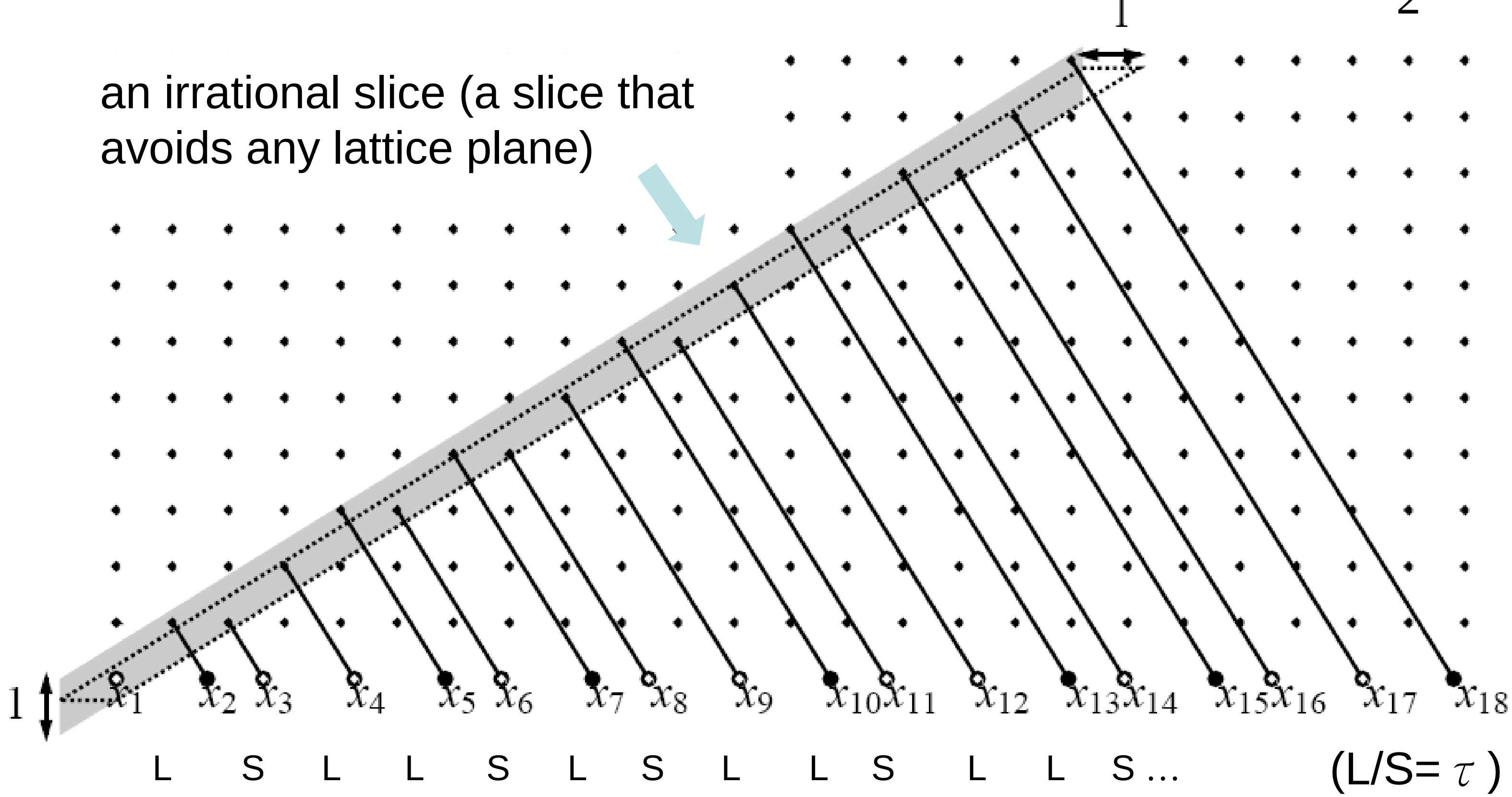
http://home.iitk.ac.in/~anandh/E-book/Quasicrystals.ppt

1D quasicrystal as a projection of 2D periodic crystal

: The "cut and project" construction

Slope=1/ τ $\tau = \frac{1+\sqrt{5}}{2} \approx 1.618$

an irrational slice (a slice that avoids any lattice plane)

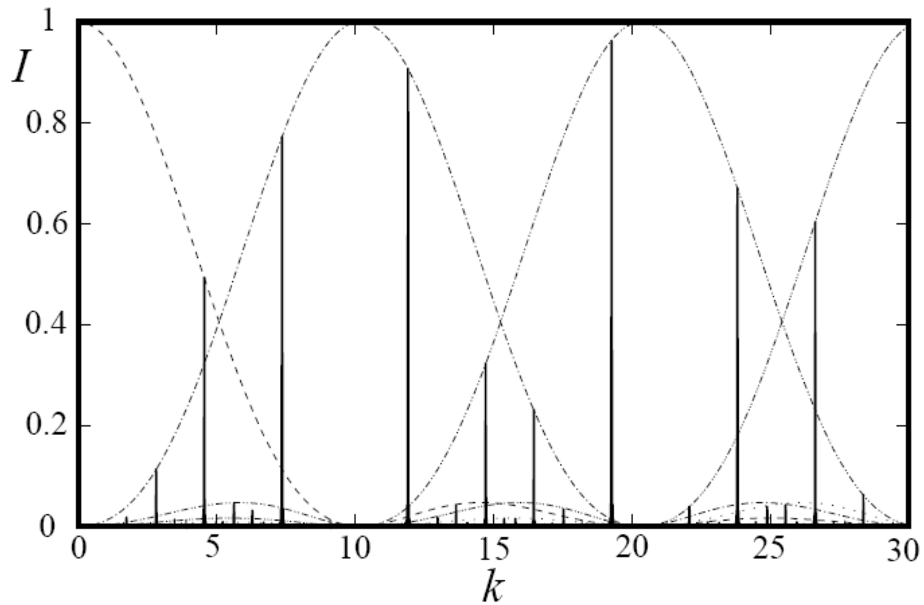


• Within a large segment, the ratio of numbers N_L/N_s approaches τ

Diffraction pattern of a Fibonacci quasicrystal

$$x_n = n + (\tau - 1)\mathrm{int}(n/\tau)$$

$$I = \left| \sum_n e^{ikx_n} \right|^2$$



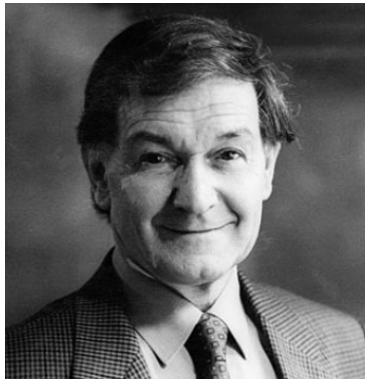
• The peaks are countably infinite and dense (in the real numbers) (aka singular continuous)

Buczek et al, Acta Physica Polonica B 2005

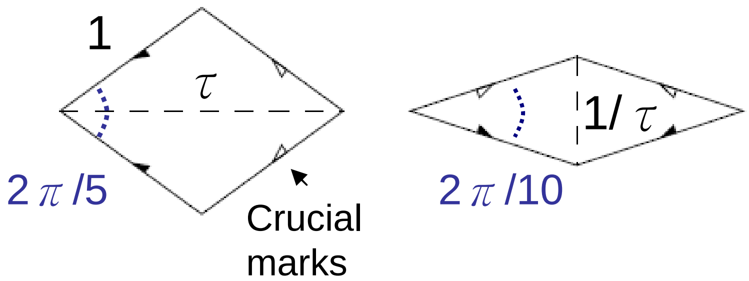
2D quasicrystal

Can one find a set of shapes that can cover the plane non-periodically?

