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Finite-temperature transitions in $\nu=2$ bilayer quantum Hall systems

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In this paper, the influence of an in-plane magnetic field $B_\parallel$ on the transitions at finite temperatures in $\nu=2$ bilayer quantum Hall systems is examined. It is found that there can exist two types of characteristic temperatures which separate different regions in the phase diagram. The first is the Kosterlitz-Thouless (KT) transition temperature, which can have an unusual *nonmonotonic* dependence on $B_\parallel$; the second type originates from the crossing of energy levels and always increases with $B_\parallel$. Based on these results, we point out that the threshold temperature observed in the inelastic light scattering experiments cannot be the KT transition temperature, because the latter shows a totally different $B_\parallel$ dependence as compared with the experimental observation. Instead, it should be the level-crossing temperature, which we found agrees with the $B_\parallel$ dependence observed. Thus an investigation of the KT transition in the bilayer quantum Hall systems awaits experimental measurements to probe the proposed canted antiferromagnetic phase more directly.

Recent theoretical works predict that, besides the fully spin polarized ferromagnetic phase (F) and the paramagnetic symmetric or spin singlet (S) phase, a novel canted antiferromagnetic (C) phase can exist in the filling factor $\nu=2$ bilayer quantum Hall (QH) systems.[1–7] In such a C phase, the electron spins in each layer are tilted away from the external magnetic field direction due to the competition between ferromagnetic ordering and singlet ordering. Encouraging experimental evidence in support of the C phase has recently emerged through inelastic light scattering spectroscopy,[8,9] transport measurements,[10,11] and capacitance spectroscopy.[12] In particular, it is observed for certain samples in the inelastic light scattering experiments[8,9] that there is a threshold temperature T_{SDE} below which the spin conserved spin-density excitation (SDE) mode (ω_0 mode) seems to lose all spectral weight. Because the $U(1)$ planar spin rotational symmetry is spontaneously broken in the C phase, there should be a finite-temperature Kosterlitz-Thouless (KT) transition with a characteristic energy scale which is about the vortex-antivortex binding energy.[13] It is claimed that the observed threshold temperature is the predicted KT transition temperature T_{KT} in the C phase.[2,3]

While the predicted value of the KT transition temperature ($T_{KT}\approx 1.8$ K in the Hartree-Fock theory[2,3]) is reasonably close to that of the threshold temperature ($T_{SDE}\approx 0.52$ K) in the inelastic light scattering experiment under normal magnetic fields,[8] which seems to support the identification between these two temperature scales, we point out that such an interpretation cannot be used in the tilted magentic field experiment[9] for two reasons. First, it is found in Ref. 9 that the threshold temperature rises as the parallel magnetic field $B_\parallel$ increases. Nevertheless, we notice that (i) the sample used in the experiment is located near the F-C phase boundary in the quantum phase diagram (see the inset in Fig. 1); and (ii) an in-plane magnetic field *effectively* moves a sample even closer to the F-C phase boundary.[14] Hence the symmetry-breaking order parameter, and therefore T_{KT} should be reduced (cf. Figs. 8 and 11 of Ref. 3), rather than enhanced, when $B_\parallel$ increases. Thus the physical content of these two characteristic temperatures should not be the same. Second, it is questionable to regard the observed disappearance of the ω_0 mode at the threshold temperature as

