Effect of in-plane magnetic field on the spin Hall effect in a Rashba-Dresselhaus system

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In a two-dimensional electron gas with Rashba and Dresselhaus spin-orbit couplings, there are two spin-split energy surfaces connected with a degenerate point. Both the energy surfaces and the topology of the Fermi surfaces can be varied by an in-plane magnetic field. We find that, if the chemical potential falls between the bottom of the upper band and the degenerate point, then simply by changing the direction of the magnetic field, the magnitude of the spin Hall conductivity can be varied by about 100 percent. Once the chemical potential is above the degenerate point, the spin Hall conductivity becomes the constant $e/8\pi$, independent of the magnitude and direction of the magnetic field. In addition, we find that the in-plane magnetic field exerts no influence on the charge Hall conductivity.

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In spintronics devices, injection of spins efficiently from ferromagnetic leads has remained a very challenging problem. Therefore, alternative approaches to generate spin polarization current are being intensively pursued. For example, one can use an external electric field to manipulate the spin transport via spin-orbit coupling. For bulk semiconductors, Murakami *et al.* showed that one can employ the spin-orbit coupling in hole bands to generate a spin Hall current.[1] According to them, the spin Hall current is dissipationless and related to the Berry curvature, which usually vanishes in materials with both inversion and time-reversal symmetry, such as Si, but could be nonzero because of the spin-orbit coupling.

In two-dimensional electron gas (2DEG), spin-orbit coupling could arise because of structure inversion asymmetry (the Rashba mechanism),[2] bulk inversion asymmetry (the Dresselhaus mechanism),[3] or other mechanisms.[4] The existence of Rashba coupling in hetero-junctions has led to many creative proposals for its applications. This includes, for example, the current modulator proposed by Datta and Das,[5] or the spin filter based on electron focusing.[6] The effect of Rashba coupling in quantum wire,[7] quantum ring,[8] or quantum dot[9] has also been investigated.[10] Recently, Sinova *et al.* showed that in a clean and infinite Rashba system, one could generate a spin Hall current by an electric field.[11] In a clean, free-electron-like system, inclusion of both Rashba and Dresselhaus mechanisms still yield the same (up to a sign change) spin Hall conductivity (SHC) $e/8\pi$, independent of spin-orbit coupling strength and carrier density if both bands are populated.[12,13]

The robustness of the value $e/8\pi$ against factors such as disorder, finite-size effect, and electron-electron interaction is being actively studied. It is generally believed that strong disorder would destroy the spin Hall effect in 2DEG. According to some analyses, the $e/8\pi$ value could be preserved in weak disorder.[14] However, several perturbative calculations conclude that the spin Hall effect would disappear as long as disorder exists.[15] Numerical calculations thus far tend to show that the SHC is indeed robust against weak disorder, but reduces to zero at strong disorder.[16] This issue remains to be clarified. Besides disorder, it is found that the interactions between electrons could renormalize the SHC to some extent.[17,18]

For the 2DEG with both Rashba and Dresselhaus spin-orbit couplings, there are two energy surfaces connected with a degenerate point at momentum $\vec{k}_0=\vec{0}$. The SHC is a constant when both bands are populated.[12,13] If the chemical potential μ is below the minimum energy of the upper band (denoted by $E_{+,\min}$, which equals the degenerate energy E_* in this case), the SHC would depend on electron density.[11] In the presence of an in-plane magnetic field, the energy surfaces are distorted such that $\vec{k}_0\neq\vec{0}$ and $E_{+,\min}$ could be below E_*. It would be interesting to investigate the effect of distortion on the SHC. Our major finding is that, when μ is lying between $E_{+,\min}$ and E_*, one can vary the magnitude of the SHC by as much as 100 percent simply by changing the direction of the magnetic field. When μ is above E_* (rather than $E_{+,\min}$), the SHC retains the same constant $e/8\pi$, independent of the direction and magnitude of the magnetic field. Even though the in-plane magnetic field has significant effect on the SHC, it exerts no influence on the charge Hall conductivity, which remains zero as the topology of (one-dimensional) Fermi surfaces are changed due to the rising chemical potential.