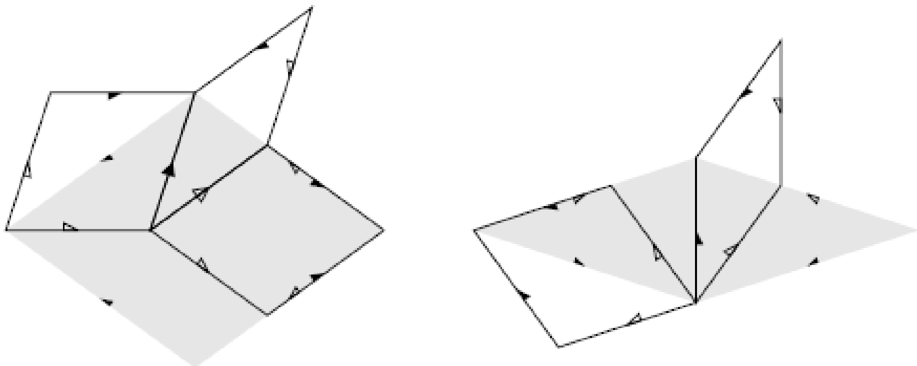
One example: Penrose tiling (1974)

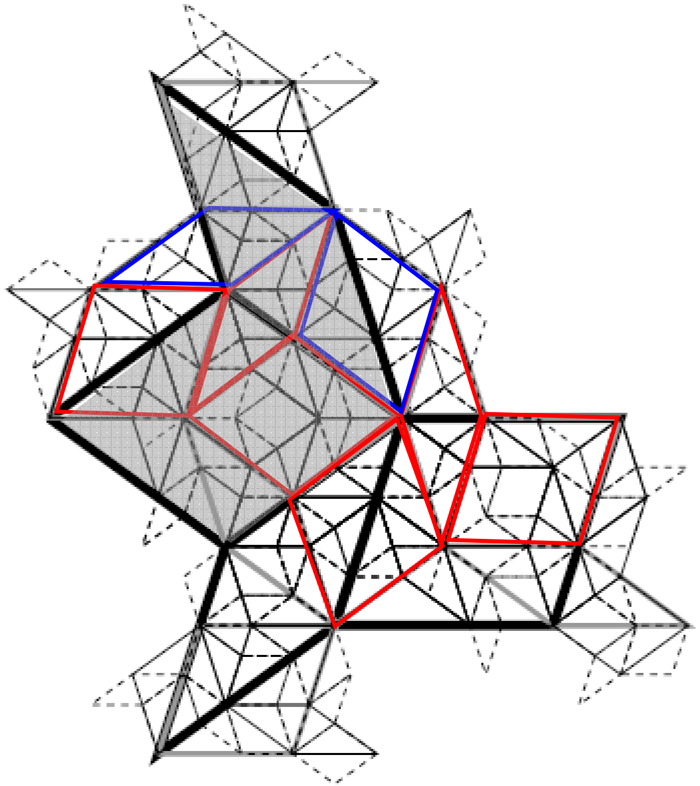


- Penrose tiles (rhombus type) 菱形



- deflation rule





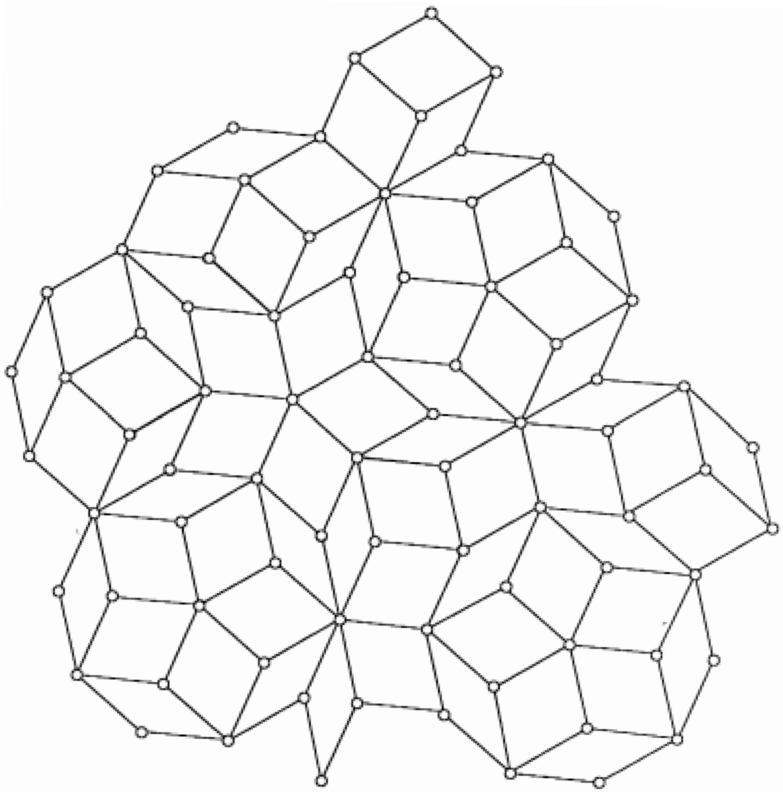


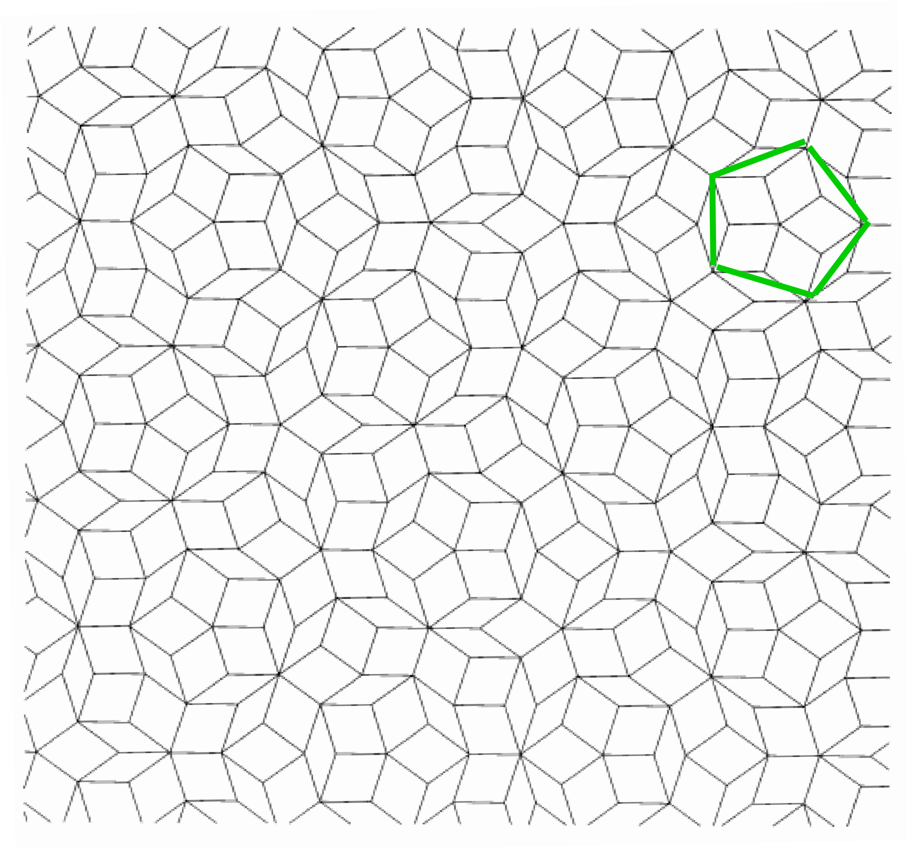
Fig. 2.8 Connecting the dots in Figure 2.7.

