Effect of Landau-level mixing for electrons in a random magnetic field

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(Received 18 March 1997)

An effective Hamiltonian approach is used to study the effect of Landau-level mixing on the energy spectrum of electrons in a smooth but random magnetic field $B(\mathbf{r})$ with a *finite* uniform component B_0. It is found that, as opposed to electrostatic disorder, the energy levels of localized electrons shift *upward* with a rate of order $O(B_0^{-1})$ when B_0 is decreased, while the extended states remain static at the same order. Therefore, there is no indication that the extended states will float out of the Fermi energy and induce a metal-insulator transition as the magnetic disorder is increased. We also find that the Zeeman term may have a significant effect on the spectral shift of low-lying Landau levels. [S0163-1829(97)03532-7]

Recently, there is intensive interest in the problem of two-dimensional electron gas in a static random magnetic field (RMF). First, this problem is related to the localization problem for the ''composite fermions'' in the fractional quantum Hall effect.[1] In the mean-field treatment, the composite fermions move in a weak effective magnetic field that contains a random component induced by the inhomogeneous electron density. Second, the study of RMF may be applied to high-T_c superconductivity systems, where the RMF is considered as a limiting case of the gauge field.[2] Third, recent experiments that measured transport properties of electrons in a static RMF also add considerable interest to this subject.[3]

Most theoretical studies in the literature focused on free electrons in a RMF with a *zero mean value*. A central issue is whether all the electrons are localized in such an environment. The results are rather controversial. Analytically, due to the zero average of the magnetic field, the field-theoretical description corresponds to a nonlinear σ model of the unitary class without a topological term, which predicts that all states are localized,[4] according to the conventional scaling theory.[5] However, Zhang and Arovas[6] have suggested that a long-range logarithmic interaction between the topological densities (due to the local fluctuations of the Hall conductance) may lead to delocalization. The numerical works on a finite lattice only add more conflicting results. Some authors claim that there may exist the mobility edge separating the localized states from the extended states;[7,8] however, other authors, while observing a strong enhancement of the localization length, find no true transition.[9,10] The controversy in the numerical works arises from the interpretation of their data. Because the localization length increases rapidly as a function of energy when the band center is approached, it is hard to distinguish whether the states are really extended or weakly localized with the localization length much longer than the sample size.

On the other hand, our understanding of electron localization in a random electric potential is more complete.[11] It is known that there are extended states at the centers of Landau bands in a strong uniform magnetic field. In order to be consistent with the conventional scaling theory for the zero-field case,[5] it is argued that the extended states will float up in energy if the magnetic field strength is reduced (or equivalently, if the strength of disorder is increased),[12] and therefore all the states below the Fermi energy are eventually localized. Although this levitation scenario is appealing, its microscopic foundation is not clear. Recently, by using a simple perturbative approach, Haldane and Yang[13] show that the levitation of the extended states can be explained as a result of Landau-level mixing, thus support the levitation scenario.

Motivated by the work of Haldane and Yang,[13] we study the spectral shift of the two-dimensional electrons in a RMF $B(\mathbf{r})=B_0+b(\mathbf{r})$ when its spatial average B_0 is reduced. As pointed out by Kalmeyer *et al.*[8] (see also Ref. 14), when $B_0\neq 0$ and $b(\mathbf{r})\ll B_0$, the random fluctuation behaves like a random scalar potential. In this case, one recovers the well-studied problem of electrons in a random potential and a uniform magnetic field, thus it is expected that there are extended states at the centers of Landau bands. If the correlation length of the disorder is much longer than the magnetic length $\ell=\sqrt{\hbar/eB_0}$, the motion of electrons can be decomposed into a fast cyclotron motion and a slow guiding-center motion.[10] The guiding centers move along the contours of $b(\mathbf{r})$ with the local drift velocity $\mathbf{V}_d=(e\xi^2/2m)\nabla b\times\hat{\mathbf{z}}$, where ξ is the cyclotron radius and m is the electron mass.[15] (See Fig. 1.) Around hills or valleys of $b(\mathbf{r})$, the contours are closed and the corresponding states are localized. The extended states occur only at the percolation contour whose energy is determined by the saddle points of $b(\mathbf{r})$, similar to the semiclassical theory for electrostatic disorder. Due to this similarity, Lee *et al.*[10] propose that the extended states will

