Solid state physics 簡答

便箋標題 2011/11/8

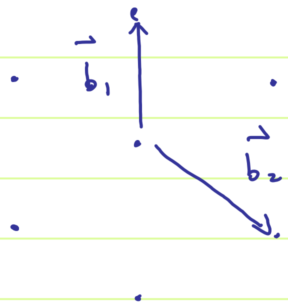
1. (a)

$$\begin{cases} \vec{a}_1 = a\,\hat{x} \\ \vec{a}_2 = \frac{a}{2}\,\hat{x} + \frac{\sqrt{3}}{2}\,a\,\hat{y} \end{cases}$$

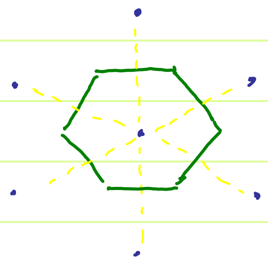
$$\vec{b}_1 \cdot \vec{a}_1 = 2\pi \ , \quad \vec{b}_1 \cdot \vec{a}_2 = 0$$

$$\vec{b}_2 \cdot \vec{a}_1 = 0 \ , \quad \vec{b}_2 \cdot \vec{a}_2 = 2\pi$$

$$\rightarrow \begin{cases} \vec{b}_1 = \frac{2\pi}{a}\,\hat{x} - \frac{2\pi}{\sqrt{3}\,a}\,\hat{y} \\ \vec{b}_2 = \frac{4\pi}{\sqrt{3}}\,a\,\hat{y} \end{cases}$$

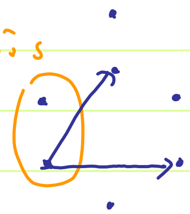


(b)



(c)

$$\begin{cases} \vec{d}_1 = 0 \\ \vec{d}_2 = \frac{a}{\sqrt{3}}\,\hat{y} \end{cases}$$



$$S(\vec{G}) = f_a \sum_j e^{-\vec{G}\cdot\vec{d}_j}$$

$$= f_a\,(1 + e^{-i\vec{G}\cdot\vec{d}_2})$$

$$\vec{G}\cdot\vec{d}_2 = (h\,\vec{b}_1 + k\,\vec{b}_2)\cdot\vec{d}_2$$

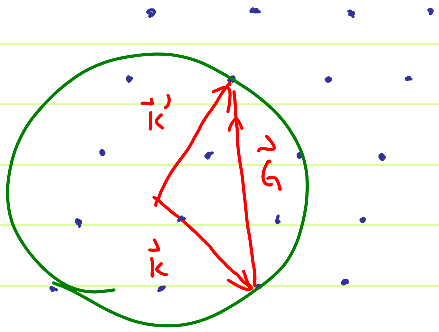
$$= \left[\frac{2\pi}{a}\,h\,\hat{x} + \left(\frac{4\pi}{\sqrt{3}\,a}\,k - \frac{2\pi}{\sqrt{3}\,a}\,h\right)\hat{y}\right]\cdot\frac{a}{\sqrt{3}}\,\hat{y}$$

$$= \frac{2\pi}{3}(2k-h)$$

$$\therefore \quad S_{kh} = f_a\,(1 + e^{-2\pi i\,\frac{2k-h}{3}})$$

$$|S_{kh}|^2 = 2|f_a|^2\left[1 + \cos\frac{2\pi}{3}(2k-h)\right] = \begin{cases} 4\,|f_a|^2 & \text{for } (h,k) = (0,0),\,(2,1),\,\ldots \\ |f_a|^2 & (1,0),\,(0,1),\,\ldots \end{cases}$$

2 (a)



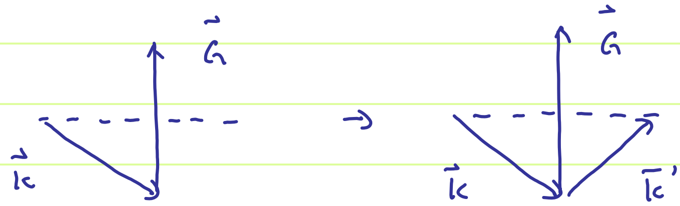
attach $\vec{k}$ to a lattice point
draw a circle (see fig.)
if the circle (or sphere, in 3D) intersects w/ a point, then we find a $\vec{k}'$ that satisfies the Laue condition.

