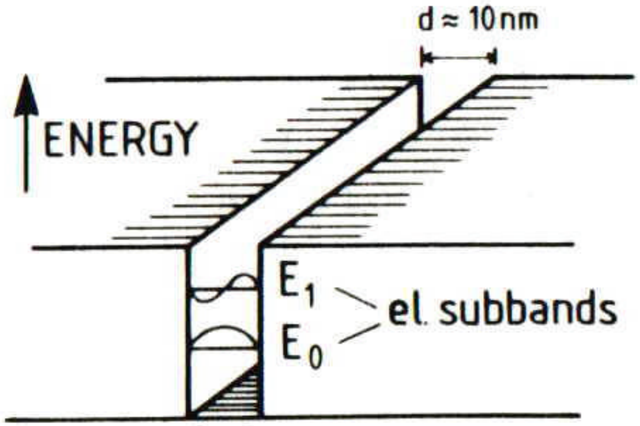
Quantum Hall effect requires

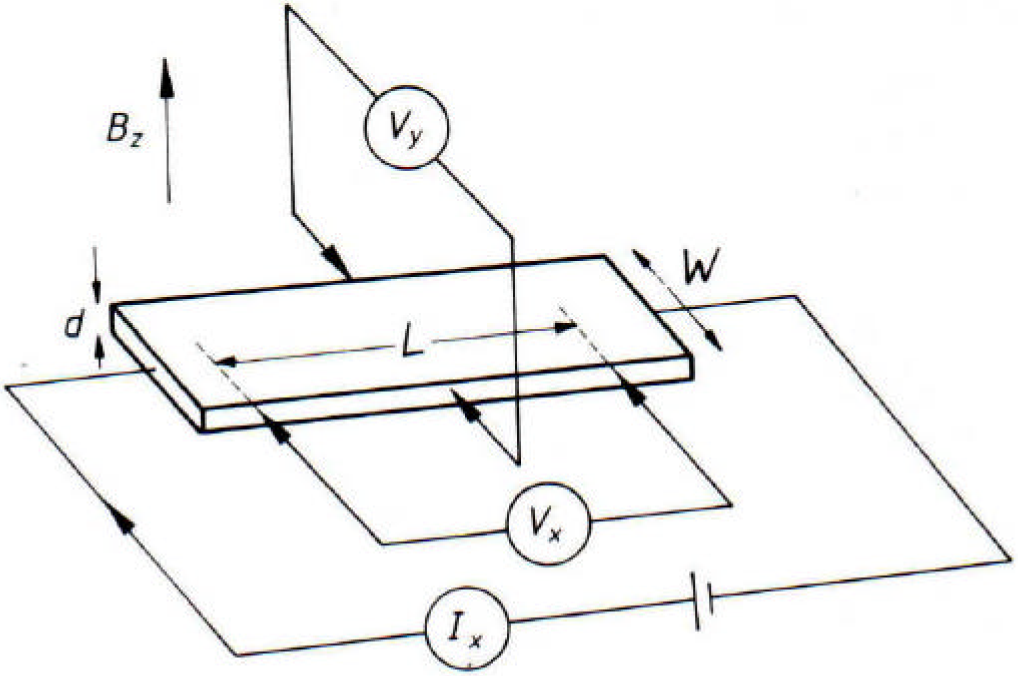
1. Two-dimensional electron gas
2. Very low temperature (< 4 K)
3. Very strong magnetic field (~ 10 Tesla)