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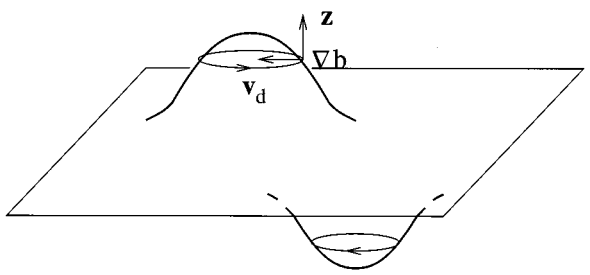

FIG. 1. Schematic diagram of the guiding-center orbits of electrons circling around the hill and valley of the random magnetic field. Note that the sense of rotation is opposite for the two paths in the figure.

levitate in energy with decreasing B_0, and hence all states below the Fermi energy should be localized when $B_0=0$. However, by using the same perturbative approach used in Ref. 13, we find that the leading term of the effective Hamiltonian will cause the *localized states*, rather than the extended states, to float up in energy as B_0 decreases. Thus, the levitation scenario of extended states in the RMF case has *no* firm support, and, therefore, gives *no* implication to electron localization in a RMF. Furthermore, we show that the Zeeman term may have a significant effect on the spectral shift of low-lying Landau levels.

For the two-dimensional electron gas in a RMF with a *nonzero* average, the Hamiltonian H is composed of three parts,

$$H_0=\frac{1}{2m}(\mathbf{p}+e\mathbf{A})^2, \tag{1}$$

$$H_1=\frac{e}{2m}[(\mathbf{p}+e\mathbf{A})\cdot\mathbf{a}+\mathbf{a}\cdot(\mathbf{p}+e\mathbf{A})], \tag{2}$$

$$H_2=\frac{e^2}{2m}\mathbf{a}^2, \tag{3}$$

where $\mathbf{A}$ and $\mathbf{a}$ are the vector potentials for $B_0\hat{\mathbf{z}}$ and $b(\mathbf{r})\hat{\mathbf{z}}$, respectively. Using the Coulomb gauge for the fluctuating vector potential, we can write

$$\mathbf{a}(\mathbf{r})=\frac{1}{\mathcal{A}}\sum_{\mathbf{q}\neq 0} i\mathbf{q}\times\hat{\mathbf{z}}\frac{b(\mathbf{q})}{q^2}e^{i\mathbf{q}\cdot\mathbf{r}}, \tag{4}$$

where $b(\mathbf{q})$ is the Fourier components of $b(\mathbf{r})$, and (quasi-) periodic boundary condition is imposed on the area $\mathcal{A}$ that contains an integer number of magnetic flux quanta. For a smooth and weak disorder, it is convenient to decompose the position of an electron into a fast cyclotron motion $\mathbf{r}-\mathbf{R}$ and a slow guiding-center motion $\mathbf{R}=(x-\Pi_y/eB_0)\hat{\mathbf{x}}+(y+\Pi_x/eB_0)\hat{\mathbf{y}}$, where $\mathbf{\Pi}=\mathbf{p}+e\mathbf{A}$ is the canonical momentum operator. It can be shown that the fast and the slow parts commute with each other and decouple nicely.[16] The velocity of the guiding center at the nth Landau level can be obtained by the Heisenberg equation of motion. To lowest order, the result is

$$\frac{d}{dt}\langle n|\mathbf{R}|n\rangle=\frac{1}{i\hbar}\langle n|[\mathbf{R},H]|n\rangle\simeq\frac{e}{m}\left(n+\frac{1}{2}\right)\ell^2\langle n|\nabla b\times\hat{\mathbf{z}}|n\rangle. \tag{5}$$

This form coincides with the classical expression $(e\xi^2/2m)\nabla b\times\hat{\mathbf{z}}$, since the cyclotron radius ξ at the nth Landau-level is given by $\sqrt{\langle n|(\mathbf{r}-\mathbf{R})^2|n\rangle}=\sqrt{2n+1}\,\ell$.