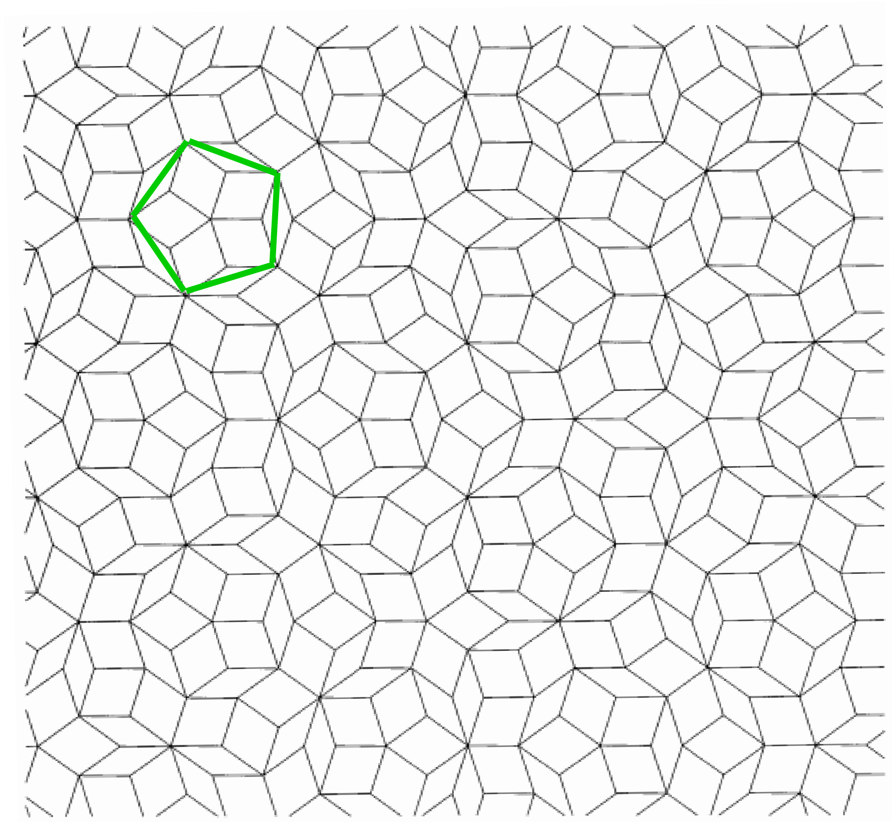
M. Senechal, Quasicrystals and Geometry, p.54

- A shifted copy will never match the original exactly.
- Any finite region in a tiling appears infinitely many times.

Are they the same?

local 5-fold symmetry





M. Senechal, Quasicrystals and Geometry, p.200

They are different.

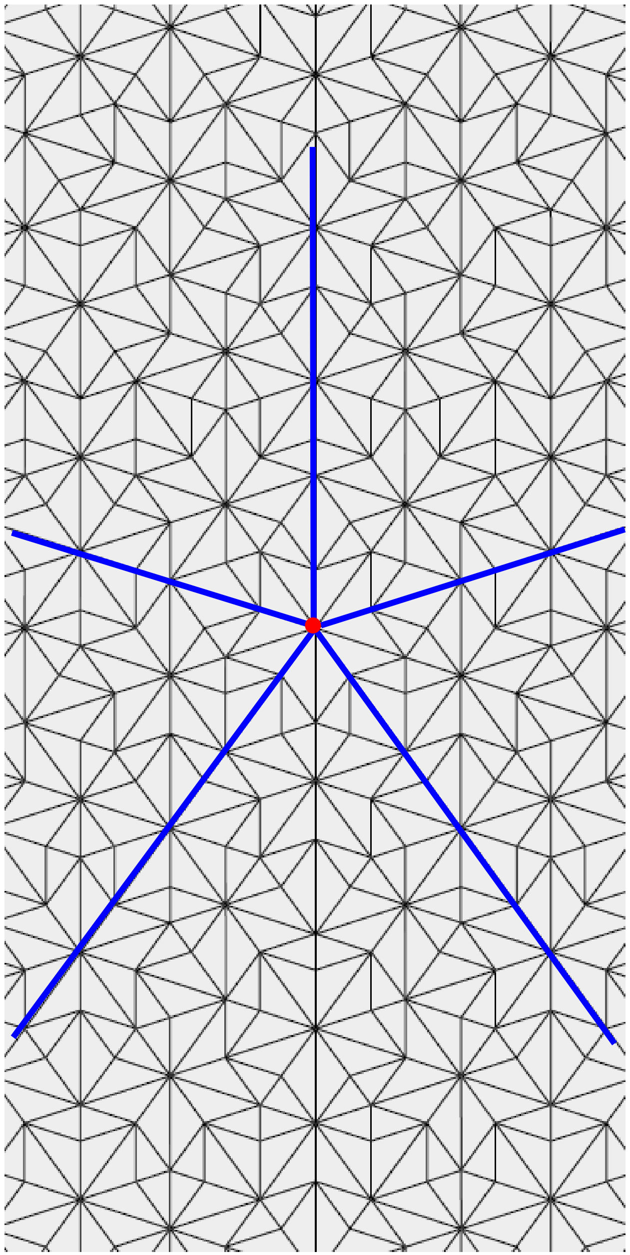
• A finite patch appears infinitely many times in a tiling and, in any other tiling.

