Berry phase in solid state physics

– a selected overview

Ming-Che Chang

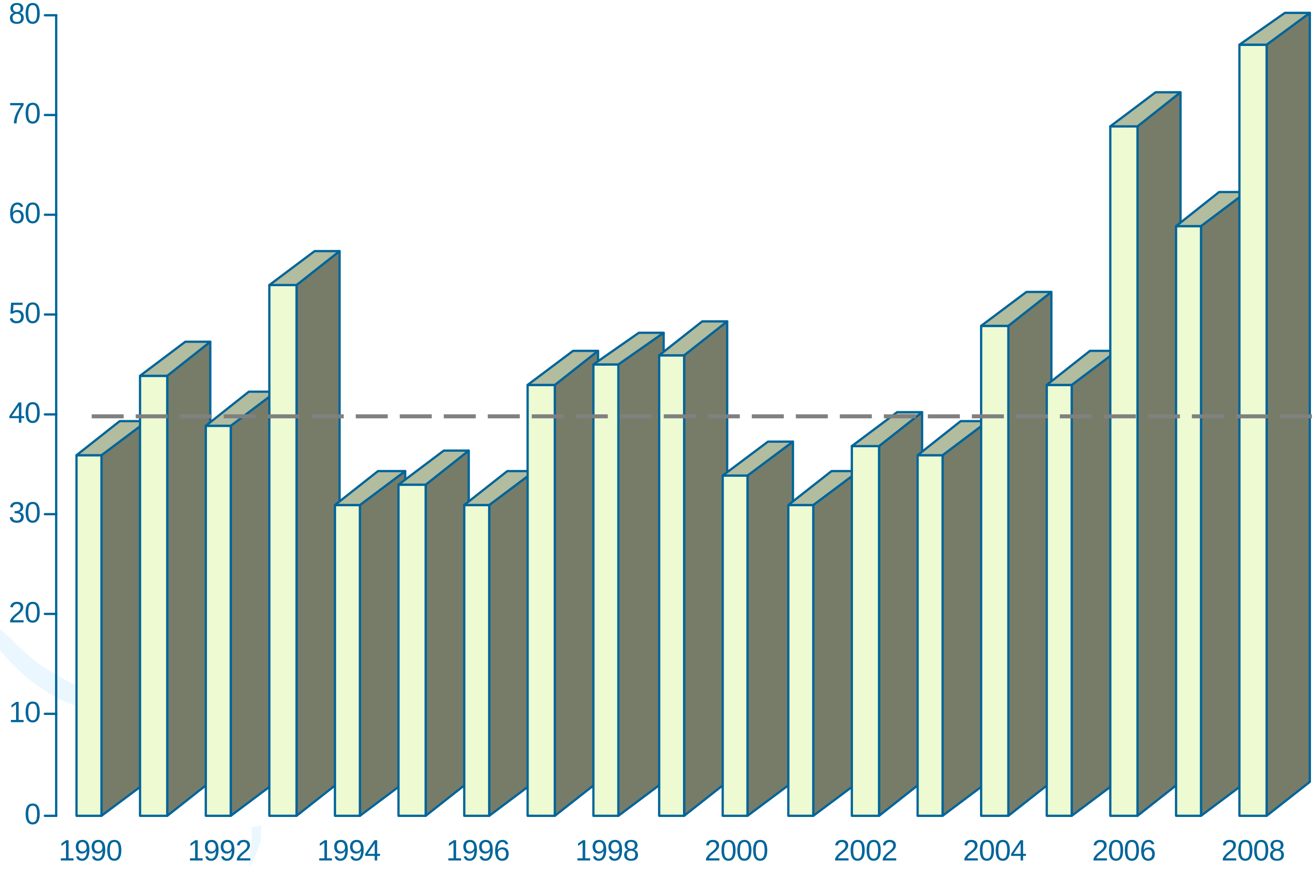
Department of Physics
National Taiwan Normal University

Qian Niu

Department of Physics
The University of Texas at Austin


Taiwan

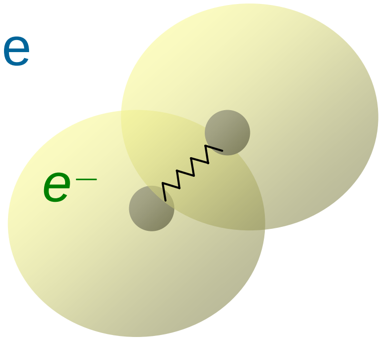
Paper/year with the title "Berry phase" or "geometric phase"



- Introduction (30-40 mins)
  - Quantum adiabatic evolution and Berry phase
  - Electromagnetic analogy
  - Geometric analogy
- Berry phase in solid state physics

Fast variable and slow variable

H^+_2 molecule



$$H(\vec{r},\vec{p};\{\vec{R}_i,\vec{P}_i\})$$

electron; {nuclei}

nuclei move thousands of times slower than the electron

Instead of solving time-dependent Schroedinger eq., one uses

Born-Oppenheimer approximation

- "Slow variables R_i" are treated as *parameters* $\lambda(t)$

(Kinetic energies from P_i are neglected)

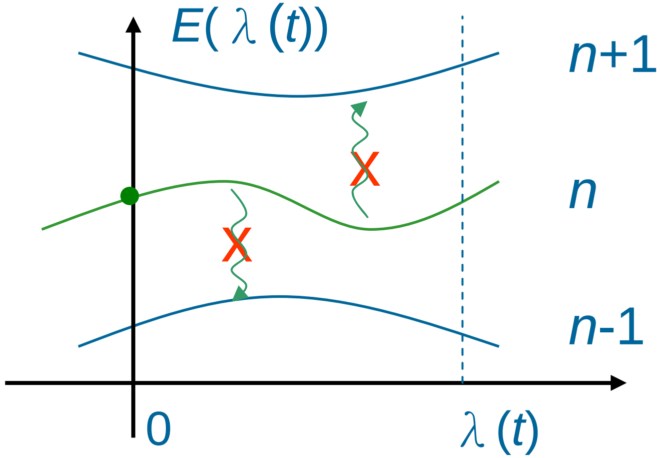
- solve time-*in*dependent Schroedinger eq.

$$H(\vec{r},\vec{p};\vec{\lambda})\psi_{n,\vec{\lambda}}(\vec{x}) = E_{n,\vec{\lambda}}\psi_{n,\vec{\lambda}}(\vec{x})$$

"snapshot" solution

Adiabatic evolution of a quantum system $H(\vec{r},\vec{p};\vec{\lambda})$

- Energy spectrum:



- After a *cyclic* evolution

$$\vec{\lambda}(T)=\vec{\lambda}(0)$$

$$\psi_{n,\vec{\lambda}(T)}=e^{-i\int_0^T dt' E_n(t')}\psi_{n,\vec{\lambda}(0)}$$

Dynamical phase

- Phases of the snapshot states at different λ's are independent and can be arbitrarily assigned

$$\psi_{n,\vec{\lambda}(t)}\to e^{i\gamma_n(\vec{\lambda})}\psi_{n,\vec{\lambda}(t)}$$

- Do we need to worry about this phase?

No!