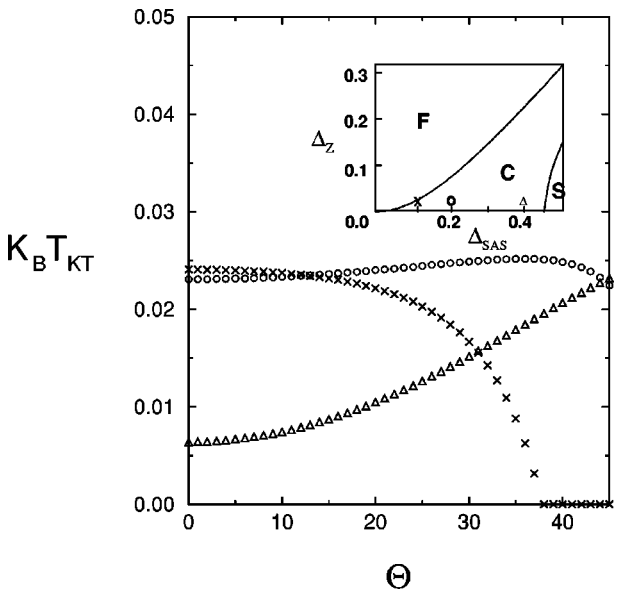

FIG. 1. k_BT_{KT} as a function of the tilted angle Θ (in units of degrees) of the applied magnetic field with a fixed $B_\perp$. The length and the energy units are chosen to be the magnetic length l and the intralayer Coulomb energy $e^2/\epsilon l$. The Zeeman energy caused by $B_\perp$ is $\Delta_z=0.008$. The interlayer separation is $d=1.45$ and the layer thickness is $b=1.0$. Crosses, circles, and triangles correspond to $\Delta_{\mathrm{SAS}}=0.1$, 0.2, and 0.4, respectively. Their locations in the $\Delta_z-\Delta_{SAS}$ quantum phase diagram calculated by the Hartree-Fock theory with $\Theta=0°$ are shown in the inset. Notice that the cross symbol represents the experimental sample of Ref. 9, and its location is very close to the F-C phase boundary in the quantum phase diagram.

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the transition to the C phase, if one is reminded that the spectral weight of the ω_0 mode (which does not involve any spin flip) is also greatly suppressed in the F phase, where almost all spin-up (down) states are occupied (empty).[15] When temperature $T \gtrsim T_{KT}$ for the systems near the F-C quantum phase boundary, it is expected that, although the expectation value of the in-plane spin component vanishes and the $U(1)$ planar spin rotational symmetry is restored, the spin component S_z along the direction of the external magnetic field may still be nonzero. That is, these systems at $T \gtrsim T_{KT}$ should behave somewhat like the F phase at finite temperatures. Thus, higher temperatures are necessary for these systems to lose all their spin polarizations, so that the ω_0 mode can be observed.

In this paper, the transitions at finite temperatures in $\nu = 2$ bilayer quantum Hall systems are investigated. As discussed above, no one has yet reached the correct theoretical understanding for the reported threshold temperature. Hence we focus our attention on solving this issue. Since the aforementioned arguments are quite general, the same qualitative results should be obtained irrespective of which kind of approximation methods being employed.[16] For simplicity, we use the Hartree-Fock approximation in the following. We show that the KT transition temperature in the C phase[2,3] can have an unusual *nonmonotonic* dependence on the tilted angle of the applied magnetic field. That is, T_{KT} can either rise or fall as $B_\parallel$ is turned on, depending on whether the samples are initially located near the C-S or the F-C phase boundary in the quantum phase diagram. By using the sample parameters in the tilted field experiment,[9] we show that T_{KT} *decreases* as the tilted angle increases. Thus the KT transition scenario fails to explain the $B_\parallel$ dependence of the threshold temperature in Ref. 9. Instead, in order to link with the observed threshold temperature, we propose another characteristic temperature T_X caused by the crossing of energy levels, since its variation with respect to $B_\parallel$ agrees qualitatively with the reported threshold temperature. Nevertheless, the value of T_X we obtained ($T_X \approx 17$ K) by using the actual experimental sample parameters is considerably larger than the observed T_{SDE}. This quantitative discrepancy between T_X and T_{SDE} may be reduced when one goes beyond the simple mean-field treatment, and our interpretation for the observed enhancement of T_{SDE} should remain vaild. Based on the dependence of T_{KT} and T_X on the tunneling-induced symmetric-antisymmetric energy gap and the in-plane magnetic field, a detailed finite-temperature phase diagram is shown.

As shown in Ref. 3, the KT transition temperature is estimated to be[17] $T_{KT} \approx 0.9\rho_s/k_B$, with the spin stiffness $\rho_s = c_A\rho_s^A + c_E\rho_s^E$, where

$$\rho_s^{A/E} = \frac{l^2}{4\pi}\sum_p p^2 V_{A/E}(p,0), \tag{1}$$

and the analytical forms of the constants c_A and c_E can be written down explicitly by minimizing the Hartree-Fock variational energy functional directly:[7]

$$c_A = \frac{\Delta_{\rm SAS}^2[(\Delta_{\rm SAS}^2-\Delta_z^2)^2-(2U_-\Delta_z)^2][(2U_-\Delta_{\rm SAS})^2-(\Delta_{\rm SAS}^2-\Delta_z^2)^2]}{(2U_-)^2(\Delta_{\rm SAS}^2-\Delta_z^2)^4}, \tag{2}$$

$$c_E = \frac{[(2U_-\Delta_z)^2-(\Delta_{\rm SAS}^2-\Delta_z^2)^2][\Delta_z^2(\Delta_{\rm SAS}^2-\Delta_z^2)^2-\Delta_{\rm SAS}^2(2U_-\Delta_z)^2]}{(2U_-)^2(\Delta_{\rm SAS}^2-\Delta_z^2)^4}. \tag{3}$$