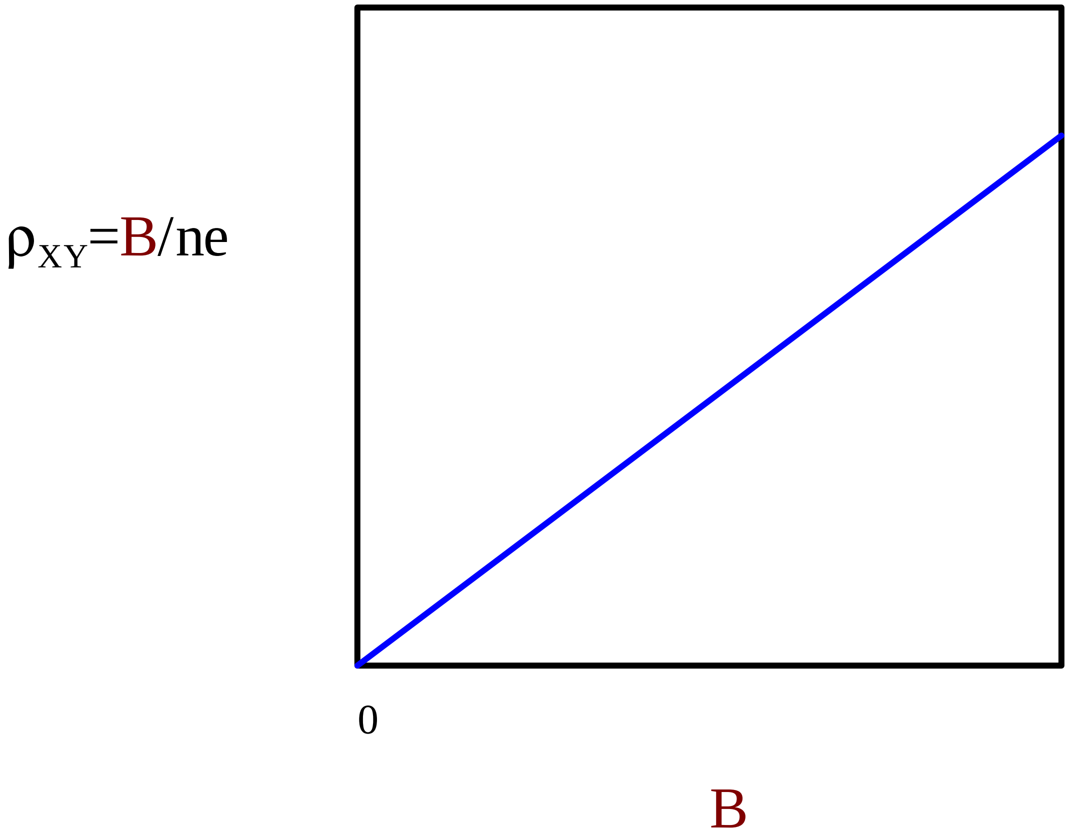
$$E = E_i + \frac{h^2 k_{\parallel}^2}{2m} \; ; \; k_{\parallel}^2 = k_x^2 + k_y^2$$