We consider the following Hamiltonian,

$$H=\frac{p^2}{2m^*}+\frac{\alpha}{\hbar}(\sigma_x p_y-\sigma_y p_x)+\frac{\gamma}{\hbar}(\sigma_x p_x-\sigma_y p_y)+\beta_x\sigma_x+\beta_y\sigma_y, \tag{1}$$

in which α and γ represent the Rashba coupling and the Dresselhaus coupling, $\vec{\beta}=(g^*/2)\mu_B\vec{B}$, where g^* is the effective g-factor (assumed isotropic and field-independent) and $\vec{B}$ is the in-plane magnetic field. The finite thickness of the 2DEG layer has been neglected so the magnetic field couples only to the electron spin. The eigenenergies of the Hamiltonian are

$$E_\lambda(\vec{k})=E_0(\vec{k})+\lambda\sqrt{(\gamma k_x+\alpha k_y+\beta_x)^2+(\alpha k_x+\gamma k_y-\beta_y)^2}, \tag{2}$$

where $E_0(\vec{k})=\hbar^2k^2/2m^*$ and $\lambda=\pm$, with the corresponding eigenstates,

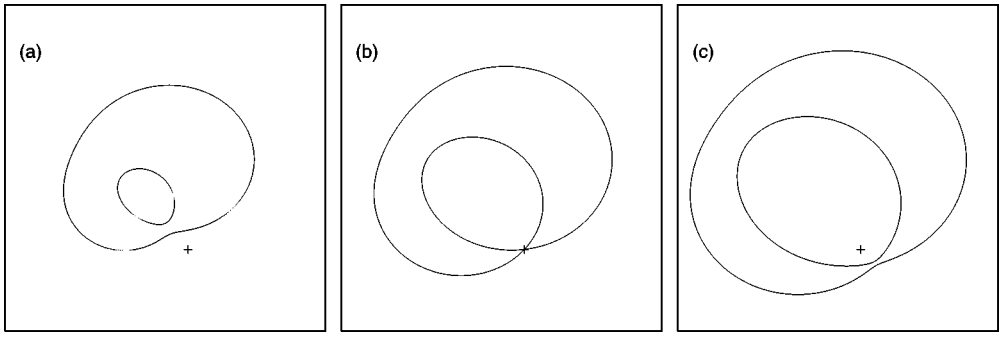

FIG. 1. Schematic diagrams of the energy contours for energies located (a) below, (b) at, and (c), above the degenerate energy E_*. The degenerate point $\vec{k}_0$ is marked with a cross. There is no rotational symmetry for the contours when α, γ, and $\vec{\beta}$ are all nonzero.

$$|\vec{k},+\rangle = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 \\ -ie^{i\theta} \end{pmatrix}, \quad |\vec{k},-\rangle = \frac{1}{\sqrt{2}}\begin{pmatrix} -ie^{-i\theta} \\ 1 \end{pmatrix}, \tag{3}$$

where $\tan\theta=(\gamma k_x+\alpha k_y+\beta_x)/(\alpha k_x+\gamma k_y-\beta_y)$. There are several legitimate but different choices of the eigenstates. Usually, the exponential factors are placed at the same lower (or upper) position for the two spinors. The choice in Eq. (3) ensures that the eigenstates are free of phase ambiguity at the degenerate point $\vec{k}_0$ in the presence of an infinitesimal and perpendicular magnetic field.[19] This choice is crucial in obtaining the correct Hall conductance, as will be explained in more details later.

It can be seen from Eq. (2) that the two energy surfaces are degenerate at the point $\vec{k}_0=[(\gamma\beta_x+\alpha\beta_y)/(\alpha^2-\gamma^2), -(\alpha\beta_x+\gamma\beta_y)/(\alpha^2-\gamma^2)]$. When $\alpha=\gamma$, there is no degeneracy in general. But if β_x also equals β_y, then there is a line degeneracy along $k_x-k_y=\beta_x/\alpha$, similar to the case for $\vec{\beta}=\vec{0}$.[12,13] The energy contours for energies located below, at, and above the degenerate point are shown in Fig. 1.[20] The degenerate point is moving from outside of the contours to inside as the energy E is increasing. We find that the density of states $g(E)$ undergoes a finite jump when E is crossing a band bottom, and it can be proved that $g(E)=m/(\pi\hbar^2)$, same as the free electron value, as long as E is above $E_*=E(\vec{k}_0)$.