Here $\Delta_{\rm SAS}$ is the tunneling-induced symmetric-antisymmetric energy separation, Δ_z is the Zeeman energy, and $U_- = (U_A - U_E)/2$ with $U_{A/E} = \Sigma_p V_{A/E}(p,0)$ being the exchange energy of the intralayer/interlayer Coulomb interaction. The matrix elements $V_{A/E}(p_1,p_2)$ of the intralayer/interlayer Coulomb interaction are

$$V_{A/E}(p_1,p_2) = \frac{1}{\Omega}\sum_{\mathbf{q}} v_{A/E}(q)\,\delta_{p_1,q_y} e^{-q^2l^2/2} e^{iq_x p_2 l^2}, \tag{4}$$

where Ω is the area of the system. $v_A(q) = (2\pi e^2/\epsilon q)F_A(q,b)$ and $v_E(q) = v_A(q)F_E(q,b)e^{-qd}$ are the Fourier transforms of the intralayer and the interlayer Coulomb interaction potentials. ϵ is the dielectric constant of the system, and d is the interlayer separation. We have also included the finite-well-thickness correction by introducing the form factor $F_{A/E}(q,b)$ in the intralayer/interlayer Coulomb potential, where $F_A(q,b) = 2/bq - 2(1-e^{-qb})/b^2q^2$, $F_E(q,b) = 4\sinh^2(qb/2)/b^2q^2$, and b is the width of a quantum well.[14,18] Since we know ρ_s exactly within the microscopic Hartree-Fock approximation, the KT transition temperature can be easily determined. As shown in Refs. 2 and 3, the KT transition temperature along with the symmetry-breaking order parameter drops continuously to zero as the phase boundaries are approached from within the C phase.

Now we consider the tilted magnetic field case, where a parallel magnetic field $B_\parallel$ and a perpendicular field $B_\perp$ both appear with the tilted angle $\Theta = \tan^{-1}(B_\parallel/B_\perp)$. The effect of the parallel magnetic field on T_{KT} can be incorporated by the following replacements:[14,19]

$$\Delta_{\rm SAS} \to \bar{\Delta}_{\rm SAS} = \Delta_{\rm SAS} e^{-Q^2l^2/4},$$

$$\Delta_z \to \bar{\Delta}_z = \Delta_z\sqrt{1+(B_\parallel/B_\perp)^2}, \tag{5}$$

$$V_E(p_1,p_2) \to \bar{V}_E(p_1,p_2) = V_E(p_1,p_2)e^{\pm iQp_1l^2},$$

with $Q=B_{\parallel}d/B_{\perp}l^2$ and the magnetic length $l=\sqrt{\hbar c/eB_{\perp}}$. In Fig. 1 we show the transition temperature T_{KT} as a function of the tilted angle Θ for some typical sample parameters. Since it is relatively easy to tune $\Delta_{\rm SAS}$ in fabrication, we vary its value with other system parameters being fixed. Three possible situations are depicted in Fig. 1: (i) if the system begins in the C phase and near the C-S phase boundary (triangle), then T_{KT} (and ρ_s) grows as $B_{\parallel}$ is turned on; (ii) if the system is in the C phase and near the F-C phase boundary (cross), then T_{KT} (and ρ_s) is reduced as $B_{\parallel}$ is turned on; (iii) when the system lies between the two phase boundaries (circle), T_{KT} (and ρ_s) can have an unusual *nonmonotonic* dependence on $B_{\parallel}$, that is, it can increase and then decrease as $B_{\parallel}$ increases. We find that the enhancement (suppression) of T_{KT} as Θ increases can be large for the system near the C-S (F-C) phase boundary. However, when the system lies between the two phase boundaries, the magnitude of T_{KT} may have roughly the same value. Since the crossed line is the predicted T_{KT} for the sample studied in Ref. 9, which has a decreasing dependence on Θ, the experimentally observed enhancement of T_{SDE} can not be explained by the result of T_{KT}.