- Fock, Z. Phys 1928
- Schiff, Quantum Mechanics (3rd ed.) p.290

Pf : Consider the *n*-th level,

$$\Psi_{\vec{\lambda}}(t) = e^{i\gamma_n(\vec{\lambda})} e^{-i\int_0^t dt' E_n(t')} \psi_{n,\vec{\lambda}}$$

Stationary, snapshot state

$$H\Psi_{\vec{\lambda}}(t) = i\hbar \frac{\partial}{\partial t} \Psi_{\vec{\lambda}}(t)$$

$$H\psi_{n,\vec{\lambda}} = E_n \psi_{n,\vec{\lambda}}$$

$$\dot{\gamma}_n = i\left\langle \psi_{n,\vec{\lambda}} \middle| \frac{\partial}{\partial \vec{\lambda}} \middle| \psi_{n,\vec{\lambda}} \right\rangle \cdot \dot{\vec{\lambda}} \neq 0$$

$\equiv \mathbf{A}_n(\lambda)$

Redefine the phase,

$$\psi'_{n,\vec{\lambda}} = e^{i\phi_n(\vec{\lambda})} \psi_{n,\vec{\lambda}}$$

$$\mathbf{A}_n'(\lambda) = \mathbf{A}_n(\lambda) - \frac{\partial \phi_n}{\partial \vec{\lambda}}$$

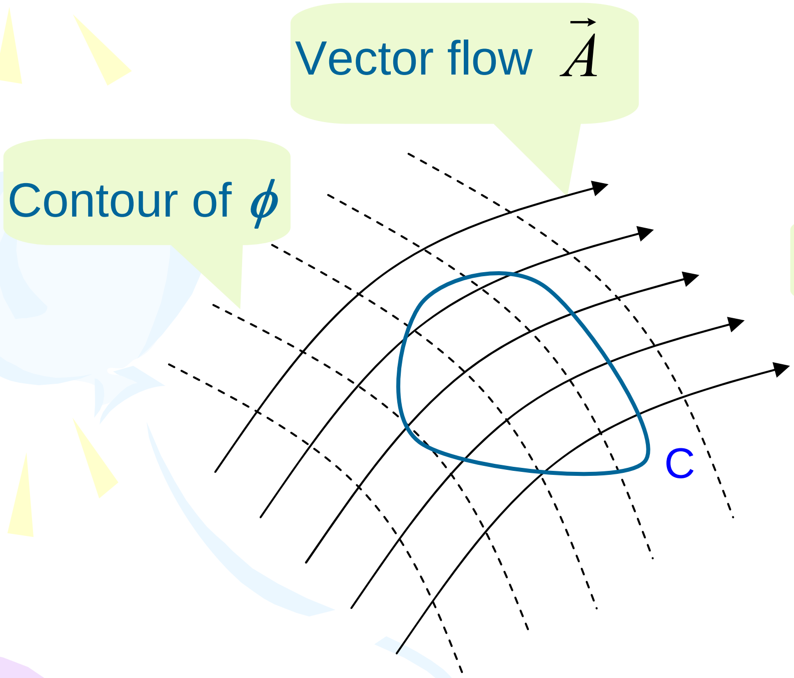
Choose a $\phi(\lambda)$ such that,

$\mathbf{A}_n'(\lambda)=0$

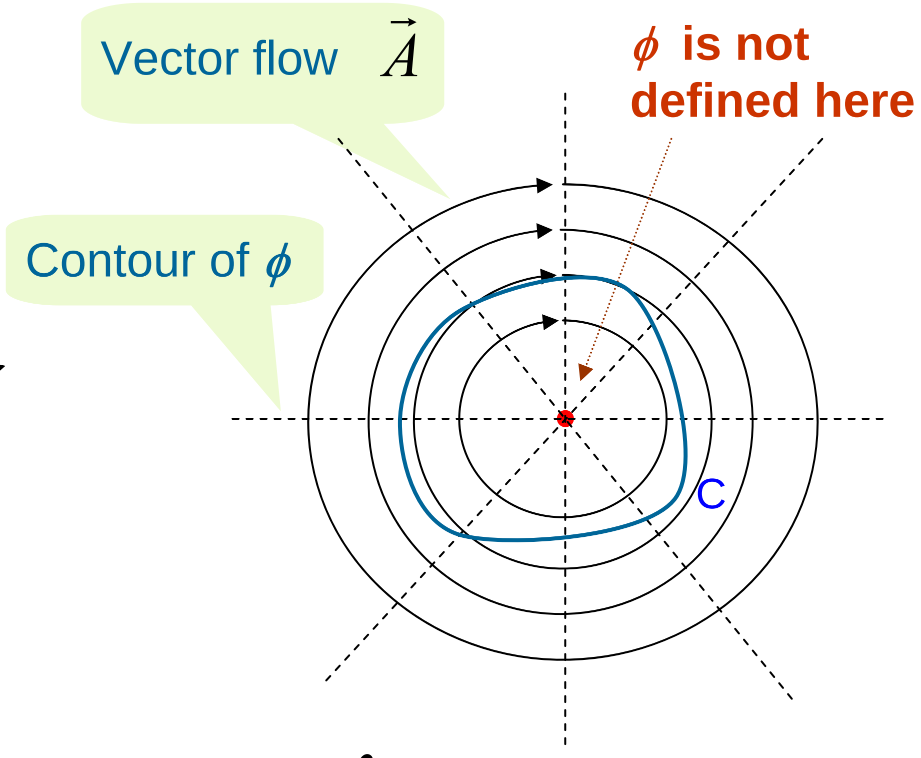
Thus removing the extra phase

One problem:

$\nabla_{\vec{\lambda}} \phi = \vec{A}(\vec{\lambda})$ does not always have a well-defined (global) solution

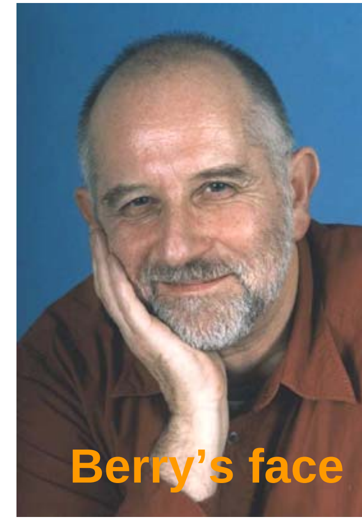


$$\oint_C \vec{A} \cdot d\vec{\lambda} = 0$$



$$\oint_C \vec{A} \cdot d\vec{\lambda} \neq 0$$

M. Berry, 1984 :
Parameter-dependent phase NOT always removable!



$$\psi_{\vec{\lambda}(T)} = e^{i\gamma_C} e^{-i\int_0^T dt' E(t')} \psi_{\vec{\lambda}(0)}$$

Index *n* neglected

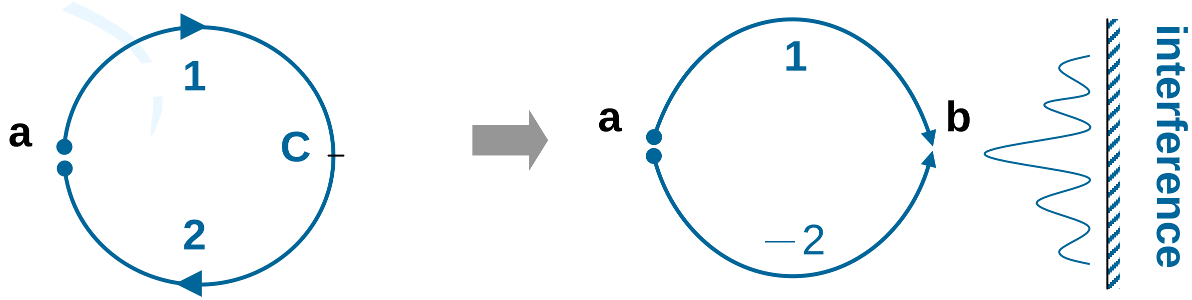
- Berry phase (path dependent)

$$\gamma_C = \oint_C \left\langle \psi_{\vec{\lambda}} \middle| i\frac{\partial}{\partial \vec{\lambda}} \middle| \psi_{\vec{\lambda}} \right\rangle \cdot d\vec{\lambda} \neq 0$$