$$E_1 - E_0 > kT, \Gamma, E_F \rightarrow 2\,\mathrm{DEG}$$

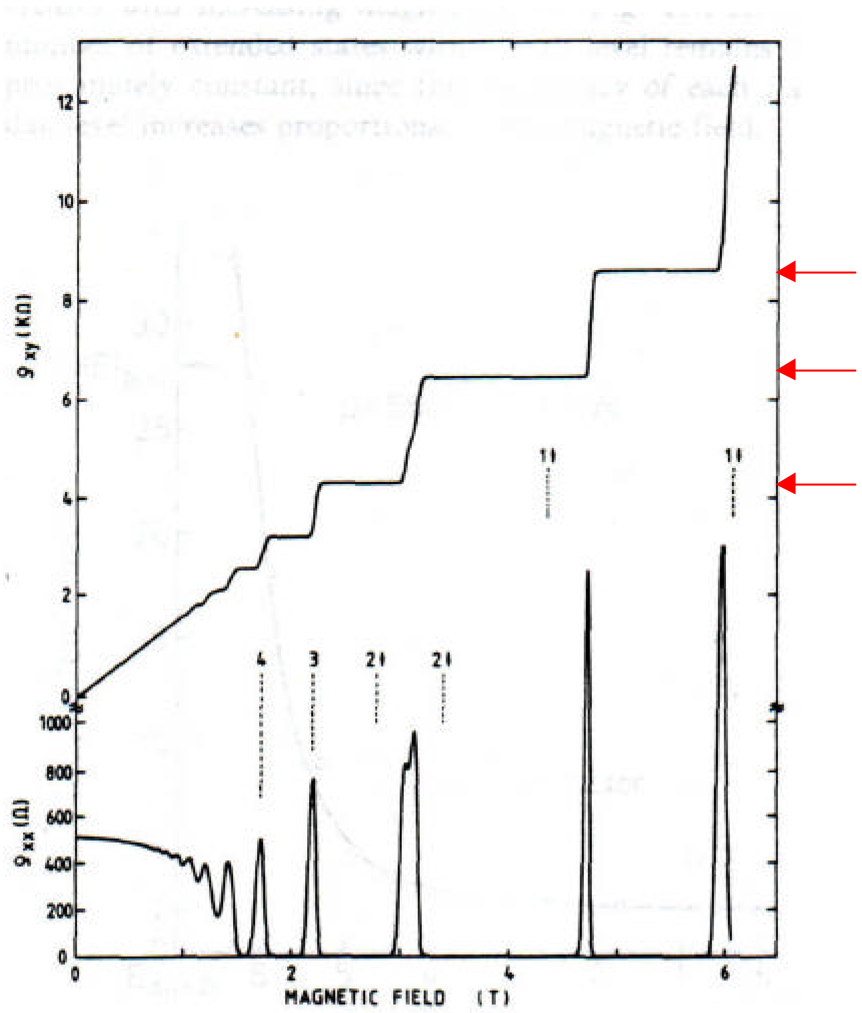
Measurement of Hall resistance



Classical prediction: Hall resistivity $\propto$ B



What’s actually been observed at 8 mK



Plateaus at $(h/e^2)/n$, $h/e^2=25.8128$ kΩ

R_H deviates from $(h/e^2)/n$ by less than 3 ppm
on the very first report of the quantum Hall effect
(von Klitzing, Phys. Rev. Lett. 1980)

- This result is independent of the shape/size of sample.
- Different materials lead to the same effect
  (Si MOSFET, GaAs heterojunction…)

→ a very convenient resistance standard

→ a very accurate way to measure a

$$\alpha^{-1} = h/e^2c \approx 137.036 \text{ (unit-indep.)}$$

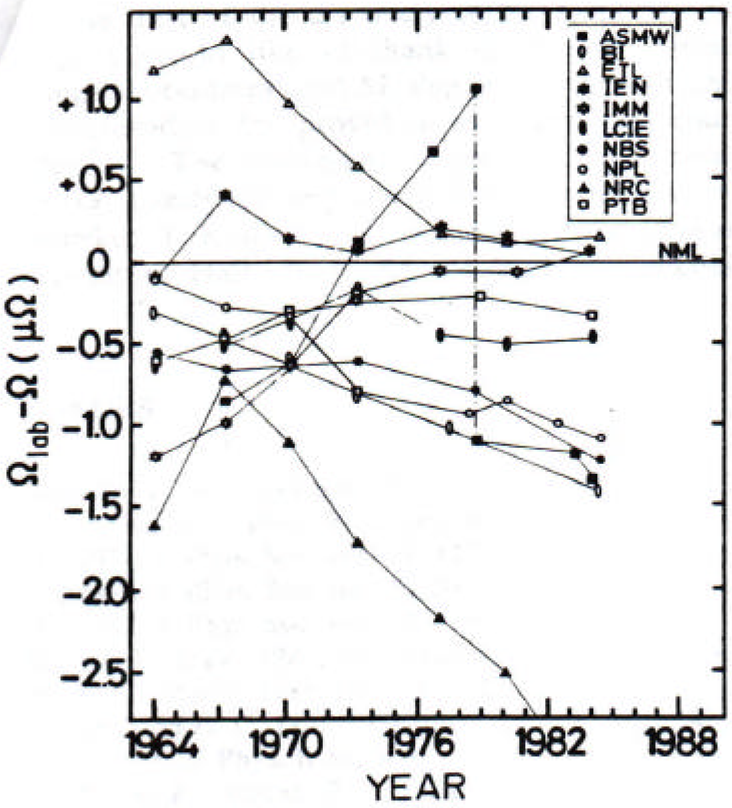


FIG. 26. Time dependence of the 1-Ω standard resistors maintained at the different national laboratories.