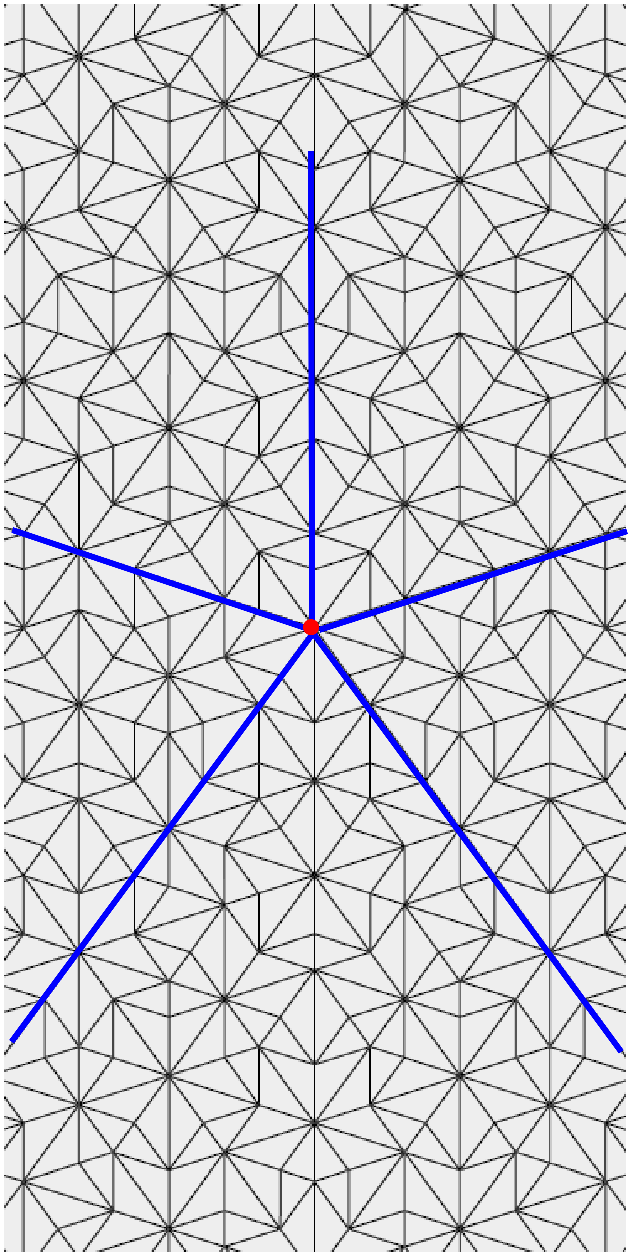
Therefore, a finite patch cannot differentiate between the uncountably many Penrose tilings.

http://www.alienscientist.com/penrose.html

Only 2 Penrose tiling have global 5-fold symmetry

At most one point of global 5-fold symmetry

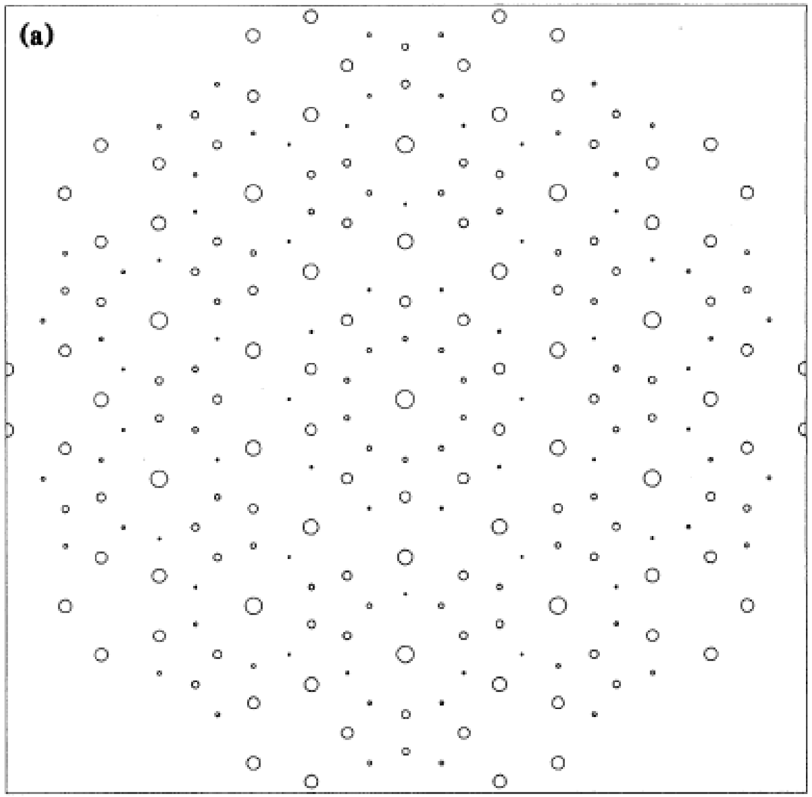




From Thomas Fernique's lectures

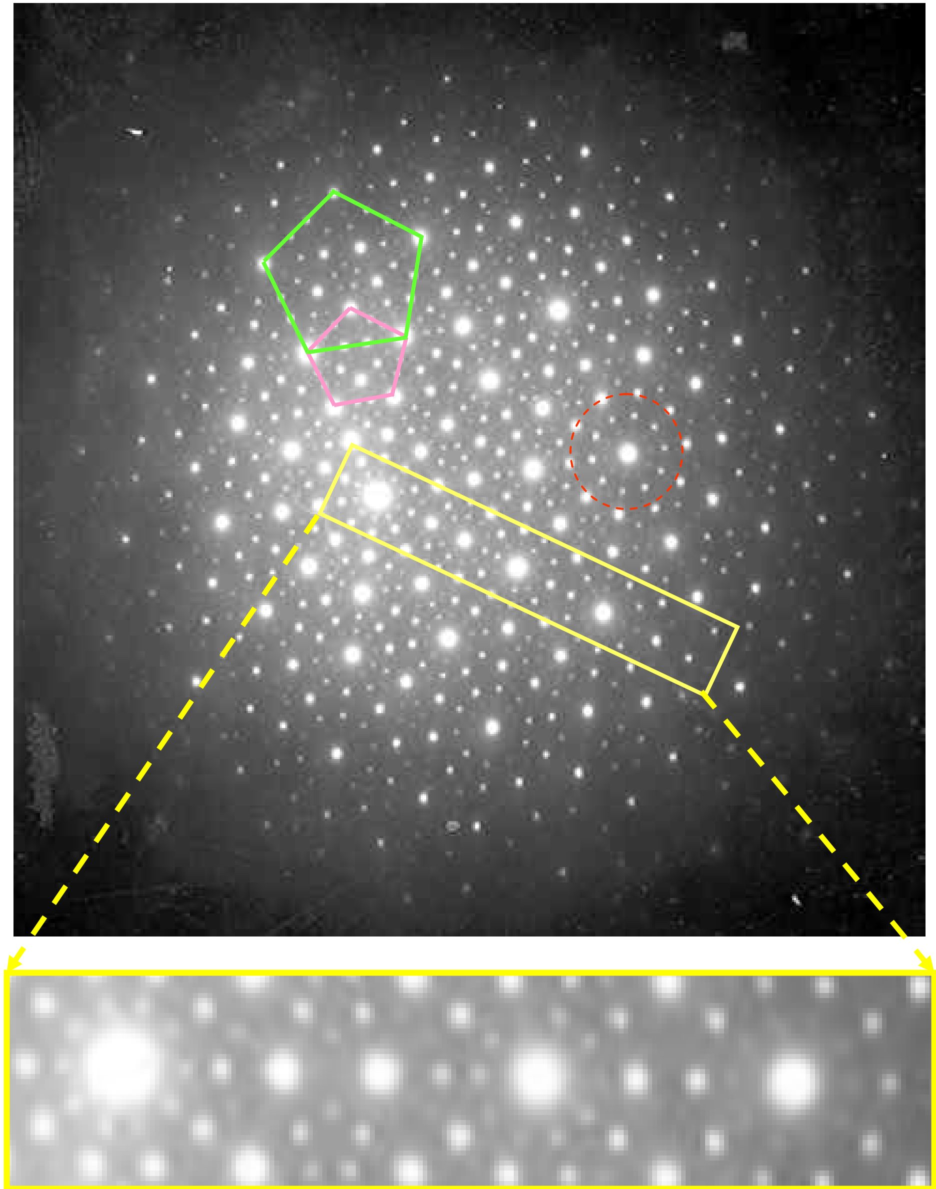
DIFFRACTION PATTERN

Indication of long range rotational order



Computed diffraction pattern for an ideal icosahedral quasicrystal (in a plane normal to a fivefold axis), displaying only peaks above some given intensity.
(Levine and Steinhardt, PRL 1985)