Motivated by the above results, we look for another characteristic temperature above which the spectral weight of the ω_0 mode indeed becomes significant. As mentioned before, for the systems near the F-C phase boundary in the quantum phase diagram, a nonvanishing spin polarization is possible when $T\gtrsim T_{KT}$. Consequently, the mean-field Hamiltonian of the self-consistent Hartree-Fock theory at finite temperatures, which takes this fact into account, reduces to

$$H^{HF}=-\frac{\Delta_{\rm SAS}+\delta_{\rm SAS}}{2}\sum_{\tau,k,\sigma}\tau c^{\dagger}_{\tau,k,\sigma}c_{\tau,k,\sigma}$$

$$-\frac{\Delta_z+\delta_z}{2}\sum_{\tau,k,\sigma}\sigma c^{\dagger}_{\tau,k,\sigma}c_{\tau,k,\sigma}, \qquad (6)$$

where $\delta_{\rm SAS}=(U_E/2)\Sigma_{\tau,\sigma}\tau f(E_{\tau,\sigma})$ and $\delta_z=(U_A/2)\Sigma_{\tau,\sigma}\sigma f(E_{\tau,\sigma})$. Here the Landau gauge is assumed, and $c^{\dagger}_{\tau,k,\sigma}$ creates an electron at the lowest Landau level orbital k in the symmetric ($\tau=1$) or the antisymmetric ($\tau=-1$) subbands with spin $\sigma/2(\sigma=\pm1)$. The thermal averages $\langle c^{\dagger}_{\tau,\sigma}c_{\tau,\sigma}\rangle=f(E_{\tau,\sigma})$, where $f(E)$ is the Fermi-Dirac distribution function and the energy eigenvalues of this mean-field Hamiltonian are

$$E_{\tau,\sigma}=-\frac{\tau}{2}(\Delta_{\rm SAS}+\delta_{\rm SAS})-\frac{\sigma}{2}(\Delta_z+\delta_z). \qquad (7)$$

We assume that the translational symmetry is not broken, thus these expectation values have no intra-Landau level dependence. Since we consider only the case of $T>T_{KT}$, the order parameters for the C phase are dropped. Moreover, because of the symmetry in the energy levels, the chemical potential is fixed at zero for all temperatures. We see that the four energy levels for the noninteracting electrons are shifted by the self-consistent mean fields, $\delta_{\rm SAS}$ and δ_z, both of which have temperature dependence.

For $T>T_{KT}$, if we assume that the effective Zeeman energy, $\Delta_z+\delta_z$, is initially larger than the effective symmetric-antisymmetric energy gap, $\Delta_{\rm SAS}+\delta_{\rm SAS}$, the crossing of energy levels can occur at a higher temperature $T=T_X>T_{KT}$, because the mean field δ_z is a monotonic decreasing function of T. Thus at $T=T_X$, where level crossing occurs, one has

$$\Delta_{\rm SAS}+\delta_{\rm SAS}=\Delta_z+\delta_z, \qquad (8)$$

$$\frac{1}{2}\sum_{\tau,\sigma}\sigma f(E_{\tau,\sigma})=\frac{1}{2}\sum_{\tau,\sigma}\tau f(E_{\tau,\sigma})=f(E_{+1,+1})-\frac{1}{2}. \qquad (9)$$

By solving Eqs. (8) and (9) with Eq. (7), the level-crossing temperature T_X can be determined. Combining the knowledge of the KT transition temperature T_{KT},[2,3] a detailed phase diagram at finite temperatures can be obtained, as shown in Fig. 2. The shift of these two characteristic temperatures due to an in-plane component of a magnetic field tilted by $\Theta=30°$ is also presented. Because of the thermal population difference between the Zeeman split spin-up and spin-down states, $\langle S_z\rangle$ is always nonvanishing at finite temperatures, thus there is no symmetry distinction between the two regions (I and II) separated by T_X. Closer examination of the variation of free energy reveals that this is a crossover, rather than a phase transition, at the level-crossing temperature T_X. From the relative positions of the four energy levels in Eq. (7), one may call the region I (II) to be F-like (S-like). Note that for $\Delta_{\rm SAS}$ slightly larger than $0.23e^2/\epsilon l$, the C phase transits directly to the S-like region at finite temperatures. It can be seen that an in-plane magnetic field moves the boundaries to the right, due to the effective modification of the sample parameters given in Eq. (5). With a fixed $\Delta_{\rm SAS}$, the change of the transition temperatures for a tilted magnetic field with $\Theta=30°$ can be read out directly from Fig. 2. This phase diagram indicates that T_X is an increasing function of Θ. By using Eq. (5), the tilted-field dependence of T_X is