- Interference due to the Berry phase

$$\text{if } \oint_C = \int_1 + \int_2 \neq 0, \text{ then } \int_1 - \int_{-2} \left(= \int_1 + \int_2 \right) \neq 0$$

Phase difference



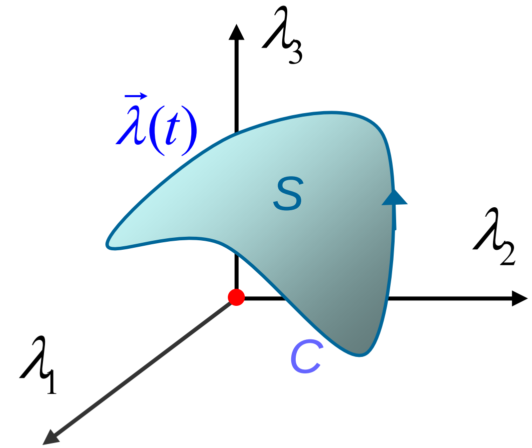
Some terminology

- **Berry connection** (or Berry potential)

$$\vec{A}(\vec{\lambda}) \equiv i \left\langle \psi_{\vec{\lambda}} \middle| \nabla_{\lambda} \middle| \psi_{\vec{\lambda}} \right\rangle$$

- **Stokes theorem** (3-dim here, can be higher)

$$\gamma_C = \oint_C \vec{A} \cdot d\vec{\lambda} = \int_S \nabla_{\vec{\lambda}} \times \vec{A} \cdot d\vec{a}$$



- **Berry curvature** (or Berry field)

$$\vec{F}(\vec{\lambda}) \equiv \nabla_{\lambda} \times \vec{A}(\vec{\lambda}) = i \left\langle \nabla_{\lambda} \psi_{\vec{\lambda}} \middle| \times \middle| \nabla_{\lambda} \psi_{\vec{\lambda}} \right\rangle$$

- **Gauge transformation** **(Nonsingular gauge, of course)**
  - $\left| \psi_{\vec{\lambda}} \right\rangle \rightarrow e^{i\phi(\vec{\lambda})} \left| \psi_{\vec{\lambda}} \right\rangle$
  - $\vec{A}(\vec{\lambda}) \rightarrow \vec{A}(\vec{\lambda}) - \nabla_{\lambda} \phi$
  - $\vec{F}(\vec{\lambda}) \rightarrow \vec{F}(\vec{\lambda})$
  - $\gamma_C \rightarrow \gamma_C$

Redefine the phases of the snapshot states

Berry curvature nd Berry phase not changed

Analogy with magnetic monopole

Berry potential (in parameter space)

$$\vec{A}(\vec{\lambda}) \equiv i\left\langle \psi_{\vec{\lambda}} \middle| \nabla_{\lambda} \middle| \psi_{\vec{\lambda}} \right\rangle$$

Vector potential (in real space)

$$\vec{A}(\vec{r})$$

Berry field (in 3D)

$$\vec{F}(\vec{\lambda}) \equiv \nabla_{\lambda} \times \vec{A}(\vec{\lambda})$$

Magnetic field

$$\vec{B}(\vec{r}) \equiv \nabla \times \vec{A}(\vec{r})$$

Berry phase

$$\gamma_C = \oint_C \vec{A}(\vec{\lambda}) \cdot d\vec{\lambda} = \int_S \vec{F} \cdot d\vec{a}$$

Magnetic flux

$$\Phi = \oint_C \vec{A}(\vec{r}) \cdot d\vec{r} = \int_S \vec{B} \cdot d\vec{a}$$

Chern number

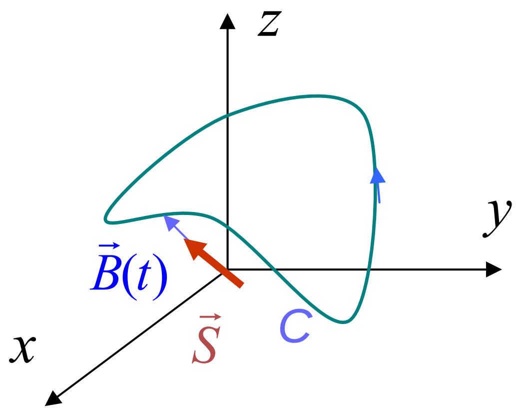
$$\frac{1}{2\pi} \oint_S \vec{F}(\vec{\lambda}) \cdot d\vec{a} = \text{integer}$$

Dirac monopole

$$\frac{1}{4\pi} \oint_S \vec{B}(\vec{\lambda}) \cdot d\vec{a} = \text{integer}$$

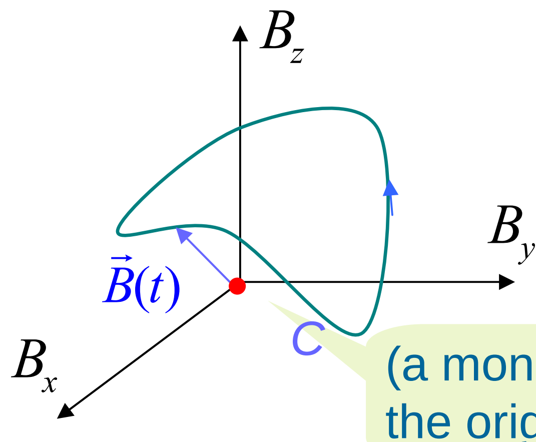
Example: spin-1/2 particle in slowly changing *B* field

- **Real space**



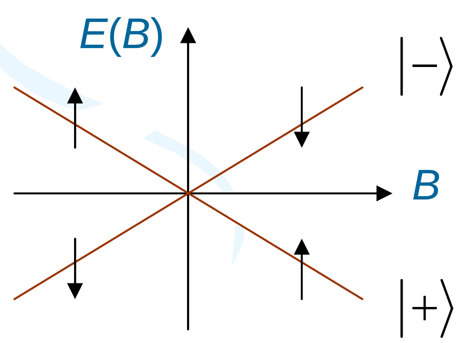
$$H_{\vec{\lambda}=\vec{B}} = \mu_B \vec{B}\cdot\vec{\sigma}$$

- **Parameter space**



(a monopole at the origin)