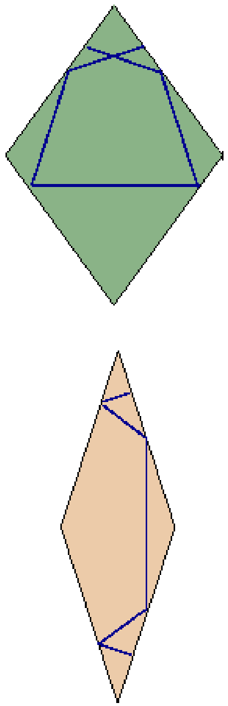
5-fold diffraction pattern from $Mg_{23}Zn_{68}Y_9$ alloy (icosahedral)



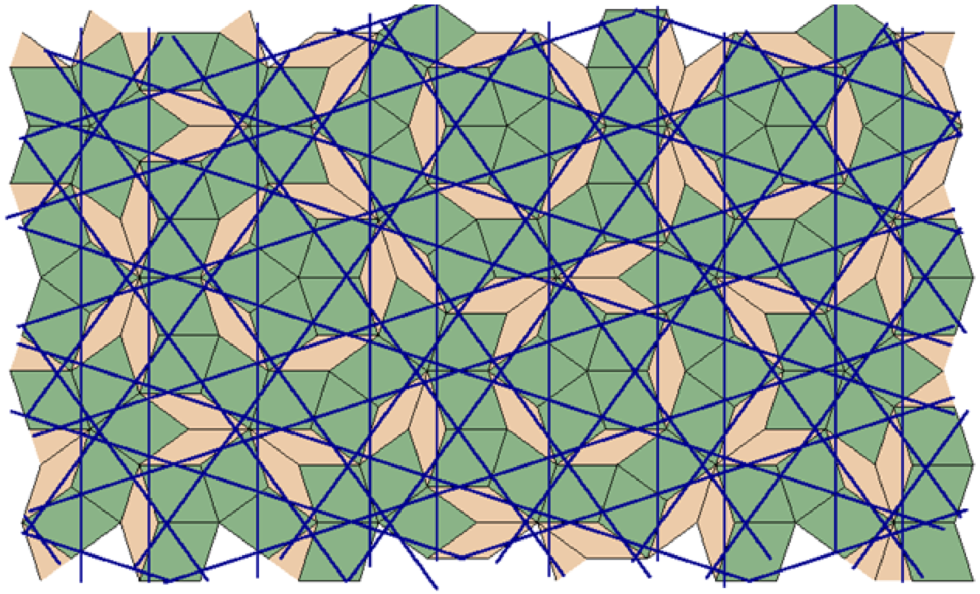
http://home.iitk.ac.in/~anandh/E-book/Quasicrystals.ppt

Another hidden order in Penrose tiling

Decoration lines



Ammann lines



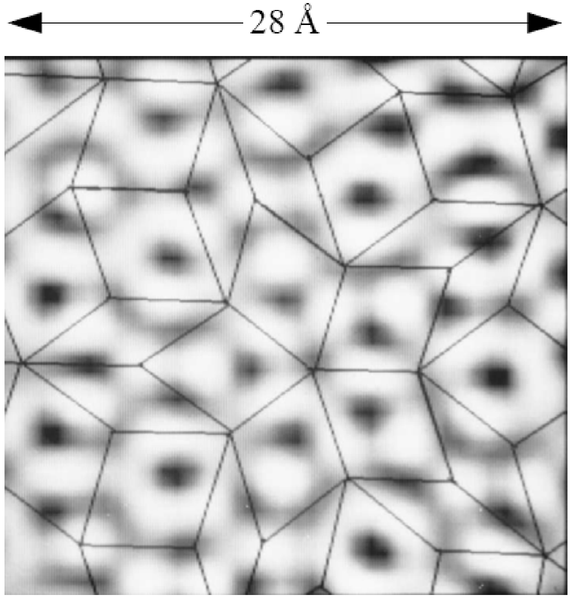
- The spacings of lines in any given direction is described by 1-dim Fibonacci sequence!

- De Bruijn (1981) showed that Penrose tilings can be viewed as two-dimensional slices of five-dimensional hypercubic structures. (wiki)

(note that there are 5 bundles of parallel lines above)

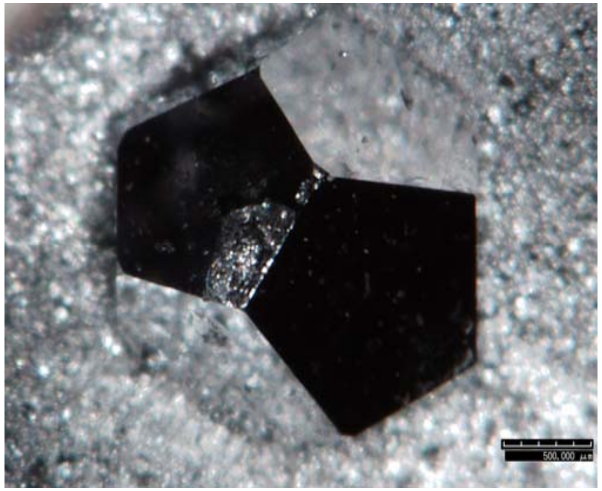
Real (artificial) quasicrystals

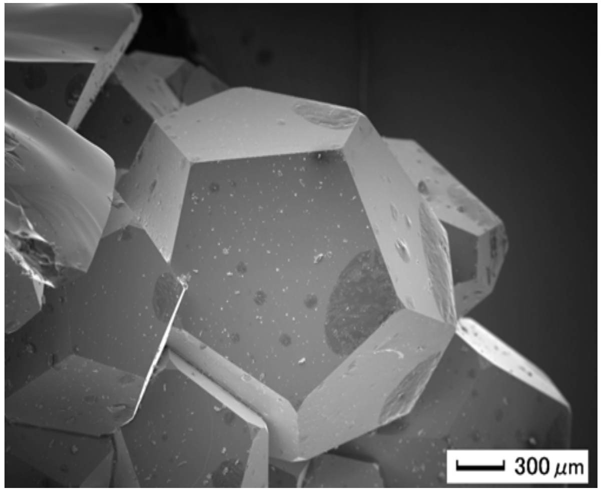
• Quasicrystals are found most often in aluminium alloys



Penrose tiling

Scanning tunneling microscope image of the 2D quasicrystal $Al_{65}Cu_{15}Co_{20}$