In the presence of the random field $b(\mathbf{r})$, the Landau-level index n is no longer a good quantum number and different levels couple with each other. If the magnetic field B_0 is very strong, we need only consider the projected Hamiltonian in the subspace of a given Landau level. However, in general, (virtual) transitions between different Landau levels will renormalize the potential seen by the electrons in this Landau level. By the perturbative renormalization in terms of powers of $\mathbf{a}$, the effective Hamiltonian for electrons in the nth Landau band can be written as

$$\langle n|H_{\rm eff}^{(n)}(\mathbf{r})|n\rangle=\left(n+\frac{1}{2}\right)\hbar\Omega+\sum_{k\geqslant 1}\langle n|V_k^{(n)}(\mathbf{r})|n\rangle, \tag{6}$$

where $\hbar\Omega=\hbar eB_0/m$ is the cyclotron energy and $|n\rangle$ is the eigenstate of H_0. The effective potential propotional to $\mathbf{a}$ is

$$\langle n|V_1^{(n)}(\mathbf{r})|n\rangle=\langle n|H_1|n\rangle, \tag{7}$$

and the correction that is quadratic in $\mathbf{a}$ is given by

$$\langle n|V_2^{(n)}(\mathbf{r})|n\rangle=\langle n|H_2|n\rangle+\sum_{n'\neq n}\frac{\langle n|H_1|n'\rangle\langle n'|H_1|n\rangle}{\hbar\Omega(n-n')}. \tag{8}$$

To first order, a direct calculation yields[8,14,17]

$$V_1^{(n)}(\mathbf{r})=-\frac{e\hbar}{m\ell^2}\frac{1}{\mathcal{A}}\sum_{\mathbf{q}\neq 0}\frac{b(\mathbf{q})}{q^2}g^{(n)}(\mathbf{q})e^{i\mathbf{q}\cdot\mathbf{r}}, \tag{9}$$

where

$$g^{(n)}(\mathbf{q})=\frac{\dfrac{\partial}{\partial\lambda}U_{nn}(\mathbf{q}\lambda)|_{\lambda=1}}{U_{nn}(\mathbf{q})}, \tag{10}$$

in which $U_{nn}(\mathbf{q})$ is the diagonal part of $U_{nn'}(\mathbf{q})=\langle n|e^{i\mathbf{q}\cdot(\mathbf{r}-\mathbf{R})}|n'\rangle$. For brevity, the projection by $|n\rangle$ and its adjoint will be neglected from now on. Bear in mind that the equality holds only in the projected subspace of the nth Landau level. For the slowly varying $b(\mathbf{r})$ (compared to the magnetic length), only the small $\mathbf{q}$ components in Eq. (9) make a significant contribution, hence one can expand $g^{(n)}(\mathbf{q})$ into a power series in $q\ell$. Up to the order of $(q\ell)^4$ for $g^{(n)}(\mathbf{q})$, $V_1^{(n)}(\mathbf{r})$ can be written as[18]

$$V_1^{(n)}(\mathbf{r})\simeq\left(n+\frac{1}{2}\right)\frac{\hbar e}{m}b(\mathbf{r})-\frac{n(n+1)}{4}\frac{\hbar e}{m}\ell^2\nabla^2 b(\mathbf{r}). \tag{11}$$

Both terms in Eq. (11) lead to broadening of the Landau levels: The first term lifts the energy degeneracy for electrons drifting along different contours of $b(\mathbf{r})$; the second term gives a positive (negative) contribution to energy for electrons drifting along the hills (valleys) of $b(\mathbf{r})$, which have a negative (positive) curvature of $b(\mathbf{r})$, thus broadening

the level further. However, neither gives a net shift to the overall profile of the density of states.

By using the algebra of $\mathbf{\Pi}$, the second term of $V_2^{(n)}(\mathbf{r})$ can be expressed as

$$\frac{e^2}{m\ell^2}\frac{1}{\mathcal{A}^2}\sum_{\mathbf{q}\neq 0,\mathbf{q}'\neq 0}\frac{b(\mathbf{q})}{q^2}\frac{b(\mathbf{q}')}{q'^2}f^{(n)}(\mathbf{q},\mathbf{q}')e^{i(\mathbf{q}+\mathbf{q}')\cdot\mathbf{r}}, \quad (12)$$

where

$$f^{(n)}(\mathbf{q},\mathbf{q}')=\frac{e^{-i\hat{\mathbf{z}}\cdot\mathbf{q}\times\mathbf{q}'\ell^2/2}}{U_{nn}(\mathbf{q}+\mathbf{q}')} \times\sum_{n'\neq n}\frac{\frac{\partial}{\partial\lambda}U_{nn'}(\mathbf{q}\lambda)\Big|_{\lambda=1}\frac{\partial}{\partial\lambda}U_{n'n}(\mathbf{q}'\lambda)\Big|_{\lambda=1}}{n-n'}. \quad (13)$$