The spin Hall conductivity is given by the Kubo formula,[11]

$$\sigma^{\eta}_{\mu\nu} = \frac{1}{i\hbar}\sum_{\vec{k}} \sum_{\lambda,\lambda'(\lambda\neq\lambda')} \frac{f_{\vec{k},\lambda} - f_{\vec{k},\lambda'}}{\omega^2_{\lambda\lambda'}(\vec{k})} \langle\vec{k},\lambda|j^{\eta}_{\mu}|\vec{k},\lambda'\rangle\langle\vec{k},\lambda'|j_{\nu}|\vec{k},\lambda\rangle, \tag{4}$$

where $j^{\eta}_{\mu}=(\hbar/4)(v_\mu\sigma_\eta+\sigma_\eta v_\mu)$ is the generally accepted definition of the spin current, $j_\nu=-ev_\nu$ is the electric current, $v_\mu=\partial\tilde{H}(\vec{k})/\partial(\hbar k_\mu)$, $\tilde{H}(\vec{k})\equiv e^{-i\vec{k}\cdot\vec{r}}He^{i\vec{k}\cdot\vec{r}}$, and $\hbar\omega_{\lambda\lambda'}(\vec{k})\equiv E_\lambda(\vec{k})-E_{\lambda'}(\vec{k})$. It can be shown that both σ^x_{xy} and σ^y_{xy} are zero in the absence of disorder, and

$$\sigma^z_{xy} = -\frac{2e}{m\hbar}\sum_{\vec{k}} (f_{\vec{k},-} - f_{\vec{k},+}) \frac{k_x^2(\alpha^2-\gamma^2) - k_x(\alpha\beta_y+\gamma\beta_x)}{\omega^3_{+-}}, \tag{5}$$

where $\hbar\omega_{+-}=2[(\alpha k_x+\gamma k_y-\beta_y)^2+(\gamma k_x+\alpha k_y+\beta_x)^2]^{1/2}$. A typical result for the SHC as a function of energy is plotted in Fig. 2. For the special case of $\gamma=0$, the degenerate point $\vec{k}_0=\vec{\beta}\times\hat{z}/\alpha$, and $E_*=(\hbar^2/2m^*)(\beta/\alpha)^2$. For $\beta>\alpha^2/2$, which is the case for Fig. 2, the bottoms of upper and lower bands are at $E_{\pm,\min}=-(m^*\alpha/\hbar^2)^2\pm\beta$. Numerical values of these energies are given in the figure caption. It can be seen that the SHC becomes nonzero when E is above the bottom of the lower band. It remains monotonic when E is between the bottoms of lower and upper bands, and may rise over the value $e/8\pi$. For most angels of the magnetic field, the abrupt turn of σ^z_{xy} at $E=E_{+,\min}$ forms a cusp. Once the energy reaches E_*, the SHC becomes the constant $e/8\pi$, independent of the direction of $\vec{\beta}$.

Notice that, according to our choice, the transverse spin current is along the x direction and the longitudinal electric field is along the y direction. When the energy E equals $E_{+,\min}$, the magnitude of σ^z_{xy} can be changed by roughly 100 percent when $\vec{B}$ rotates from the transverse direction (minimum) to the direction of the electric field (maximum). As the angle increases further, the SHC again reaches minimum along the $-x$ direction and rises to maximum along the $-y$ direction. That is, when $\gamma=0$, there is a twofold symmetry in the plot of $\sigma^z_{xy}(\vec{B})$.

In our model [Eq. (1)], in the most general cases when both α and γ are nonzero, the SHC still remains the constant $e/8\pi$ when E is above E_*. We sketch the proof below to show that this is indeed true for all ranges of the parameters