Level crossing at *B*=0



Berry curvature

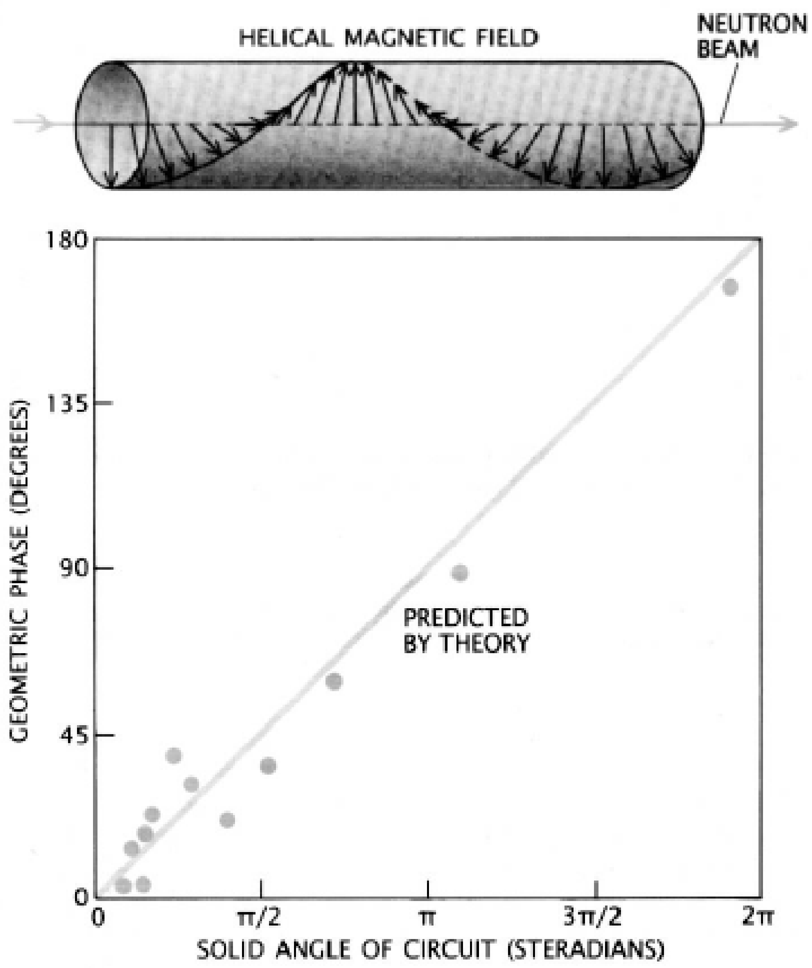
$$\vec{F}_\pm(\vec{B}) = i\left\langle \nabla_B \psi_{\pm,\vec{B}} \middle| \times \middle| \nabla_B \psi_{\pm,\vec{B}} \right\rangle = \mp \frac{1}{2}\frac{\hat{B}}{B^2}$$

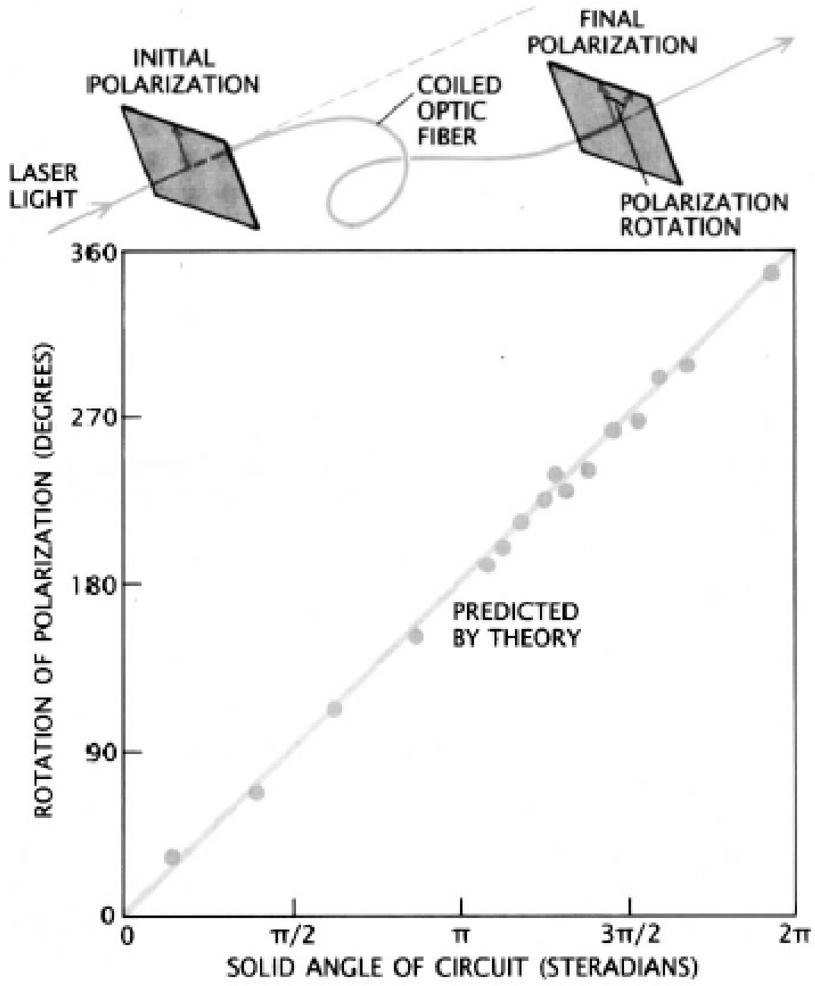
Berry phase

$$\gamma_\pm = \int_S \vec{F}_\pm \cdot d\vec{a} = \mp\frac{1}{2}\Omega(C)$$

spin × solid angle

Experimental realizations :



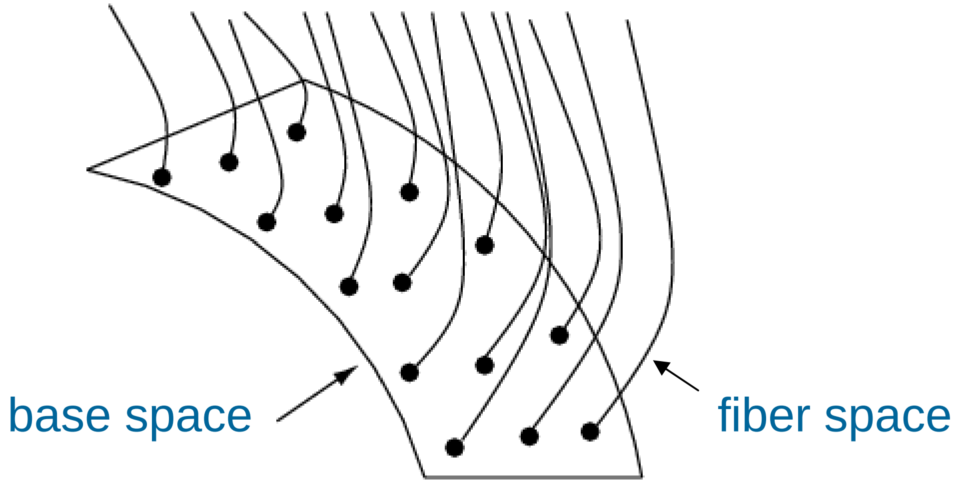


Bitter and Dubbers , PRL 1987

Tomita and Chiao, PRL 1986

Geometry behind the Berry phase

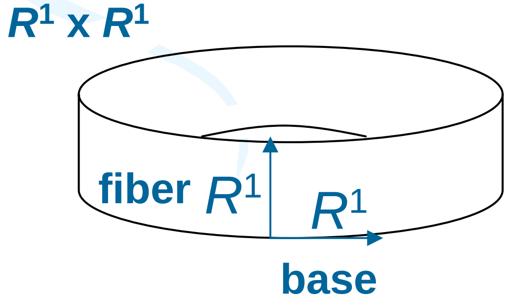
Why Berry phase is often called geometric phase?