The expansion of $f^{(n)}(\mathbf{q},\mathbf{q}')$ in powers of $q\ell$ is given by

$$f^{(n)}(\mathbf{q},\mathbf{q}')=\frac{\mathbf{q}\cdot\mathbf{q}'}{2}\ell^2-\left(n+\frac{1}{2}\right)\frac{\mathbf{q}\cdot\mathbf{q}'}{2}(q^2+q'^2)\ell^4 +O(q^6\ell^6). \quad (14)$$

The first term in Eq. (14) gives $-(e^2/2m)\mathbf{a}^2$ to $V_2^{(n)}(\mathbf{r})$, which is negative for all states and cancels the first term in Eq. (8). Thus, up to the order of $(q\ell)^4$ for $f^{(n)}(\mathbf{q},\mathbf{q}')$,

$$V_2^{(n)}(\mathbf{r})\simeq\frac{e^2}{m}\left(n+\frac{1}{2}\right)\ell^2\mathbf{a}\cdot\nabla b\times\hat{\mathbf{z}}. \quad (15)$$

Notice that the effective potential $V_2^{(n)}(\mathbf{r})$ is not manifestly gauge invariant.[19] However, this lack of gauge invariance does not appear in the energy expectation value of the electronic states. Under the semiclassical approximation, for electrons circling a closed orbit $\mathcal{C}$ with a constant energy $\mathcal{E}$, the energy expectation value altered by $V_2^{(n)}(\mathbf{r})$ is proportional to the following integral:

$$\langle V_2^{(n)}\rangle\propto\int d^2\mathbf{r}\,\delta[\mathcal{E}^{(n)}(\mathbf{r})-\mathcal{E}]\frac{e^2}{m}\left(n+\frac{1}{2}\right)\ell^2\mathbf{a}\cdot\nabla b\times\hat{\mathbf{z}} =\oint_{\mathcal{C}}\frac{dl}{|\nabla\mathcal{E}^{(n)}(\mathbf{r})|}\frac{e^2}{m}\left(n+\frac{1}{2}\right)\ell^2\mathbf{a}\cdot\nabla b\times\hat{\mathbf{z}}\simeq\frac{1}{B_0}\oint_{\mathcal{C}}d\mathbf{l}\cdot\mathbf{a}, \quad (16)$$

where we have used the fact that the local energy of the nth Landau band $\mathcal{E}^{(n)}(\mathbf{r})\simeq(n+1/2)\hbar\Omega+(n+1/2)(\hbar e/m)b(\mathbf{r})$ [see Eqs. (6) and (11)], and $d\mathbf{l}$ is in the direction of $\mathbf{V}_d$. That is, $\langle V_2^{(n)}\rangle$ is proportional to the magnetic flux of $b(\mathbf{r})$ enclosed by $\mathcal{C}$ and is *positive* for *both* of the orbits circling the hill and the valley. (See Fig. 1.) Hence $\langle V_2^{(n)}\rangle$ is gauge invariant (as it should be) and gives an *upward* shift in energy for the localized states. For the extended states, the shift is determined by the saddle points of $b(\mathbf{r})$,[10] where $\nabla b(\mathbf{r})=0$. Therefore, $V_2^{(n)}(\mathbf{r})$ vanishes (thus also gauge invariant), and the energy of the extended states remains static at this order.

It is quite interesting to compare our result with that of the electrostatic disorder case.[13] For electrostatic disorder, it is found that the energies of the localized states shift downward and that of the extended states is static at order $O(B_0^{-2})$.[13] The downward movement is a manifestation of the generic ''level-repulsion'' effect at the second order perturbation. At the order of $O(B_0^{-3})$, which is from the $(q\ell)^4$ term of the second order perturbation, the energy of the extended states shifts upward in stronger disorder and this behavior supports the levitation scenario[12] to explain the metal-insulator transition. However, the spectral shift in the RMF case is very different: the energies of the localized states shift *upward*, and that of the extended states remains static at the same order. In a relative sense, the extended states move downward with respect to the other states. Therefore, it seems unlikely that the extended states will float out of the Fermi energy at strong disorder and induce a metal-insulator transition. It might appear that the result presented here contradicts the generic level-repulsion effect, which would result in lowering of the levels (especially the lowest Landau level). This is not so. The level-repulsion effect due to the level mixing should come from the second term in Eq. (8). As indicated in Eq. (14), the leading contribution $-(e^2/2m)\mathbf{a}^2$ indeed contributes to downward movement. However, this downward movement is canceled by the diamagnetic term $(e^2/2m)\mathbf{a}^2$ that comes from the first term in Eq. (8). This cancellation is unique in the magnetic disorder problem.