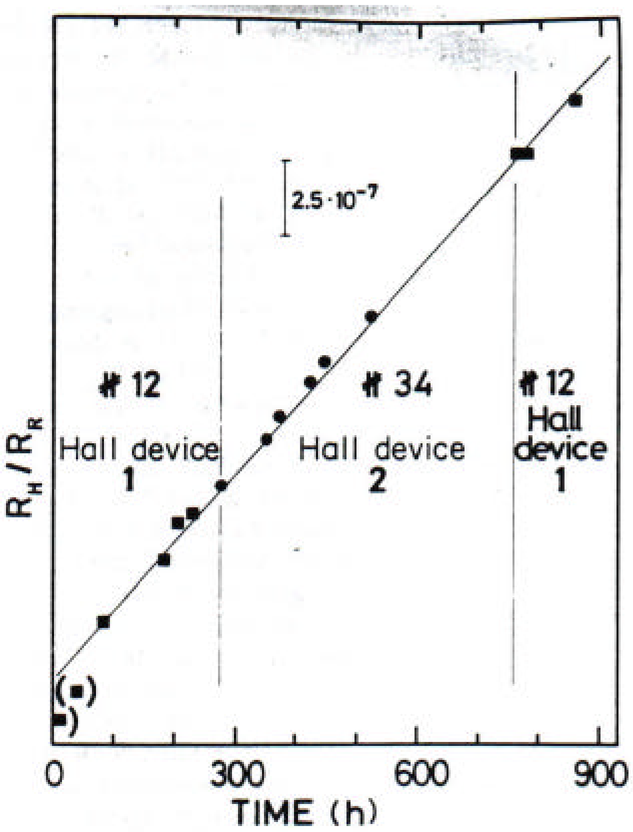
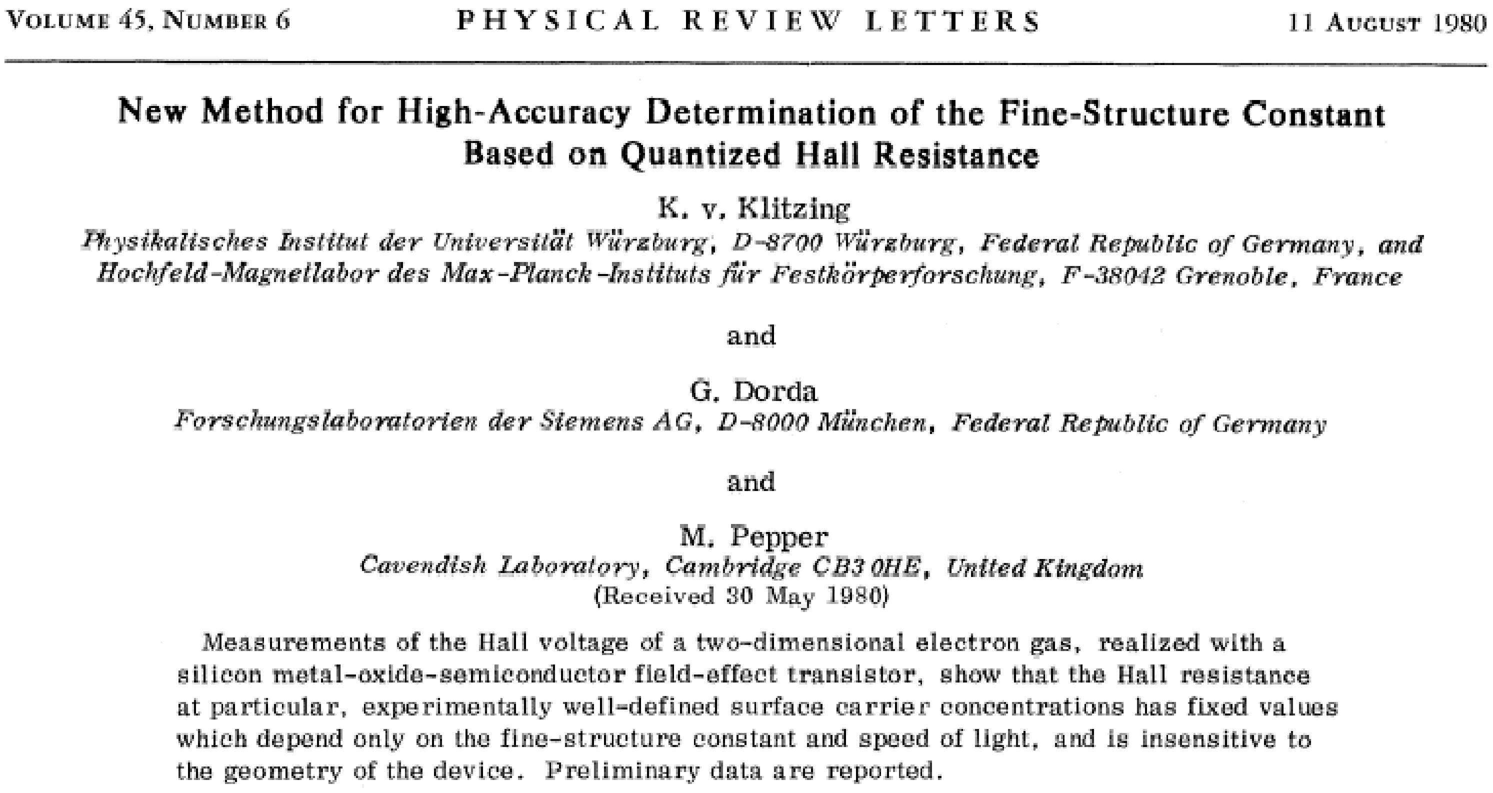