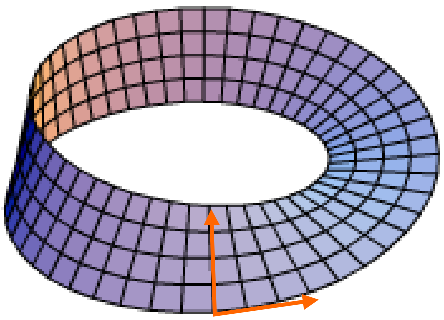
Fiber bundle

Examples:

- **Trivial** fiber bundle
(= a product space)



- **Nontrivial** fiber bundle
Simplest example: Möbius band

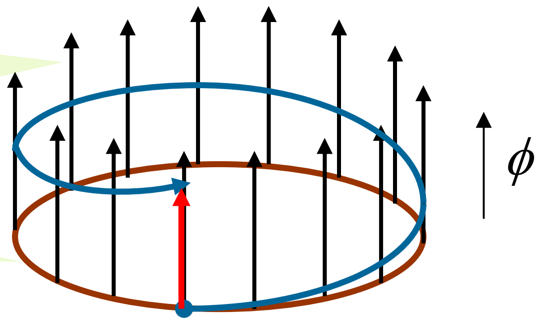


Fiber bundle and quantum state evolution

(Wu and Yang, PRD 1975)

Fiber space:
inner DOF, eg., U(1) phase

Base space:
parameter space



- Berry phase = Vertical shift along fiber
  (U(1) anholonomy)

- Chern number n

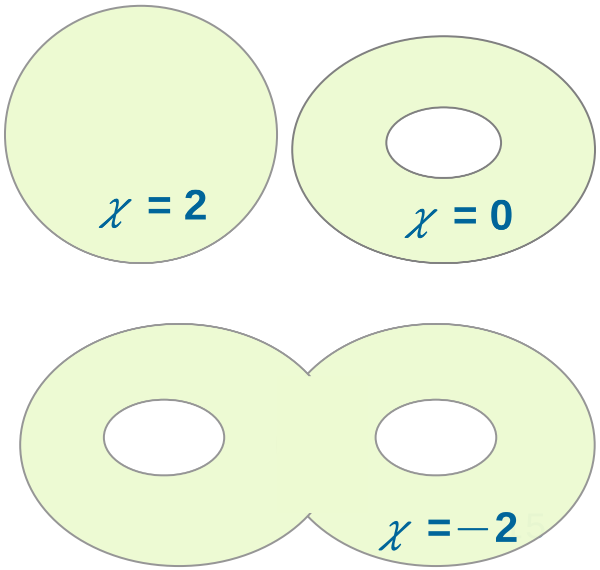
$$n = \frac{1}{2\pi}\int_S da\ F$$

For fiber bundle

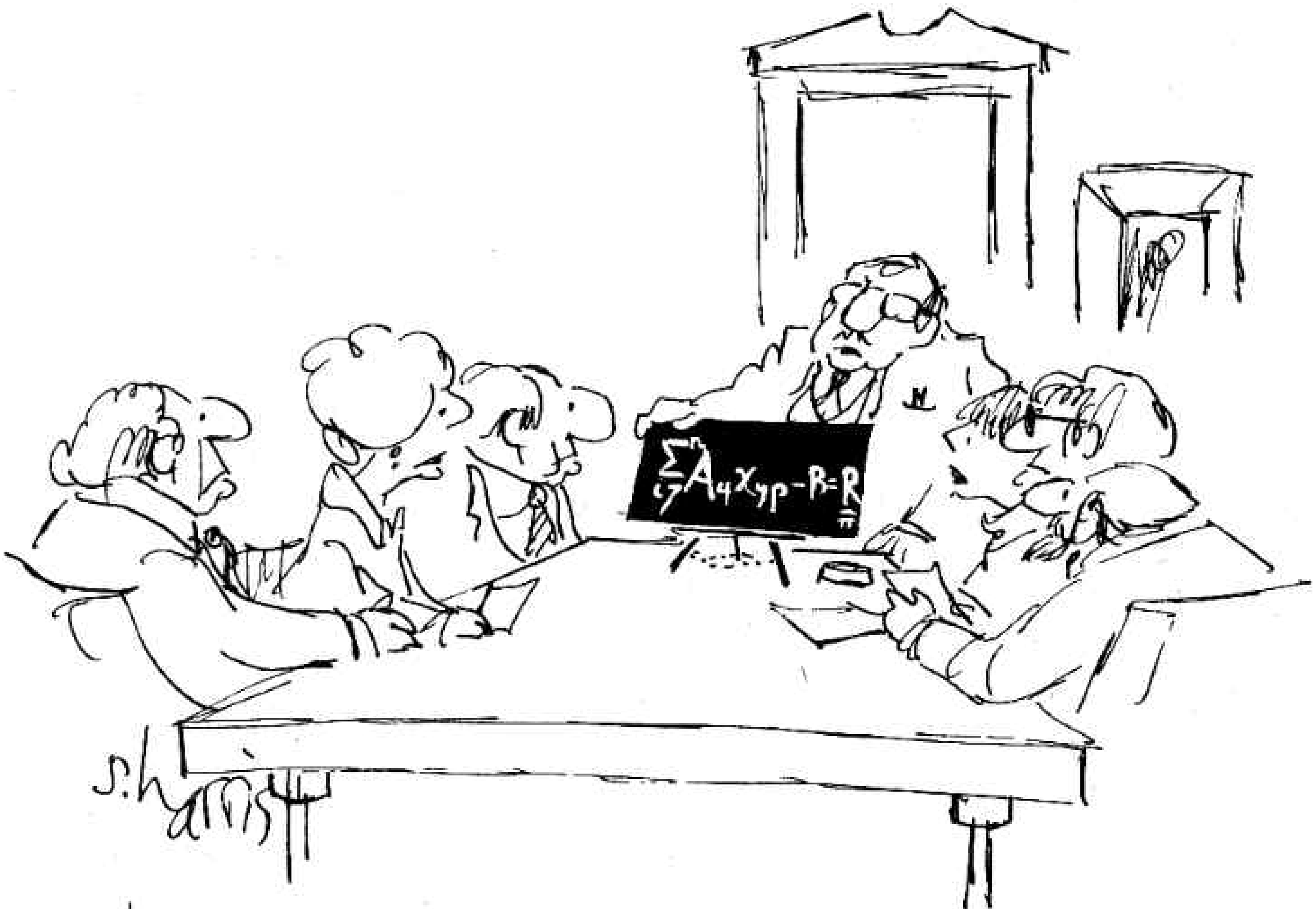
$\sim$ Euler characteristic χ

$$\chi = \frac{1}{2\pi}\int_S da\ G$$

For 2-dim closed surface



- Introduction
- Berry phase in solid state physics



"LADIES AND GENTLEMEN, OUR RESEARCH DEPARTMENT HAS COME UP WITH THIS. WHAT DO WE WISH TO DO WITH IT?"