In the following, we would like to discuss briefly the influence of the Zeeman term on the spectral shift. Besides contributing a constant shift in energy, $\pm(g/4)\hbar\Omega$ (g is the electron g factor), as it does for the electrostatic disorder problems, the Zeeman term adds a $b(\mathbf{r})$-dependent part $H_z=-(g\hbar e/4m)\sigma_3 b(\mathbf{r})$ to Eq. (2), where σ_3 is the Pauli matrix. Consequently, the inclusion of the Zeeman term leads to the following changes in the perturbative calculation:[20] for the first order calculation, we get an extra term $-(g\hbar e/4m)\sigma_3 b(\mathbf{r})$ to $V_1^{(n)}(\mathbf{r})$; while the additional contribution to $V_2^{(n)}(\mathbf{r})$ is given by

$$\sum_{n'\neq n}\left\{\frac{\langle n|H_1|n'\rangle\langle n'|H_z|n\rangle}{\hbar\Omega(n-n')}+\text{H.c.}\right\} +\sum_{n'\neq n}\frac{\langle n|H_z|n'\rangle\langle n'|H_z|n\rangle}{\hbar\Omega(n-n')}. \quad (17)$$

A straightforward calculation shows that the first term in the equation above contributes $-(ge^2/4m)\sigma_3\mathbf{a}\cdot\nabla b\times\hat{\mathbf{z}}$ to Eq. (15); while the second term is of a higher order in $q\ell$ and can be neglected. Note that, apart from a multiplicative constant, this term has the same form as the term in Eq. (15). Consequently, the conclusion that the extended states are not shifted, because $\nabla b(\mathbf{r})=0$ at the saddle points, remains valid. Also note that the additional contribution is dependent on spin but independent of n. Therefore, the spectrum may shift differently between low-lying states and higher levels: for Landau levels with $(n+1/2)>g/4$, the localized states always move upward, but it may become *downward* for spin-*up* electrons, if $(n+1/2)<g/4$. In particular, for spin-up electrons at the lowest Landau level (LLL), if $g=2$, then the $b(\mathbf{r})$-dependent effective potentials in Eqs. (11) and (15) are

canceled by these extra terms due to the Zeeman term. If fact, it is not difficult to prove that the cancellation is exact to all orders of $q\ell$ in $V_1^{(n)}(\mathbf{r})$ and $V_2^{(n)}(\mathbf{r})$. This cancellation is consistent with the Aharonov-Casher theorem,[21] which states that the LLL of spin-up electrons with $g=2$ will not be broadened by magnetic disorder, no matter how strong the disorder is.

Finally, some comments are in order: First, our result seems to be against the proposal that all states below the Fermi energy are localized when $B_0=0$.[4,9,10] However, since our perturbative approach is valid only for weak $b(\mathbf{r})$ (compared to B_0), it is not sufficient to predict whether the extended states will remain static when $B_0\to 0$ and become the delocalized states suggested in Refs. 6–8. Therefore, to settle down the localization problem for the $B_0=0$ case, an alternative approach that is applicable to the $B_0\ll 1$ limit is urgently needed. Second, the calculation presented here may be related to the $1/3\to 1/2$ transition of the quantum Hall systems (i.e., the $1\to 0$ transition of the composite fermions) by tuning the external field at a given magnetic disorder — if the ubiquitous electrostatic disorder in real systems does not dominate the spectral shift. As mentioned above, depending on the magnitude of the g factor, the Zeeman term may lead to a different spectral shift between spin-up and spin-down electrons. It would be interesting to observe this subtle behavior in future experiments.

The authors would like to thank C. Y. Mou for many valuable discussions. This work is supported by the National Science Council of Taiwan under Contract Nos. NSC 86-2112-M-007-026 and 86-2112-M-002-028.

[1] J. K. Jain, Phys. Rev. Lett. **63**, 199 (1989).

[2] L. B. Ioffe and A. I. Larkin, Phys. Rev. B **39**, 8988 (1989); P. A. Lee, Phys. Rev. Lett. **63**, 680 (1989); M. Grilli and G. Kotliar, *ibid.* **64**, 1170 (1990); N. Nagaosa and P. A. Lee, *ibid.* **64**, 2450 (1990); J. M. Wheatley and T. M. Hong, Phys. Rev. B **43**, 6288 (1991).

[3] A. K. Geim, S. J. Bending, and I. V. Grigorieva, Phys. Rev. Lett. **69**, 2252 (1992); A. Smith *et al.*, Phys. Rev. B **50**, 14 726 (1994); F. B. Mancoff *et al.*, *ibid.* **51**, 13 269 (1995); **53**, R7599 (1996).