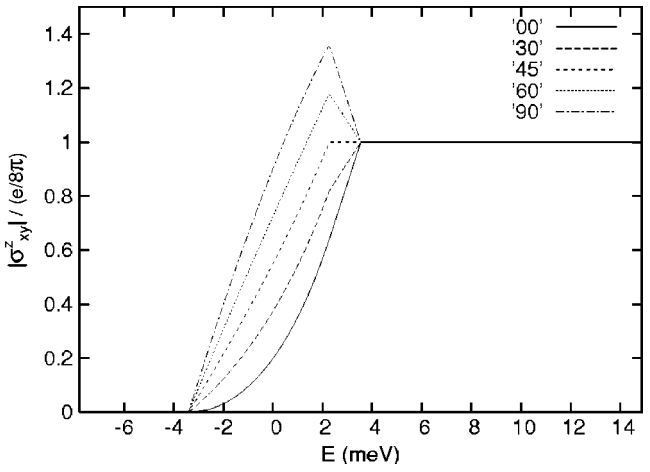

FIG. 2. The spin Hall conductivity as a function of energy for different directions of the magnetic field. The angles between $\vec{B}$ and x axis are 0, 30, 45, 60, and 90°. The parameters are $m^*=0.024m_e$, $\alpha=6\times10^{-9}$ eV cm, $\gamma=0$, and $B=6.52$ T ($g^*=15$). The bottoms of lower or upper bands and the degenerate energy are at $E_{-,\min}=-3.4$ meV, $E_{+,\min}=2.27$ meV, and $E_*=3.54$ meV.

α, γ, and $\vec{\beta}$.[21] While evaluating the integral in Eq. (5), it is more convenient to shift the origin to $\vec{k}_0$. Then the $\vec{\beta}$ dependence of ω_{+-} in the denominator can be eliminated,

$$\hbar\omega_{+-} = 2k(\alpha^2 + \gamma^2 + 2\alpha\gamma \sin 2\phi)^{1/2} \equiv 2kg(\phi), \qquad (6)$$

where the magnitude k and polar angle ϕ are relative to the new origin $\vec{k}_0$. The k integration now can be carried out analytically,

$$\sigma_{xy}^z = -\frac{e\hbar^2}{4m}\frac{\alpha^2-\gamma^2}{(2\pi)^2}\int_0^{2\pi} d\phi \frac{(k_-(\phi)-k_+(\phi))\cos^2\phi}{g^3(\phi)} - \frac{e\hbar^2}{4m}\frac{\alpha\beta_y+\gamma\beta_x}{(2\pi)^2}\int_0^{2\pi} d\phi \frac{\ln(k_-(\phi)/k_+(\phi))\cos\phi}{g^3(\phi)}, \qquad (7)$$

where $k_\pm$ are the lower and upper bounds of the k integration. As long as the energy is above E_*, the values of k_- and k_+ are unique [see Fig. 1(c)],

$$k_\lambda(\phi) = -k_0\cos(\phi-\phi_0) - \frac{\lambda}{2}g(\phi) + \sqrt{[k_0\cos(\phi-\phi_0)+\lambda g(\phi)/2]^2 + E - E_*}, \qquad (8)$$

where ϕ_0 is the angle between $\vec{k}_0$ and the x axis. From Eq. (8), one may not expect that the SHC would be a constant in energy, since both k_- and k_+ depend on energy E explicitly. Nevertheless, we can show that the integrals in Eq. (7) are independent of energy. Substitute the difference,

$$k_-(\phi) - k_+(\phi) = g(\phi) + \sqrt{[k_0\cos(\phi-\phi_0) - g(\phi)/2]^2 + E - E_*} - \sqrt{[k_0\cos(\phi-\phi_0)+g(\phi)/2]^2 + E - E_*}, \qquad (9)$$

to the first integral in Eq. (7), we find that the first term $g(\phi)$ contributes $e/8\pi$ to the SHC. The first square root in Eq. (9) is equal to the second one after the shift $\phi \to \phi+\pi$. Since both $\cos^2\phi$ and $g(\phi)$ in Eq. (7) are invariant under $\phi \to \phi+\pi$, these two terms in Eq. (9) would cancel with each other after integration. Similarly, for the second term in Eq. (7), one can show that the ratio $k_-(\phi)/k_+(\phi)$ is invariant under $\phi \to \phi+\pi$. Therefore, the second integral in Eq. (7) vanishes since the $\cos\phi$ in the numerator changes sign after a π rotation.

The proof above is valid for $E > E_*$. At the degenerate energy $E = E_*$, the denominator ω_{+-} is zero and the proof above does not apply, but it still can be proved that the SHC converges to the value $e/8\pi$ without showing any singular behavior at $E = E_*$.