Berry phase in condensed matter physics, a partial list:

- 1982 Quantized Hall conductance **(Thouless et al)**
- 1983 Quantized charge transport **(Thouless)**
- 1984 Anyon in fractional quantum Hall effect **(Arovas et al)**
- 1989 Berry phase in one-dimensional lattice **(Zak)**
- 1990 Persistent spin current in one-dimensional ring **(Loss et al)**
- 1992 Quantum tunneling in magnetic cluster **(Loss et al)**
- 1993 Modern theory of electric polarization **(King-Smith et al)**
- 1996 Semiclassical dynamics in Bloch band **(Chang et al)**
- 1998 Spin wave dynamics **(Niu et al)**
- 2001 Anomalous Hall effect **(Taguchi et al)**
- 2003 Spin Hall effect **(Murakami et al)**
- 2004 Optical Hall effect **(Onoda et al)**
- 2006 Orbital magnetization in solid **(Xiao et al)**
- …

Berry phase in condensed matter physics, a partial list:

- 1982 Quantized Hall conductance **(Thouless et al)**
- 1983 Quantized charge transport **(Thouless)**
- 1984 Anyon in fractional quantum Hall effect **(Arovas et al)**
- 1989 Berry phase in one-dimensional lattice **(Zak)**
- 1990 Persistent spin current in one-dimensional ring **(Loss et al)**
- 1992 Quantum tunneling in magnetic cluster **(Loss et al)**
- 1993 Modern theory of electric polarization **(King-Smith et al)**
- 1996 Semiclassical dynamics in Bloch band **(Chang et al)**
- 1998 Spin wave dynamics **(Niu et al)**
- 2001 Anomalous Hall effect **(Taguchi et al)**
- 2003 Spin Hall effect **(Murakami et al)**
- 2004 Optical Hall effect **(Onoda et al)**
- 2006 Orbital magnetization in solid **(Xiao et al)**
- …

Berry phase in solid state physics

- ❖ Persistent spin current
- ❖ Quantum tunneling in a magnetic cluster
- ❖ Modern theory of electric polarization
- ❖ Semiclassical electron dynamics
  - ✓ Quantum Hall effect (QHE)
  - ✓ Anomalous Hall effect (AHE)
  - ✓ Spin Hall effect (SHE)

Spin

- Persistent spin current
- Quantum tunneling

Spin and Bloch state

- AHE
- SHE

Bloch state

- Electric polarization
- QHE

Electric polarization of a periodic solid

$$\vec{P} = \frac{1}{V}\int d^3r\, \vec{r}\, \rho(\vec{r})$$

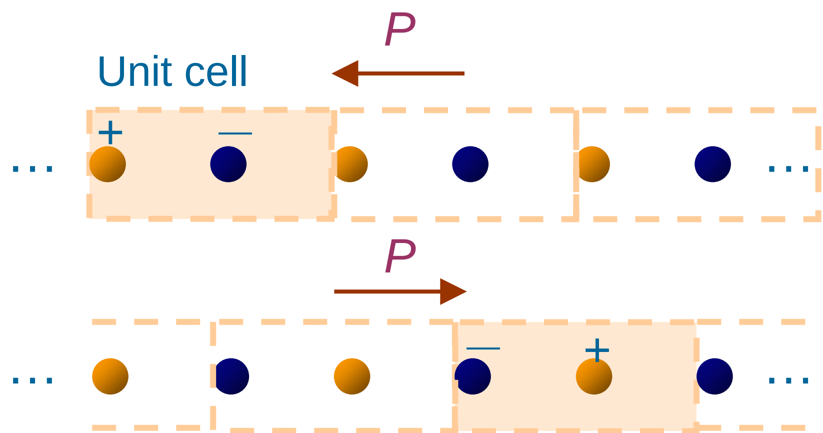
✓ well defined only for finite system (sensitive to boundary)

✓ or, for crystal with well-localized dipoles (Claussius-Mossotti theory)

- *P* is **not** well defined in, e.g., covalent crystal:

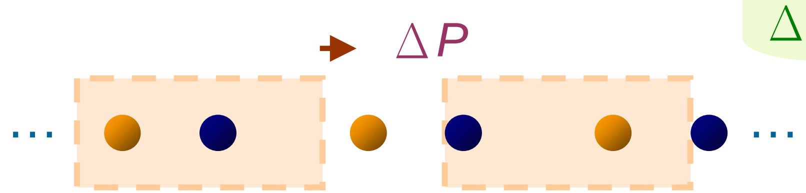
Choice 1

Choice 2



- However, the change of *P* is well-defined

Experimentally, it's ΔP that's measured



Modern theory of polarization

One-dimensional lattice (λ =atomic displacement in a unit cell)

$$P = \frac{q}{L} \sum_{nk} \langle \psi_{nk}^{\lambda} | r | \psi_{nk}^{\lambda} \rangle \qquad \psi_{nk}^{\lambda}(r) = e^{ikr} u_{nk}^{\lambda}(r)$$

Iℓℓ-defined

Resta, Ferroelectrics 1992

However, *dP/d* λ is well-defined, even for an infinite system !

$$\Delta P = \int \frac{dP}{d\lambda} d\lambda = P(\lambda_2) - P(\lambda_1)$$

King-Smith and Vanderbilt, PRB 1993

$$\text{where} \quad P(\lambda) = q \sum_n \int_{BZ} \frac{dk}{2\pi} \langle u_{nk}^{\lambda} | i \frac{\partial}{\partial k} | u_{nk}^{\lambda} \rangle$$

Berry potential

$$= q \sum_n \frac{\gamma_n}{2\pi}$$

- For a one-dimensional lattice **with** *inversion* symmetry

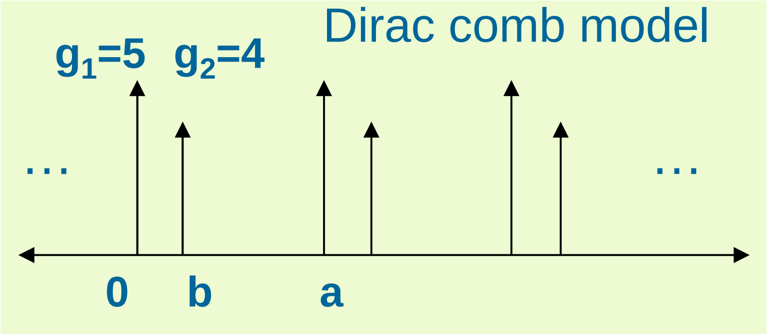
(*if* the origin is a symmetric point)

$$\gamma_n = 0 \quad \text{or} \quad \pi$$

(Zak, PRL 1989)

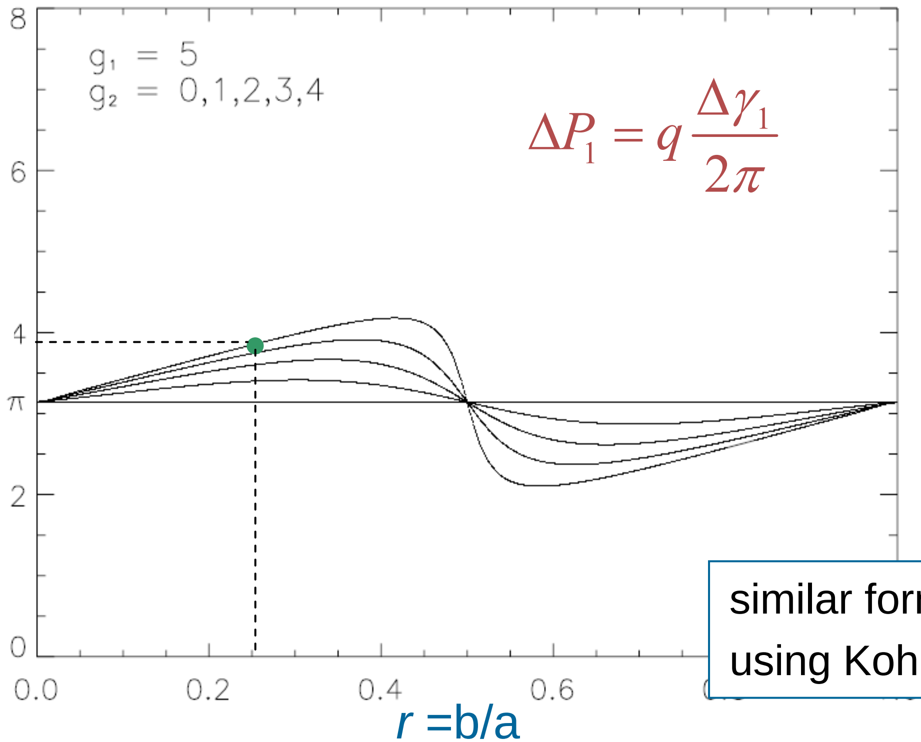
- Other values are possible **without** inversion symmetry