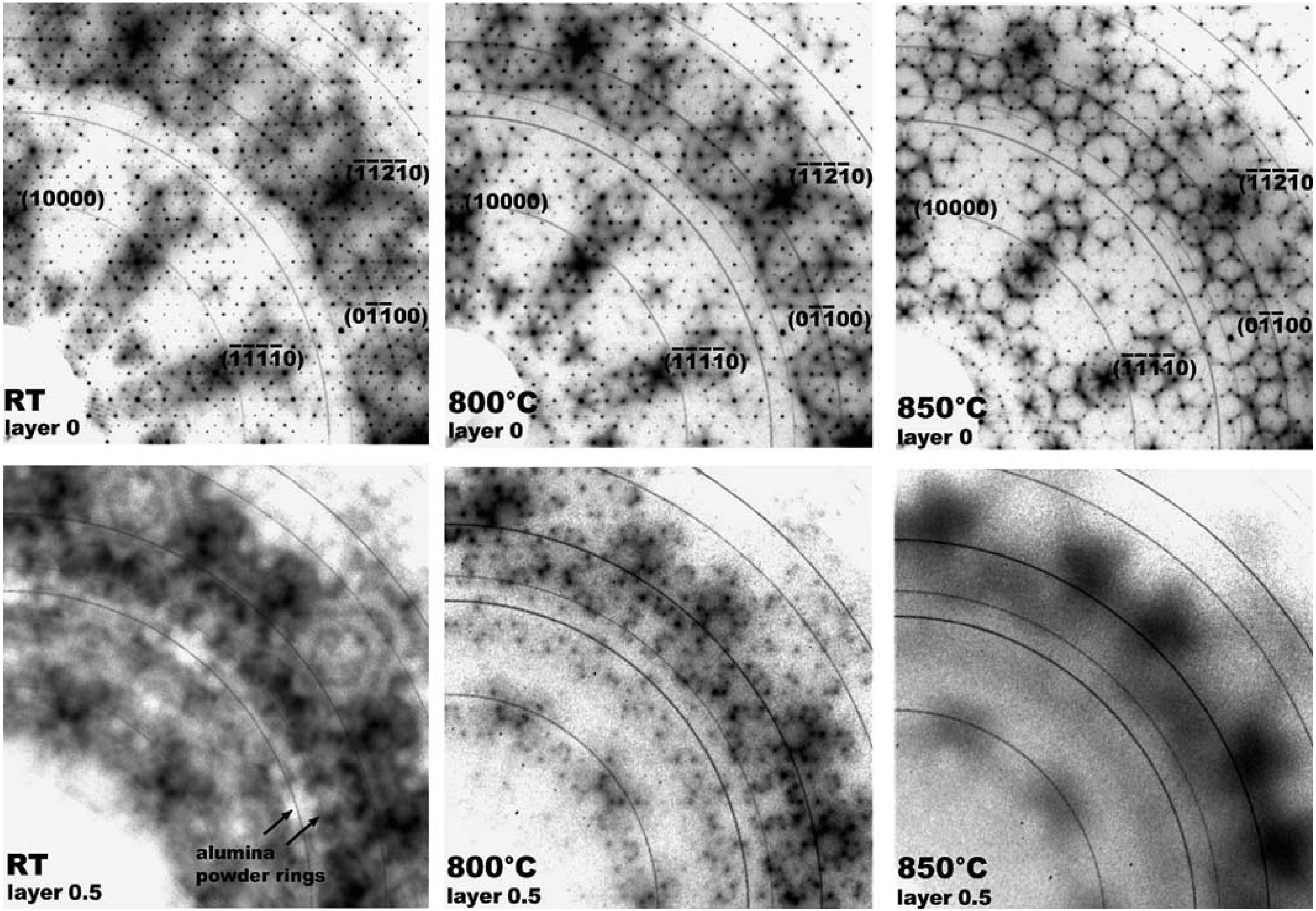


http://www.tagen.tohoku.ac.jp/labo/tsai/qc.html

Natural Quasicrystals?

See Bindi et al, Science 2009

Where are the atoms, actually?

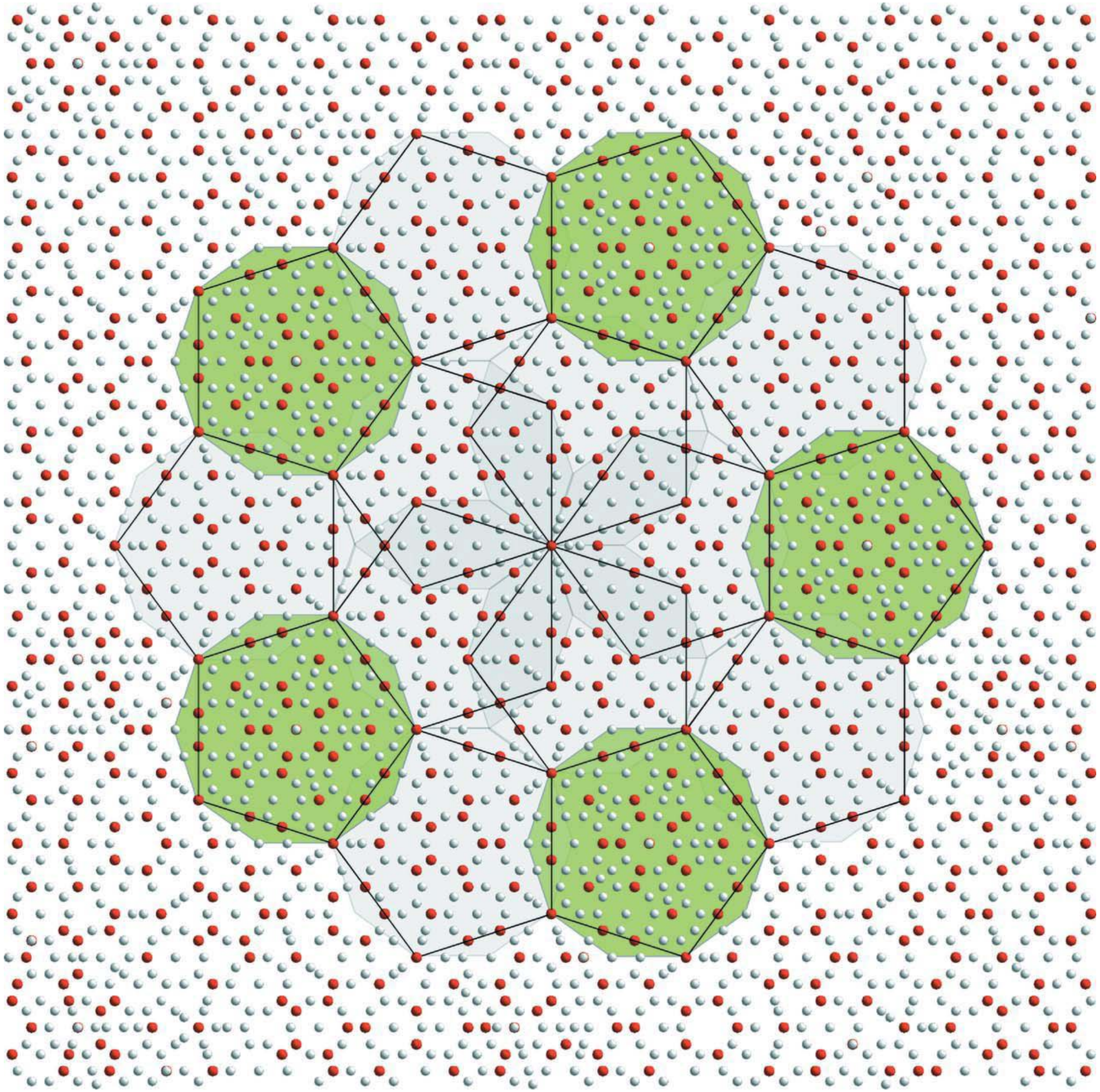


decagonal $Al_{70}Co_{12}Ni_{18}$ reconstructed from 360 image plate scanner frames at each temperature