FIG. 27. Ratio R_H/R_R between the quantized Hall resistance R_H and a wire resistor R_R as a function of time. The result is time dependent but independent of the Hall device used in the experiment.



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New Method for High-Accuracy Determination of the Fine-Structure Constant Based on Quantized Hall Resistance

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(Received 30 May 1980)

Measurements of the Hall voltage of a two-dimensional electron gas, realized with a silicon metal-oxide-semiconductor field-effect transistor, show that the Hall resistance at particular, experimentally well-defined surface carrier concentrations has fixed values which depend only on the fine-structure constant and speed of light, and is insensitive to the geometry of the device. Preliminary data are reported.

experiment

$$\left\{\begin{aligned} \alpha^{-1}(\text{q. Hall}) &= 137.035\,997\,9(32) \quad (0.024\ \text{ppm}), \\ \alpha^{-1}(\text{acJ}) &= 137.035\,977\,0(77) \quad (0.056\ \text{ppm}), \\ \alpha^{-1}(h/m_n) &= 137.036\,010\,82(524) \quad (0.039\ \text{ppm}). \end{aligned}\right.$$

theory

$$\alpha^{-1}(a_e) = 137.035\,999\,44(57) \quad (0.0042\ \text{ppm}).$$

(Kinoshita, Phys. Rev. Lett. 1995)

Why R_H has to be exactly $(h/e^2)/n$?

The importance of

1. Landau level (→ discrete levels)
2. Disorder and localization (→ plateaus)
3. Hidden gauge symmetry (→ quantization of R_H)

   (Laughlin, Phys. Rev. 23, 5632 (81))

Adventures to the 2-dim electron system

'80 discovery of QHE (von Klitzing)

connection with topology (Laughlin, Thouless)

fractional QHE (Stormer, Tusi, Gossard) }

fractional charged excitation (Laughlin) }

Superfluid analogy (CSGL theory, S.C. Zhang)

Scaling of plateau transition (Pruisken)

'90 global phase diagram (Kivelson, Lee, Zhang)

edge states as chiral Luttinger liquid ? (X.G. Wen)

composite fermion description (Jain)