Berry phase and electric polarization



Rave and Kerr, EPJ B 2005

Lowest energy band: γ_1



← g_2=0

$\gamma_1 = \pi$

similar formulation in 3-dim using Kohn-Sham orbitals

Review: Resta, J. Phys.: Condens. Matter 12, R107 (2000)

Berry phase in solid state physics

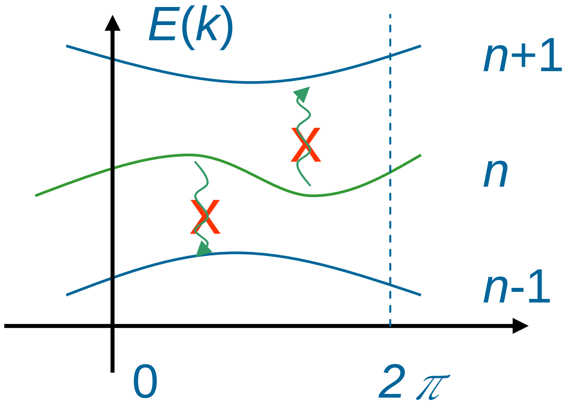
- ❖ Persistent spin current
- ❖ Quantum tunneling in a magnetic cluster
- ❖ Modern theory of electric polarization
- ❖ Semiclassical electron dynamics
  - ✓ Quantum Hall effect
  - ✓ Anomalous Hall effect
  - ✓ Spin Hall effect

Semiclassical dynamics in solid

Limits of validity: **one band approximation**

Negligible inter-band transition.

"never close to being violated in a metal"



$$\hbar \frac{d\vec{k}}{dt} = -e\vec{E} - e\dot{\vec{r}} \times \vec{B}$$

$$\frac{d\vec{r}}{dt} = \frac{1}{\hbar}\frac{\partial E_n}{\partial \vec{k}}$$

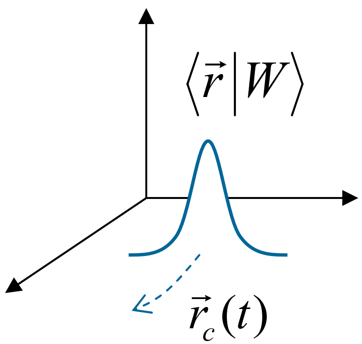
- Lattice effect hidden in $E_n(k)$
- Derivation is harder than expected

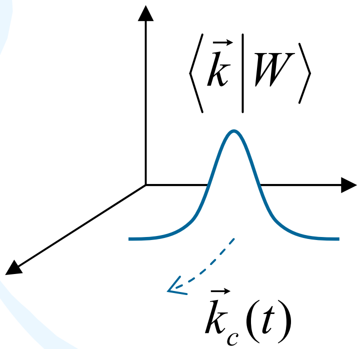
Explains (Ashcroft and Mermin, Chap 12)

- **Bloch oscillation** in a DC electric field,
  quantization → **Wannier-Stark ladders**
- **cyclotron motion** in a magnetic field,
  quantization → **LLs, de Haas - van Alphen effect**

…

Semiclassical dynamics - wavepacket approach





1. Construct a wavepacket that is localized in both *r*-space and *k*-space (parameterized by its c.m.)

2. Using the time-dependent variational principle to get the effective Lagrangian for the c.m. variables

$$L_{eff}(\vec{r}_c, \vec{k}_c; \dot{\vec{r}}_c, \dot{\vec{k}}_c) = \langle W | i\hbar \frac{\partial}{\partial t} - H | W \rangle$$

3. Minimize the action $S_{\text{eff}}[\boldsymbol{r}_{\text{c}}(t), \boldsymbol{k}_{\text{c}}(t)]$ and determine the trajectory ($\boldsymbol{r}_{\text{c}}(t)$, $\boldsymbol{k}_{\text{c}}(t)$)

$\rightarrow$ Euler-Lagrange equations

Wavepacket in Bloch band:

$$L_{eff} = \left(\hbar\vec{k}_c - e\vec{A}\right)\cdot\dot{\vec{r}}_c + \hbar\dot{\vec{k}}_c\cdot\vec{R}_n - E_n(\vec{r}_c, \vec{k}_c)$$

Berry potential

(Chang and Niu, PRL 1995, PRB 1996)

Semiclassical dynamics with Berry curvature

$$\hbar \frac{d\vec{k}}{dt} = -e\vec{E} - e\dot{\vec{r}} \times \vec{B}$$

$$\frac{d\vec{r}}{dt} = \frac{1}{\hbar}\frac{\partial E_n}{\partial \vec{k}} - \dot{\vec{k}} \times \vec{\Omega}_n(\vec{k})$$

"Anomalous" velocity

Cell-periodic Bloch state

Berry curvature

$$\vec{\Omega}_n(\vec{k}) = i\left\langle \nabla_k u_{n\vec{k}} \right| \times \left| \nabla_k u_{n\vec{k}} \right\rangle$$

Wavepacket energy

$$E_n(\vec{r}_c, \vec{k}_c) = E_n^0(\vec{k}_c) + \frac{e}{2m}\vec{L}_n(\vec{k}_c)\cdot\vec{B}$$

Bloch energy

Zeeman energy due to spinning wavepacket

$$\vec{L}_n(\vec{k}_c) = m\left\langle W \right| (\vec{r} - \vec{r}_c) \times \vec{v} \left| W \right\rangle$$

If B=0, then dk/dt // electric field

$\rightarrow$ Anomalous velocity $\perp$ electric field

Simple and Unified

- (integer) Quantum Hall effect
- (intrinsic) Anomalous Hall effect
- (intrinsic) Spin Hall effect

◆ Why the anomalous velocity is not found earlier?

In fact, it *had been found* by

• Adams, Blount, in the 50's

◆ Why it *seems* OK not to be aware of it?

For *scalar* Bloch state (*non*-degenerate band):