Steurer, Journal of Non-Crystalline Solids, 2004

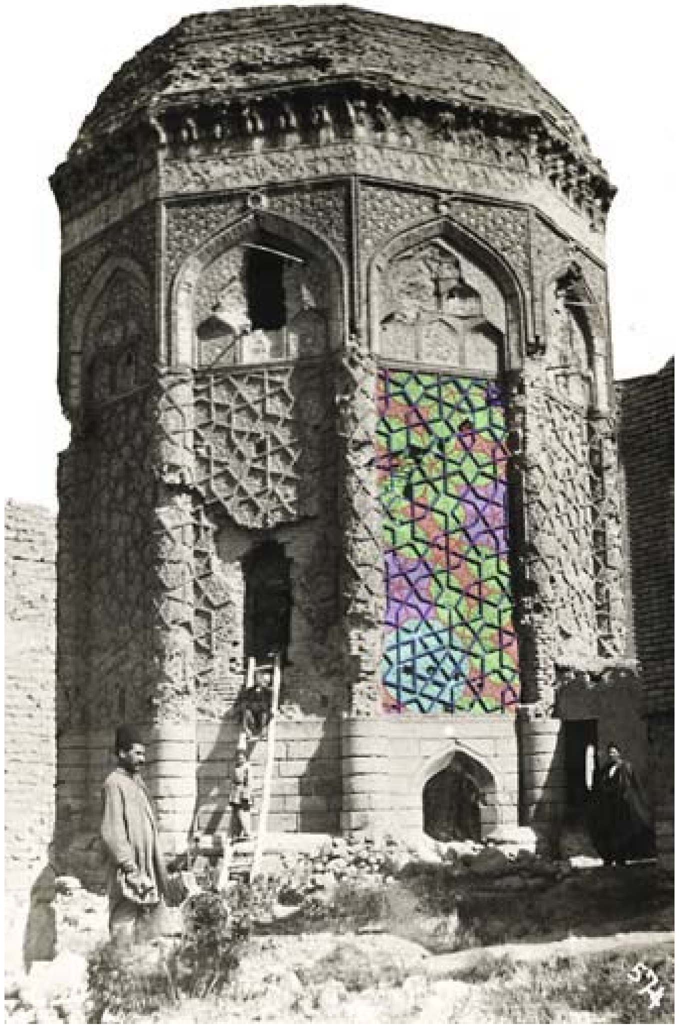
Where are the atoms, actually?

Steurer, Philos. Mag. 2007



Section perpendicular to the decagonal axis of Al-Co-Ni_{36}.

Quasi-Crystalline Tilings in Medieval Islamic Architecture

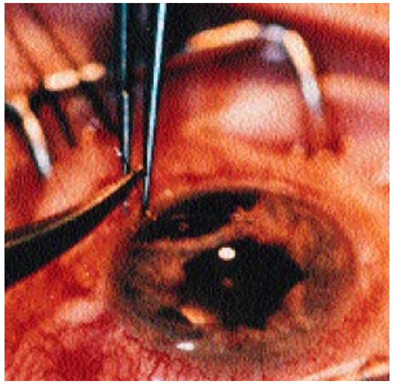


Gunbad-i Kabud tomb tower in
Maragha, Iran (1197 C.E.)

Lu and Steinhardt, Science 2007

APPLICATIONS OF QUASICRYSTALS

- hard and brittle
- low surface energy (non-stick)
- high electrical resistivity
- high thermal resistivity
- high thermoelectric power
- …



