Solid State Physics 2:10 PM - 4:30 PM (1/3/2012)

1. (20%) Consider a <u>2-dim</u> rectangular lattice formed by *divalent* atoms. The lattice parameters are 2 A and 3 A respectively.

(a) In the free electron approximation, what is the radius of the Fermi surface?

(b) In the empty lattice approximation, draw the Fermi surfaces using the reduced zone scheme. Darken the parts that are filled by electrons.

2. (20%) In the tight-binding model (using the Wannier function basis), electrons in a <u>1-dim</u> lattice with bond length *a=1* has the following Hamiltonian:

$$H=\sum_{j=0}^{2N-1}\left[t|j\rangle\langle j+1|+t|j+1\rangle\langle j|+h(-1)^j|j\rangle\langle j|\right]$$ (with periodic BC).

(a) Because of the *h*-terms, the period of the lattice is 2*a*. That is, each unit cell has 2 types of atoms. Write $|2j\rangle=|a_j\rangle, |2j+1\rangle=|b_j\rangle$, and do a Fourier transformation,

$$|a_j\rangle=\frac{1}{\sqrt{N}}\sum_k e^{ik2j}|a_k\rangle,\quad |b_j\rangle=\frac{1}{\sqrt{N}}\sum_k e^{ik(2j+1)}|b_k\rangle .$$

What is the form of the Hamiltonian in the new bases $|a_k\rangle, |b_k\rangle$?

(b) Find out the energy eigenvalues and plot the energy bands as a function of *k*.

3. (30%) Consider a <u>2-dim</u> electron gas in the jellium approximation.

(a) Calculate the average kinetic energy per particle (write it in terms of k_F^2).

(b) What is the *Hartree* energy per particle? The *Fock* energy per particle is very tricky, so we'll not do it. Instead, please calculate the pair correlation function $g^{HF}_{ss'}(\vec{r},0)$.

4. (30%) The following 2 separate questions are about phonons:

(a) For a 3-dim solid, the phonon specific heat at low temperature is proportional to T^3. In 2-dim, is it still proportional to T^3? If it is, explain why. If not, give the correct *T*-dependence. Use the heuristic argument, instead of rigorous derivations.

(b) In the study of Mössbauer effect, one needs to evaluate $I\equiv\frac{1}{N}\sum_{ks}\frac{\langle n_{ks}\rangle+1/2}{\hbar\omega_{ks}}$, where *N* is the number of ions, n_{ks} are the phonon occupation numbers. Use the Debye model (with 3 equivalent acoustic branches) to find out $I(T)$ at $T>>\ \theta$ (the Debye temperature).

Useful formulas: $\frac{1}{2\pi}\int_0^{2\pi}dx\, e^{ia\cos x}=J_0(a);\quad \int_0^a J_0(x)x\,dx=aJ_1(a)$