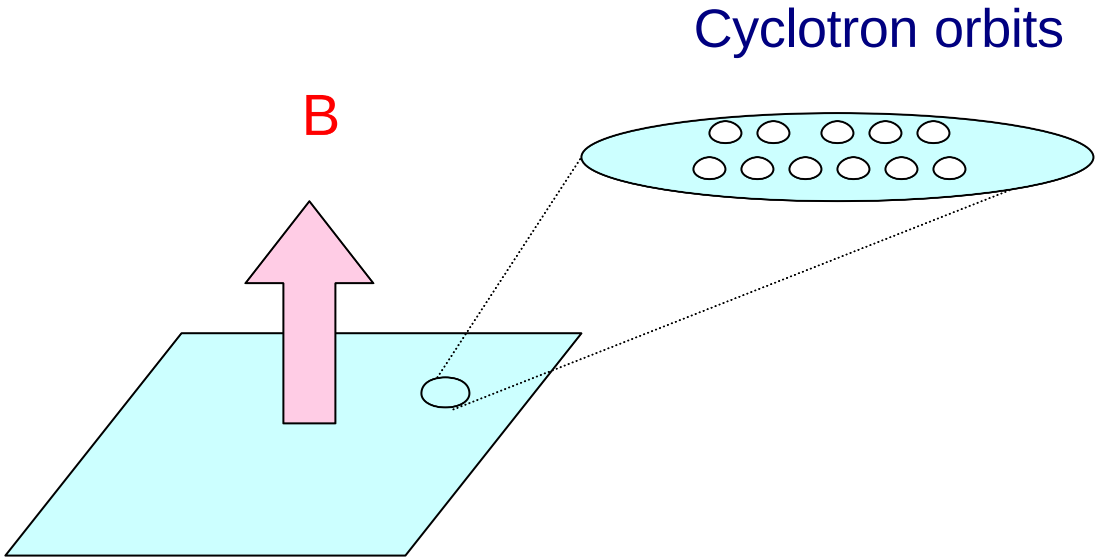
charge-vortex duality (Shahar)

stripe phase (Lilly et al)

skyrmion excitation for v=1 (Sondhi) ← spin

'00 Josephson-like effect in bilayer system (Girvin, MacDonald) ←pseudo-spin

Degeneracy of a Landau level (LL)



Number of degenerate orbitals for each LL

$D = F/F_0$ (F_0: flux quantum)

eg. for $F = 10\ \mathrm{T} \times 1\ \mathrm{cm}^2$

$D = 10^{11}$

If $N = 10^{11}/\ \mathrm{cm}^2$, then filling factor $\nu = 1$

Search and discovery, Physics Today

2000 June*: Spin and isospin: exotic order in QH ferromagnets (Girvin)

2000 May: QHE - in pentacene?

2000 Apr*: The composite fermion: a quantum particle and its quantum fluids (Jain)

1998 Dec: Physics Nobel prize goes to Tsui, Stormer, and Laughlin for the FQHE

1998 Dec: Two-dimensional electron gases continue to exhibit intriguing behavior (Charge density wave)

1997 Nov: Fractional charged quasiparticles signal their presence with noise

1997 Feb: In a QH System, is the insulator really a conductor in vortex clothing? (Duality)

1996 Sep: One-dimensional systems show signs of interacting electrons (edge states as 1-D chiral Luttinger liquid)

1995 July: In a two-dimensional electron system, the skyrmion's the limit

1994 June: Experiment reveals a new type of electron system (edge states as 1-D chiral Luttinger liquid)

1993 July: Half-filled Landau level yields intriguing data and theory

1990 Dec: Evidence accumulates, at last, for the Wigner crystal

1990 Jan: Experiments provide evidence for the fractional charge of quasiparticles

1989 Nov: Bosons condense and fermions exclude, but anyons...?

1988 Sep: Universal singular behavior is observed in QHE (localization)

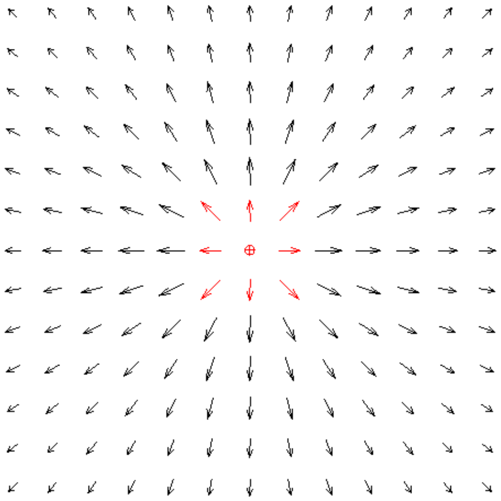
1988 Jan: QHE shows surprising even-denominator plateau