(b) Laue condition:

$$\vec{k}' = \vec{k} + \vec{G}$$

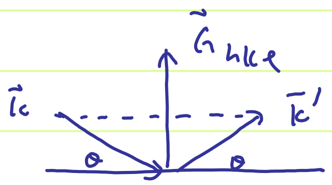
$$\to \vec{k}\cdot\hat{G} = -\frac{G}{2}$$

i.e.



$\vec{G}_{hkl} \perp (hkl)$ plane, $|\vec{k}| = |\vec{k}'|$

∴



from the geometry, we get

$$2k\sin\theta = G_{hkl} = \frac{2\pi}{d_{hkl}}n$$

$$k = \frac{2\pi}{\lambda}$$

$$\therefore \quad 2d_{hkl}\sin\theta = n\lambda$$

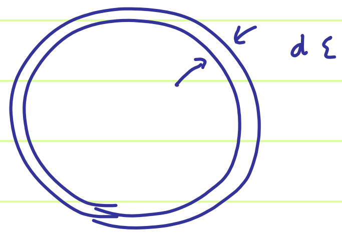
3. (a) A fermi disk has radius k_F

$$2\,\frac{\pi k_F^2}{(2\pi/L)^2} = N$$

$$\rightarrow \quad \frac{k_F^2}{2\pi} = \frac{N}{L^2} = n$$

$$\rightarrow \quad k_F = \sqrt{2\pi n}$$

(b)
$$D(\varepsilon)\,d\varepsilon = 2\cdot\frac{2\pi k\,dk}{(2\pi/L)^2}$$

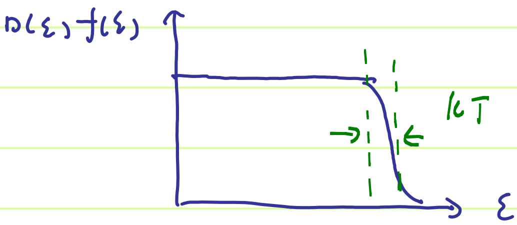


$$= \frac{L^2}{\pi}\,k\,dk$$

$$\frac{\hbar^2 k^2}{2m} = \varepsilon \quad \rightarrow \quad \frac{\hbar^2}{m}\,k\,dk = d\varepsilon$$

$$\therefore \quad D(\varepsilon)\,d\varepsilon = \frac{L^2}{\pi}\frac{m}{\hbar^2}\,d\varepsilon$$

(c) in 2-dim



thermally excited electron $\sim N\dfrac{kT}{\varepsilon_F}$

each electron has energy $\sim kT$

$\therefore$ electrons absorb thermal energy $\delta U \sim N\dfrac{(kT)^2}{\varepsilon_F}$.

specific heat $C_e = \dfrac{dU}{dT} \sim N\dfrac{k^2 T}{\varepsilon_F} \quad \left(\text{or } R\dfrac{T}{T_F}\right)$

$\propto T^1$ as in 3D

4. (a) # of k points in the 1st BZ

$$= \frac{1\ \text{mm}^3}{5\ \text{Å}^3} \qquad 1\ \text{mm} = 10^7\ \text{Å}$$

$$= 2 \times 10^{20}$$

each band can host

$$2 \times 2 \times 10^{20} = 4 \times 10^{20} \text{ electrons}$$

(b)

$$\vec{v} = \langle \psi_{nk} | \frac{\vec{p}}{m} | \psi_{nk} \rangle$$

$$= \langle u_{nk} | \frac{\vec{p} + \hbar\vec{k}}{m} | u_{nk} \rangle$$

$$= \langle u_{nk} | \frac{\partial \tilde{H}}{\hbar \partial \vec{k}} | u_{nk} \rangle , \qquad \tilde{H} = \frac{1}{2m} (\vec{p} + \hbar\vec{k})^2 + U(\vec{r})$$

$$= \frac{\partial}{\hbar \partial \vec{k}} \langle u_{nk} | \tilde{H} | u_{n\vec{k}} \rangle - \langle \frac{\partial}{\hbar \partial \vec{k}} u_{nk} | \tilde{H} | u_{nk} \rangle - \langle u_{nk} | \tilde{H} | \frac{\partial u_{nk}}{\hbar \partial \vec{k}} \rangle$$

$$= \frac{\partial \varepsilon_{nk}}{\hbar \partial \vec{k}} - \varepsilon_{nk} \frac{\partial}{\hbar \partial \vec{k}} \langle u_{nk} | u_{nk} \rangle$$

$$= \frac{\partial \varepsilon_{nk}}{\hbar \partial \vec{k}}$$