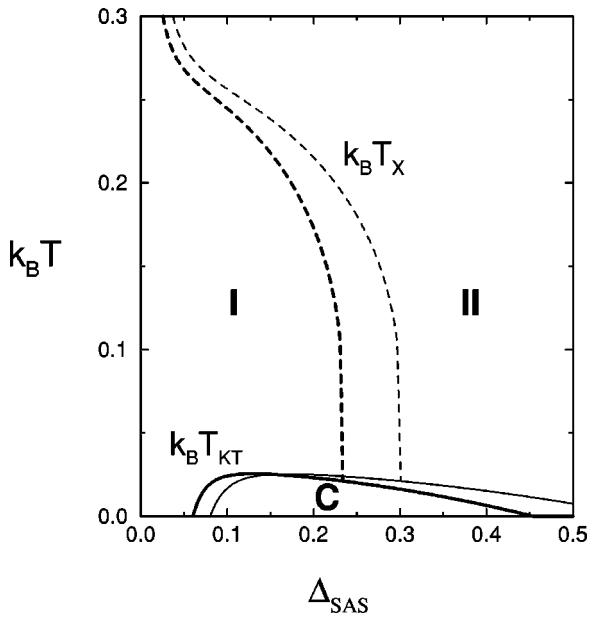

FIG. 2. $\nu=2$ bilayer phase diagrams at finite temperatures. The energy unit is $e^2/\epsilon l$. The sample parameters, Δ_z, d, and b are the same as in Fig. 1. The continuous lines are the phase boundaries for the KT transitions. The dashed lines are for the crossovers. The boundaries of the KT transitions and the crossovers under a magnetic field tilted by $\Theta=30°$ with a fixed $B_{\perp}$ are shown with thinner continuous and dashed lines, respectively.

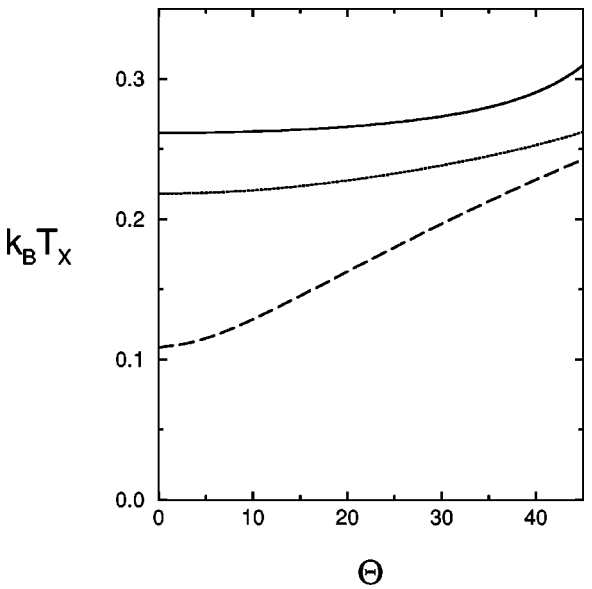

FIG. 3. $k_B T_X$ as a function of the tilted angle Θ (in units of degrees) of the applied magnetic field with a fixed $B_\perp$. The energy unit is $e^2/\epsilon l$. Continuous, dotted, and dashed lines correspond to $\Delta_{\rm SAS}=0.062$, 0.15, and 0.23, respectively. The sample parameters, Δ_z, d, and b are the same as in Fig. 1.

explicitly shown in Fig. 3. The values of $\Delta_{\rm SAS}$ are chosen such that the corresponding samples can undergo a crossover from the F-like region to the S-like region at temperatures T_X. In general, it takes a higher T_X for a sample with a smaller $\Delta_{\rm SAS}$ to transit to the S-like region, which is reasonable since the F-like region becomes more stable for a larger ratio of $\Delta_z/\Delta_{\rm SAS}$. The in-plane magnetic field always elevates T_X, in contrast to its effect on T_{KT}, which is more complicated, as shown in Fig. 1. The result shows that for systems near the $T=0$ F-C phase boundary (say, for the cross symbol in the inset of Fig. 1), the level-crossing temperature T_X indeed increases with $B_\parallel$ and should be identified as the experimentally observed T_{SDE}.