[4] A. G. Aronov, A. D. Mirlin, and P. Wölfle, Phys. Rev. B **49**, 16 609 (1994); Y. B. Kim, A. Furusaki, and D. K. K. Lee, *ibid.* **52**, 16 646 (1995).

[5] E. Abrahams, P. W. Anderson, D. C. Licciardello, and V. Ramakrishnan, Phys. Rev. Lett. **42**, 673 (1979).

[6] S. C. Zhang and D. P. Arovas, Phys. Rev. Lett. **72**, 1886 (1994).

[7] V. Kalmeyer and S. C. Zhang, Phys. Rev. B **46**, 9889 (1992); Y. Avishai, Y. Hatsugai, and M. Kohmoto, *ibid.* **47**, 9561 (1993); T. Kawarabayashi and T. Ohtsuki, *ibid.* **51**, 10 897 (1995); D. Z. Liu, X. C. Xie, S. Das Sarma, and S. C. Zhang, *ibid.* **52**, 5858 (1995); D. N. Sheng and Z. Y. Weng, Phys. Rev. Lett. **75**, 2388 (1995); K. Yang and R. N. Bhatt, Phys. Rev. B **55**, R1922 (1997).

[8] V. Kalmeyer, D. Wei, D. P. Arovas, and S. C. Zhang, Phys. Rev. B **48**, 11 095 (1993).

[9] T. Sugiyama and N. Nagaosa, Phys. Rev. Lett. **70**, 1980 (1993); D. K. K. Lee and J. T. Chalker, *ibid.* **72**, 1510 (1994); K. Yakubo and Y. Goto, Phys. Rev. B **54**, 13 432 (1996); J. A. Vergés, *ibid.* **54**, 14 822 (1996).

[10] D. K. K. Lee, J. T. Chalker, and D. Y. K. Ko, Phys. Rev. B **50**, 5272 (1994).

[11] For a review, see B. Huckestein, Rev. Mod. Phys. **67**, 357 (1995).

[12] D. E. Khmelnitskii, Phys. Lett. **106A**, 182 (1984); R. B. Laughlin, Phys. Rev. Lett. **52**, 2304 (1984). However, based on the numerical results on the *lattice* models, an entirely different scenario of the metal-insulator transition is also proposed. See Y. Tan, J. Phys.: Condens. Matter **6**, 7941 (1994); D. Z. Liu, X. C. Xie, and Q. Niu, Phys. Rev. Lett. **76**, 975 (1996); X. C. Xie, D. Z. Liu, B. Sundaram, and Q. Niu, Phys. Rev. B **54**, 4966 (1996); D. N. Sheng and Z. Y. Weng, Phys. Rev. Lett. **78**, 318 (1997).

[13] F. D. M. Haldane and K. Yang, Phys. Rev. Lett. **78**, 298 (1997).

[14] B. Huckestein, Surf. Sci. **361/362**, 75 (1996); Phys. Rev. B **53**, 3650 (1996).

[15] J. D. Jackson, *Classical Electrodynamics*, 2nd ed. (Wiley, New York, 1975), p. 585. A quantum mechanical derivation of the drift velocity will be given later in Eq. (5)

[16] For a review, see A. H. MacDonald, in *Mesoscopic Quantum Physics*, Proceedings of the Les Houches Summer School of Theoretical Physics, Les Houches, 1994, Session LXI, edited by E. Akkermans *et al.* (North-Holland, Amsterdam, 1995).

[17] M. C. Chang and Q. Niu, Phys. Rev. B **50**, 10 843 (1994).

[18] We believe that, in Eq. (2.10) of Ref. 8, the authors miss a factor 2 in the coefficients of the second and the third terms of their gradient expansion.

[19] A similar gauge noninvariance was found in the one-band effective Hamiltonian for a lattice system. See W. Kohn, Phys. Rev. **115**, 1460 (1959); L. M. Roth, J. Phys. Chem. Solids **23**, 433 (1962). In Kohn's paper, the gauge noninvariance is fixed by requiring the wave function to be real at the origin. In this paper, we follow Roth's discussion.

[20] Notice that the constant shift in energy $-\sigma_3(g/4)\hbar\Omega$ is added to H_0 and therefore gives a correction to the first term of Eq. (6).

[21] Y. Aharonov and A. Casher, Phys. Rev. A **19**, 2461 (1979).