In Fig. 3, we show the SHC as a function of the in-plane magnetic field $\vec{B}$ at a fixed electron density n. For smaller magnetic field, the degenerate energy E_* is lower than the chemical potential. Therefore, there is a plateau with $|\sigma_{xy}^z| = e/8\pi$ at the center of Fig. 3. The plateau has an elliptical boundary defined by the equation $\mu = E_*$, or, equivalently,

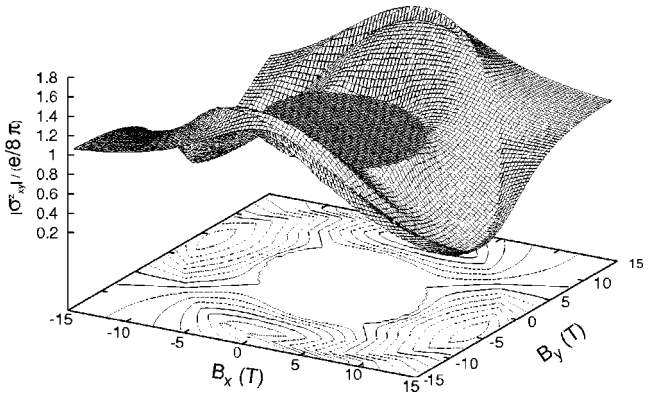

FIG. 3. Spin Hall conductivity plotted as a function of the in-plane magnetic field $\vec{B}$. There is a plateau in the middle when the magnetic field is weak. For stronger magnetic field beyond the plateau region, a circle of cusp (ridge) is clearly visible. Contours of the surface are projected onto the plane below. Relevant parameters are specified in the text.

$$\beta_x^2 + 4\alpha\gamma\beta_x\beta_y + \beta_y^2 = \zeta\frac{(\alpha^2-\gamma^2)^2}{\alpha^2+\gamma^2}, \qquad (10)$$

where ζ is a constant of the order of electron density. For larger magnetic field, the degenerate point is driven out of the Fermi sea and the SHC could be enhanced or reduced, depending on the direction of $\vec{B}$ (similar to the behavior in Fig. 2). The circle of cusp (ridge) surrounding the plateau is caused by the crossing of μ and the bottom of the upper band $E_{+,\min}$ (see Fig. 2). Split cusps are visible in Fig. 3 in over half of the ridge, which is due to the *re*-crossing of μ and $E_{+,\min}$. That is, when the magnetic field is increasing at a fixed angle, because of the complex shift of the bottoms of energy surfaces, the chemical potential $\mu(B)$ could fall below $E_{+,\min}(B)$ after going through the first ridge at $\mu(B_1) = E_{+,\min}(B_1)$, then rise to touch the bottom of the upper band again at $\mu(B_2) = E_{+,\min}(B_2)$ $(B_2 > B_1)$, where the second ridge (or cusp) is formed.

The Rashba and Dresselhaus couplings in Fig. 3 are $\alpha = 6\times10^{-9}$eV cm and $\gamma = 2\times10^{-9}$eV cm.[22] We choose the effective mass $m^* = 0.024m_e$ and effective g-factor $g^* = 15$ for electrons in bulk InAs. The electron density n is fixed at $5.7\times10^{10}/\text{cm}^2$ during the $\vec{B}$ scan. The ranges of in-plane magnetic field are 15 T in both directions. Based on these realistic parameters, such angular variation of SHC might be tested in future experiments. Different material parameters, such as m^* and g^*, could alter the range of appropriate n and B by one or two orders of magnitude. In general, it is better to choose materials with a large product of m^*g^* to reduce the magnetic field strength required.[23]

At the end of the paper, we comment briefly on whether the in-plane magnetic field could generate a charge Hall current in the Rashba-Dresselhaus system. Replacing the spin current j_μ^η in Eq. (4) by electric current j_μ, the Hall conductivity for the λ-band ($\lambda = \pm$) can be written as[24]

$$\sigma_{xy}^\lambda = \frac{e^2}{\hbar}\sum_{\vec{k}\ \text{filled}} \Omega_\lambda(\vec{k}), \qquad (11)$$

where the Berry curvature for the $(\vec{k},\lambda)$ state is

$$\Omega_\lambda(\vec{k}) = i \sum_{\lambda' \neq \lambda} \frac{\langle \vec{k},\lambda|v_x|\vec{k},\lambda'\rangle\langle \vec{k},\lambda'|v_y|\vec{k},\lambda\rangle - \langle \vec{k},\lambda|v_y|\vec{k},\lambda'\rangle\langle \vec{k},\lambda'|v_x|\vec{k},\lambda\rangle}{\omega^2_{\lambda\lambda'}(\vec{k})}. \quad (12)$$