Before concluding this paper, some remarks are in order. First, we would like to comment that the threshold temperature in the experiment[8] is $T_{SDE}\approx 0.5$ K, which is considerably lower then the calculated $T_X\approx 17$ K using the actual experimental sample parameters (see Fig. 2 for $\Delta_{\rm SAS}=0.1e^2/\epsilon l$). Therefore, the present theory is not quantitatively satisfying. The quantum fluctuations neglected in the mean-field theory should lower the calculated level-crossing temperature T_X and reduce this discrepancy. Although the above analysis is crude, it provides a starting point for interpreting the enhancement of the threshold temperature in Ref. 9. Second, we notice that, for the sample in the transport experiment[11] (say, the sample with the total density $n_t=0.7\times 10^{11}$ cm^{-2} at the balanced point), which is initially located in the C phase and near the $T=0$ F-C phase boundary, its activation energy decreases as the tilted angle increases from zero. We suggest that the energy scale set by T_{KT} (i.e., the vortex-antivortex binding energy) may be related to this activation energy, since they have similar $B_\parallel$ dependence.

In conclusion, we have investigated the dependence of the characteristic temperatures, T_{KT} and T_X, on the in-plane magnetic field, and demonstrated that it is T_X, rather than T_{KT}, that agrees qualitatively with the experimentally reported threshold temperature. Based on the Hartree-Fock approximation, the dependence of the finite-temperature phase diagram on $B_\parallel$ is also shown. A verification of these two different characteristic temperatures awaits experimental measurements to probe the C phase more directly at lower temperatures. For example, the heat capacity measurements should show power law temperature dependence in the C phase (see Fig. 12 of Ref. 3) because of the existence of the gapless Goldstone mode due to spontaneous symmetry breaking. Once that is achieved, it would be quite interesting for future experiments to confirm the predicted nonmonotonic $B_\parallel$ dependence of T_{KT}.

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[1] L. Zheng, R. J. Radtke, and S. Das Sarma, Phys. Rev. Lett. **78**, 2453 (1997).

[2] S. Das Sarma, S. Sachdev, and L. Zheng, Phys. Rev. Lett. **79**, 917 (1997).

[3] S. Das Sarma, S. Sachdev, and L. Zheng, Phys. Rev. B **58**, 4672 (1998).

[4] E. Demler and S. Das Sarma, Phys. Rev. Lett. **82**, 3895 (1999).

[5] Kun Yang, Phys. Rev. B **60**, 15 578 (1999).

[6] L. Brey, E. Demler, and S. Das Sarma, Phys. Rev. Lett. **83**, 168 (1999).

[7] A. H. MacDonald, R. Rajaraman, and T. Jungwirth, Phys. Rev. B **60**, 8817 (1999).

[8] V. Pellegrini *et al.*, Phys. Rev. Lett. **78**, 310 (1997).

[9] V. Pellegrini *et al.*, Science **281**, 799 (1998).

[10] A. Sawada *et al.*, Phys. Rev. Lett. **80**, 4534 (1998).

[11] A. Sawada *et al.*, Phys. Rev. B **59**, 14 888 (1999).

[12] V. S. Khrapai *et al.*, Phys. Rev. Lett. (to be published).

[13] J. M. Kosterlitz and D. J. Thouless J. Phys. C **6**, 1181 (1993); J. M. Kosterlitz, *ibid.* **7**, 1046 (1974).

[14] M. F. Yang and M. C. Chang, Phys. Rev. B **60**, R13 985 (1999).

[15] In contrast, the ω_0 mode has a maximum spectral weight in the S phase, where there are as many spin-up (down) empty states as there are spin-up (down) electrons at low temperatures.

[16] For example, starting from the effective boson theory, (Refs. 4 and 5), we get the same conclusions.

[17] As discussed in Refs. 2 and 3, the finite-temperature renormalizations for T_{KT} can be much larger in the vicinity of the C-S phase boundary such that this estimation can then no longer be used. Since we are interested mainly in the systems near the F-C phase boundary, our discussions may be still valid.

[18] R. Côté, L. Brey, and A. H. MacDonald, Phys. Rev. B **46**, 10 239 (1992); L. Zheng and H. A. Fertig, *ibid.* **52**, 12 282 (1995).

[19] Here we assume that two quantum wells are arbitrarily narrow, and the effect of the in-plane magnetic field on orbital degrees of freedom is ignored.