fine but strong

Technology Assessment & Transfer, Inc. 2010

http://www.mdatechnology.net/update.aspx?id=a5580

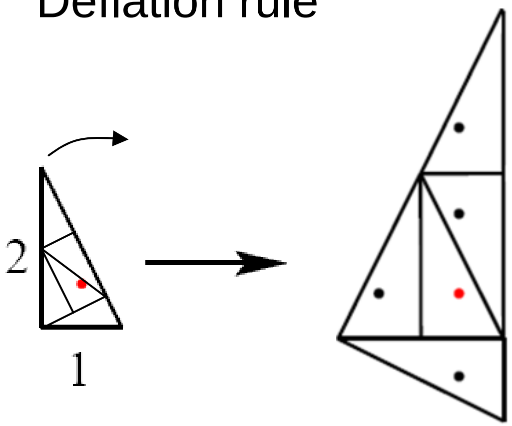
Philips and Sandvik Materials Tech

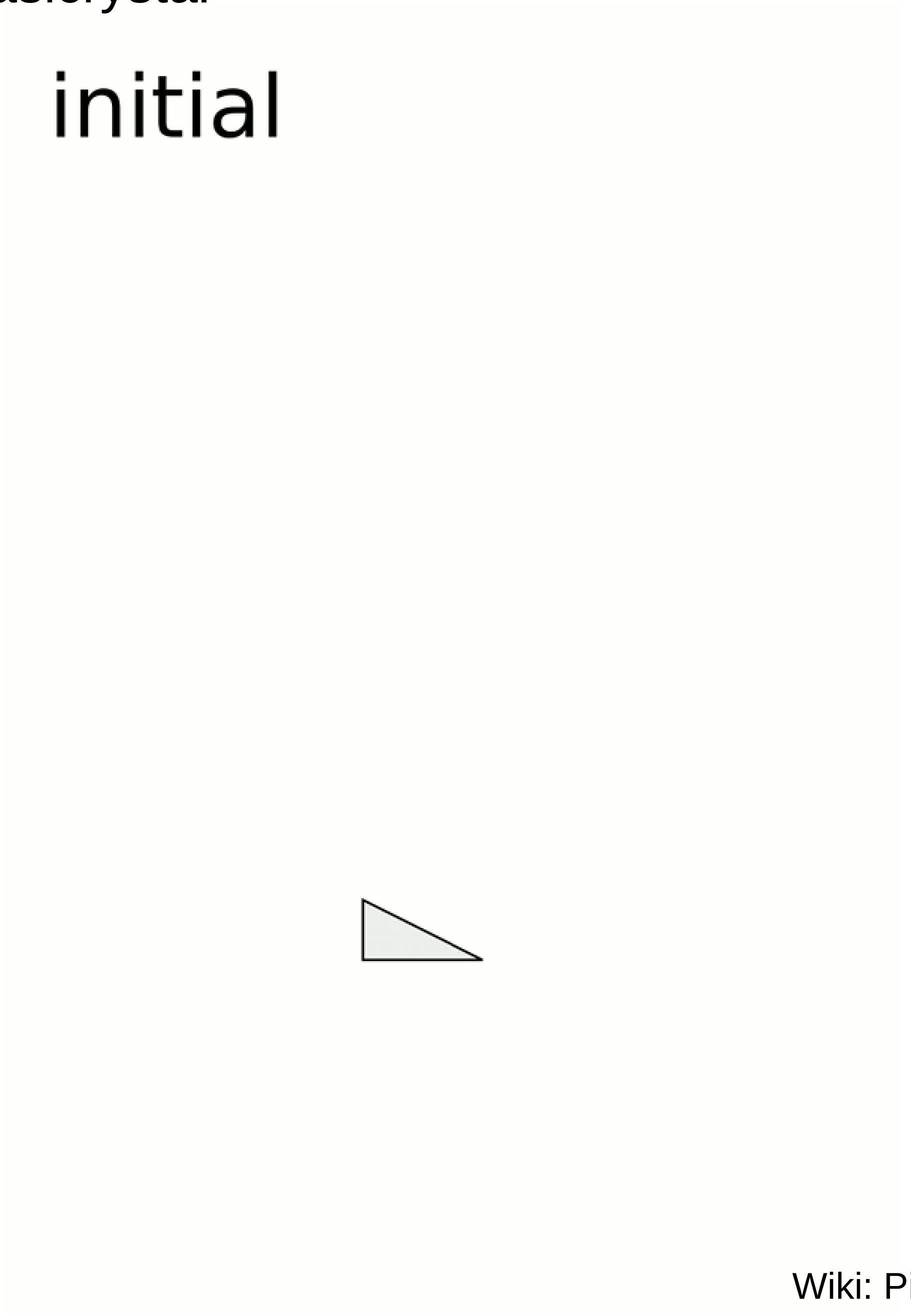
Another example of 2D quasicrystal

Pinwheel tiling

(C. Radin, 1994)

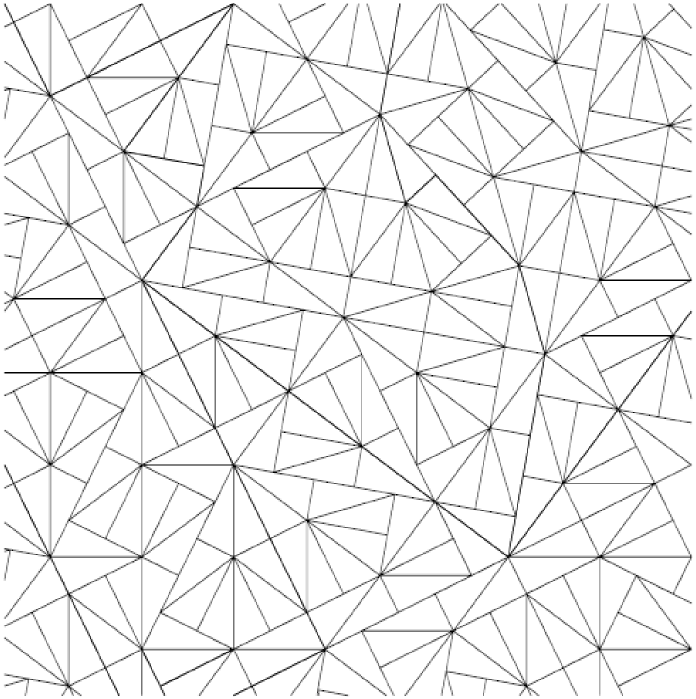
Deflation rule





Wiki: Pinwheel tiling

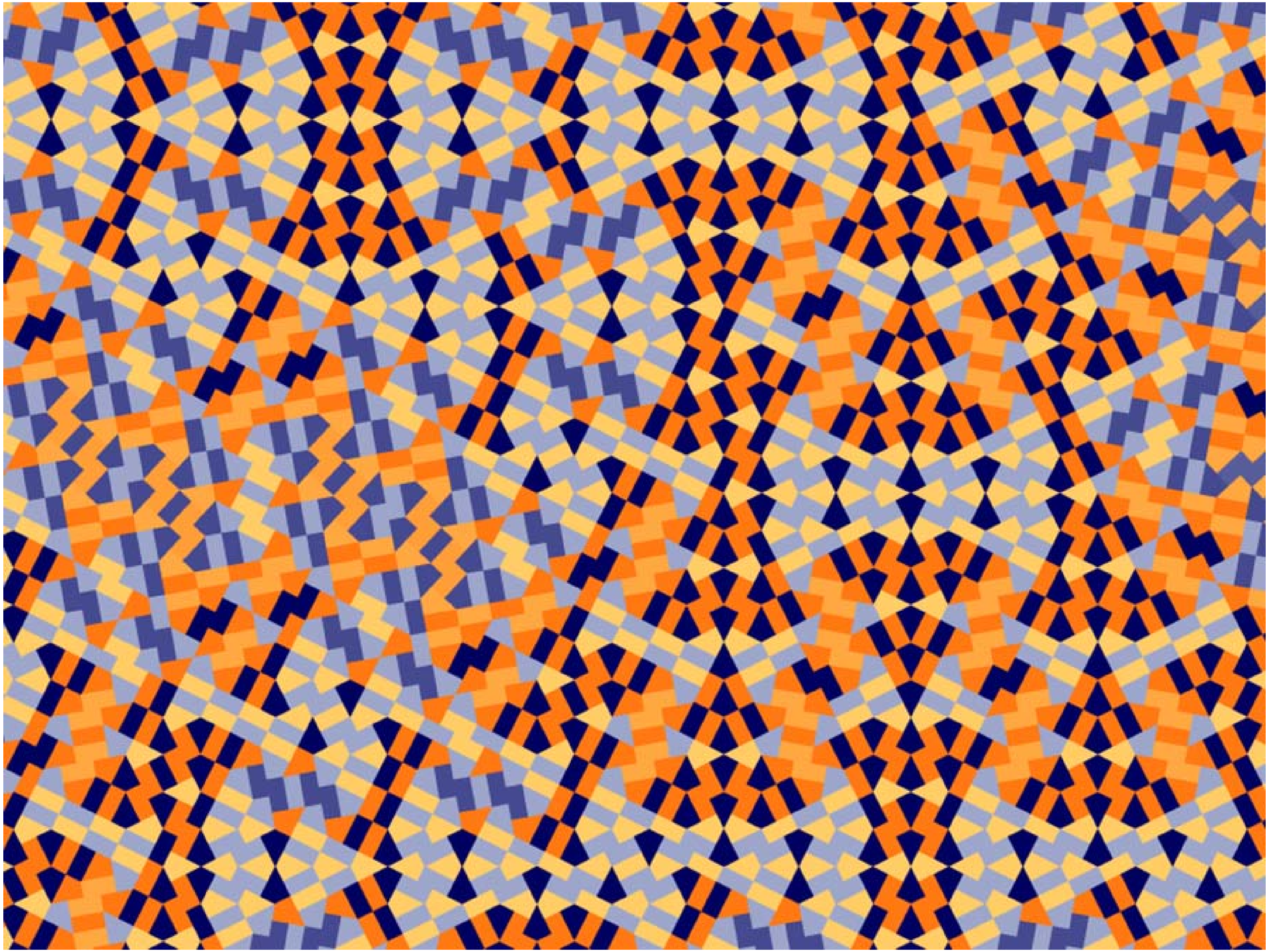
Pinwheel tiling



The Federation Square buildings in Melbourne, Australia

- Can not be obtained by the "cut and project“ construction
- Diffraction pattern is fully rotation invariant

Kite-Domino quasicrystal



And a lot more, at http://tilings.math.uni-bielefeld.de/