It is easy to show that the Berry curvature is zero at every $\vec{k}$, except at the degenerate point where the formula above does not apply. However, one can preform a line integral along an arbitrary contour that encloses the degenerate point to obtain the Berry phase,

$$\Gamma_\lambda = \oint d\vec{k} \cdot \langle \vec{k},\lambda| i\frac{\partial}{\partial \vec{k}} |\vec{k},\lambda\rangle = -\,\mathrm{sgn}(\alpha^2 - \gamma^2)\lambda\pi, \quad (13)$$

in which sgn $(\alpha^2-\gamma^2)=1$, 0, -1 when α^2 is larger than, equal to, or smaller than γ^2. This is basically the same result as that of $\vec{\beta}=0$, and reflects the fact that the Berry phase is topological in nature and is not altered by the smooth distortion of the energy surfaces. We note that the result in Eq. (13) differs slightly from that (for $\vec{\beta}=0$) in Ref. 13, in which the signs of Γ_+ and Γ_- are the same (discussed below). From the nonzero Berry phase, one can infer that the Berry curvature is singular at the degenerate point, $\Omega_\lambda(\vec{k})=-\mathrm{sgn}(\alpha^2 - \gamma^2)\lambda\pi\delta(\vec{k}-\vec{k}_0)$, which is different from the usual monopole-field-like Berry curvature. The result above is valid for the Rashba-Dresselhaus system in an arbitrary in-plane magnetic field.

Despite the nonzero Berry phase, the Hall conductance of the system remains zero for whole range of the chemical potential. To illustrate this point, let us consider the simpler "pure" Rashba system ($\gamma=0, \vec{\beta}=0$). If the chemical potential is below the degenerate point, then only the lower band is populated and the $\vec{k}$ integral in Eq. (11) is performed over an annular region. The result of integration using Eq. (11) is zero since the singularity of Berry curvature is *not* located in the annulus. If the chemical potential is above the degenerate point, then both bands are populated (degenerate point included in *both* cases), and $\sigma_{xy}=\sigma^+_{xy}+\sigma^-_{xy}=0$. Therefore, there exists no charge Hall current no matter the chemical potential is below or above the degenerate point. Different choices of bases in Eq. (3) may lead to different results such as $\sigma^\lambda_{xy}=e^2/(2h)$ (λ independent),[13] which would give $\sigma_{xy}=\sigma^+_{xy}+\sigma^-_{xy}=e^2/h$ when both bands are populated. This would imply the counter intuitive result that the system exhibits charge Hall effect even in the small spin-orbit coupling limit.[25]

For the generic situation in which both γ and $\vec{\beta}$ are nonzero, the topology of the one dimensional Fermi surface can be different from those described in the previous paragraph (see Fig. 1). But it is not difficult to see that the conclusion of null charge Hall current remains valid.

It is possible to reveal the nonzero Berry curvature by adding a very small (so that orbital quantization can be neglected) perpendicular magnetic field to remove the point degeneracy. When the chemical potential is within the Zeeman gap, only the lower band is filled but now the point of high curvature is included in the $\vec{k}$-integration in Eq. (11). Therefore, when μ is within the Zeeman gap, the Hall conductance is roughly equal to $e^2/(2h)$.[26]

In summary, crucial energy scales in the Rashba-Dresselhaus system under an in-plane magnetic field are: the bottoms of the spin-split bands $E_{\pm,\min}$, the degenerate energy E_*, and the chemical potential μ. All of these energies depend on the applied magnetic field. By shifting the chemical potential μ through $E_{+,\min}$ and E_*, the SHC could first be enhanced (or reduced) and show a cusp, then become a material-independent constant. Whether the SHC is enhanced or reduced depends on, and can be controlled by, the direction of the magnetic field. Finally, we caution that the charge Hall conductivity depends subtly on the choice of eigenstates. As far as we know, the SHC is not influenced by such subtle choice of eigenbasis.

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