1985 Dec: von Klitzing wins Nobel physics prize for QHE

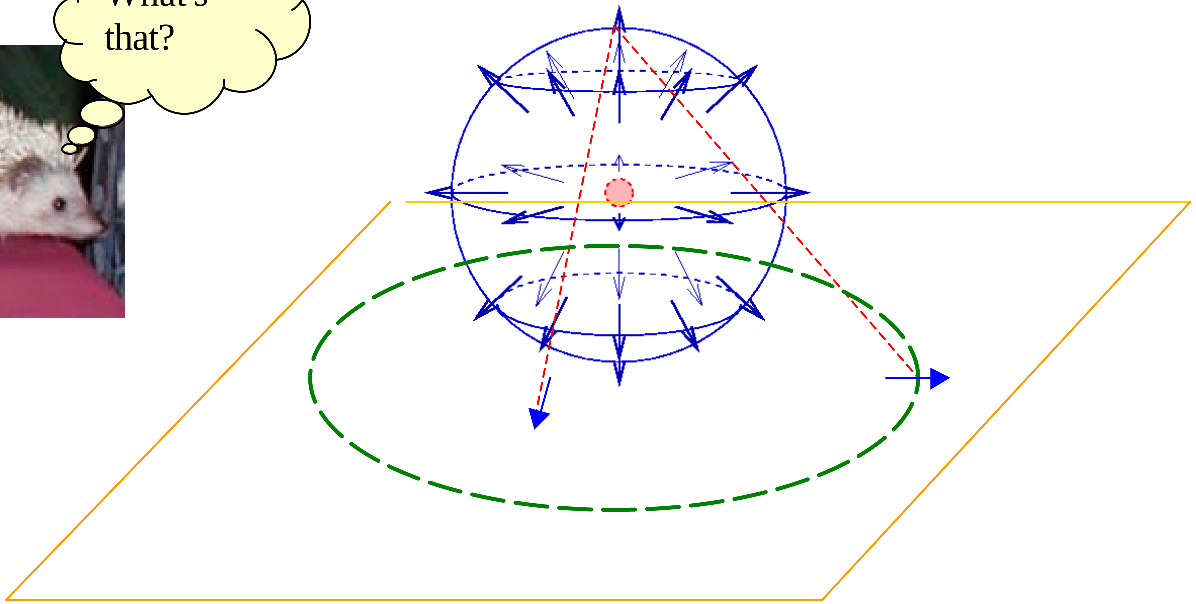
1983 July: FQHE indicates novel quantum liquid

1981 June: QHE yields e^2/h to part per million

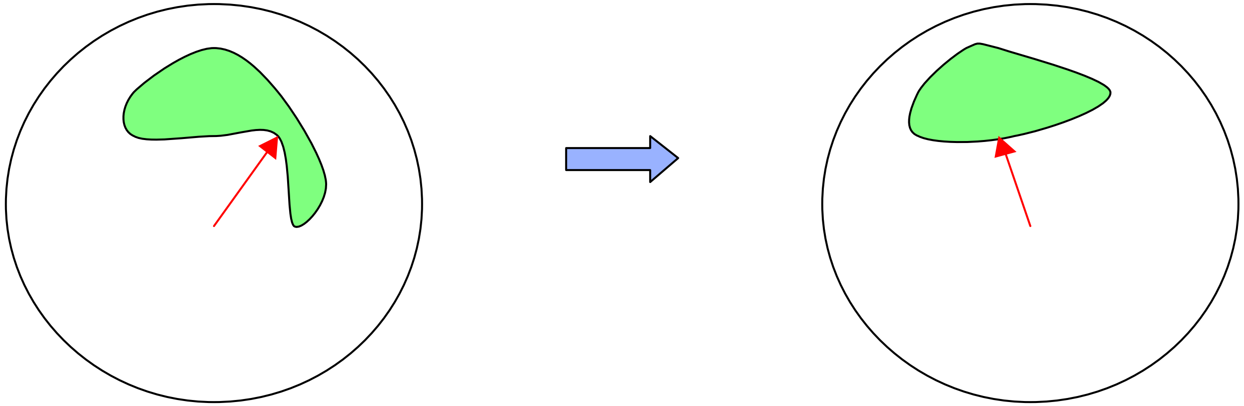
skyrmion



"Hedgehog"



Topological property of skyrmion



tip of position **r**

tip of spin **n**

- continuous **r** → **n** mapping

  "wrapping" number = integer Q_T

  robust again continuous deformation

- skyrmion charge $Q_e = \nu e Q_T$