• Space inversion symmetry $\vec{\Omega}_n(-\vec{k}) = \vec{\Omega}_n(\vec{k})$

• Time reversal symmetry $\vec{\Omega}_n(-\vec{k}) = -\vec{\Omega}_n(\vec{k})$

both symmetries $\vec{\Omega}_n(\vec{k}) = 0, \ \forall \vec{k}$

◆ When do we expect to see it? $\vec{\Omega}_n(\vec{k}) \neq 0$

• SI symmetry is broken ← **electric polarization**

• TR symmetry is broken ← **QHE**

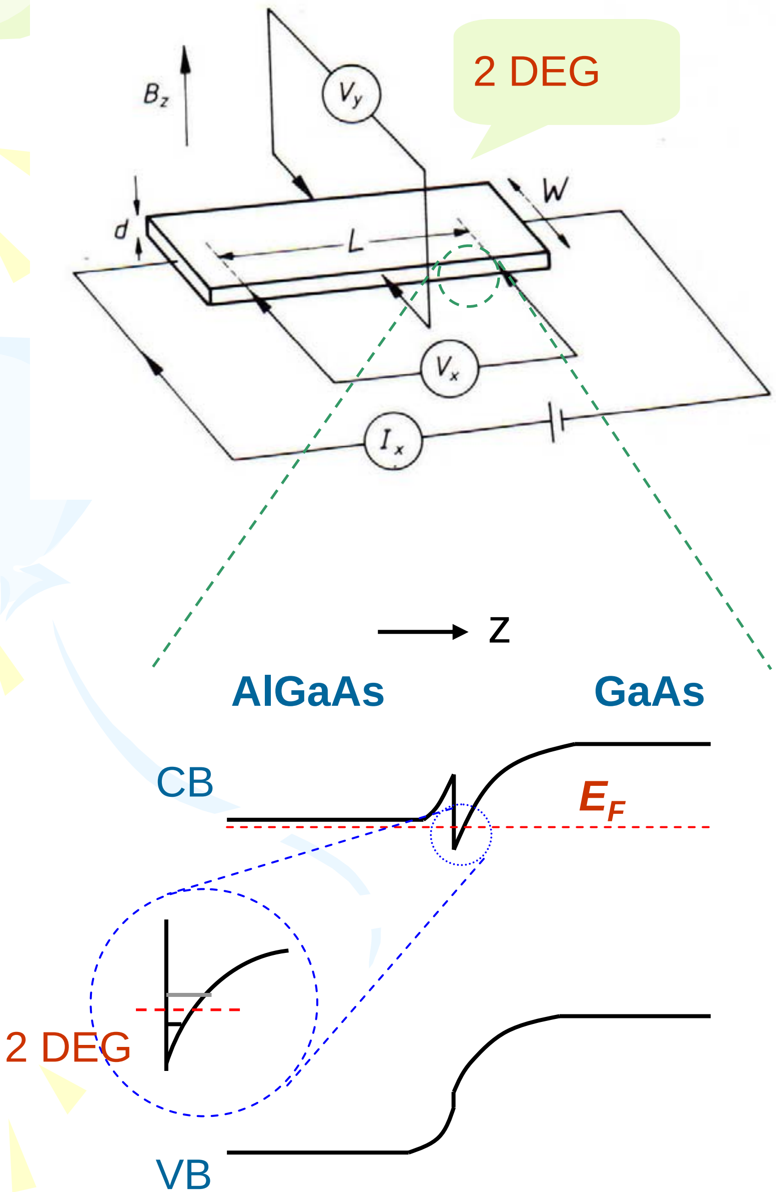
• *spinor* Bloch state (degenerate band) ← **SHE**

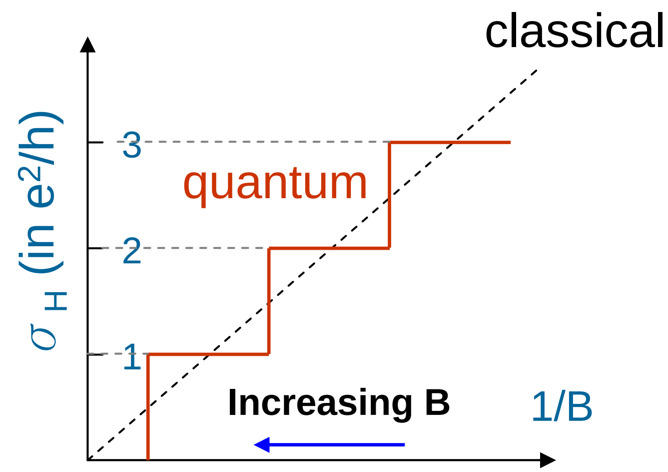
Also, **• band crossing** ← **monopole**

Berry phase in solid state physics

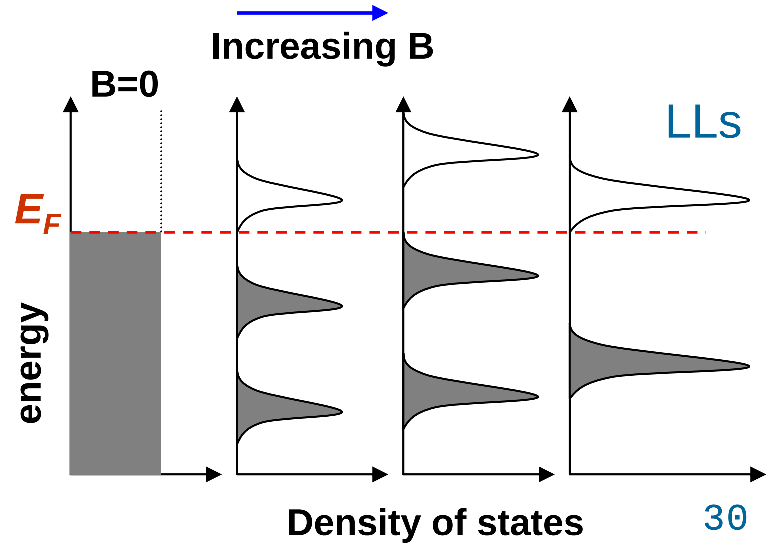
- ❖ Persistent spin current
- ❖ Quantum tunneling in a magnetic cluster
- ❖ Modern theory of electric polarization
- ❖ Semiclassical electron dynamics
    - ✓ Quantum Hall effect
    - ✓ Anomalous Hall effect
    - ✓ Spin Hall effect

Quantum Hall effect (von Klitzing, PRL 1980)





Each LL contributes one e^2/h



Semiclassical formulation

Equations of motion

(In one Landau subband)

$$\hbar \frac{d\vec{k}}{dt} = -e\vec{E}$$

$$\frac{d\vec{r}}{dt} = \frac{1}{\hbar}\frac{\partial E}{\partial \vec{k}} - \dot{\vec{k}} \times \vec{\Omega}(\vec{k})$$

Magnetic field effect is hidden here

$$\vec{J} = -e \int_{filled} \frac{d^2k}{(2\pi)^2} \dot{\vec{r}}$$

$$= 0 - \frac{e^2}{\hbar} \vec{E} \times \int_{filled} \frac{d^2k}{(2\pi)^2} \vec{\Omega}(\vec{k})$$

$$\Rightarrow J_x = -\left(\frac{e^2}{h} \frac{1}{2\pi} \int_{filled} d^2k \, \Omega_z(\vec{k}) \right) E_y$$

Hall conductance σ_H

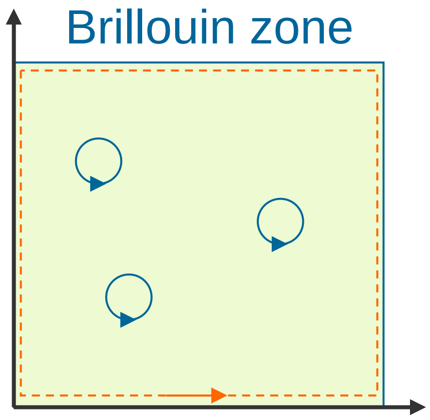
Quantization of Hall conductance (Thouless *et al* 1982)

$$\frac{1}{2\pi} \int_{BZ} d^2k \, \Omega_z(\vec{k}) = \text{integer } n$$

$$\therefore \quad \sigma_H = n \frac{e^2}{h}$$

Remains quantized even with disorder, e-e interaction
(Niu, Thouless, Wu, PRB, 1985)

Quantization of Hall conductance (II)



For a filled Landau subband

$$\int_{BZ} d^2k\, \Omega_z(\vec{k}) = \oint_{\partial BZ} d\vec{k} \cdot \vec{A}$$

Counts the amount of vorticity in the BZ

due to zeros of Bloch state
(Kohmoto, Ann. Phys, 1985)

In the language of differential geometry,

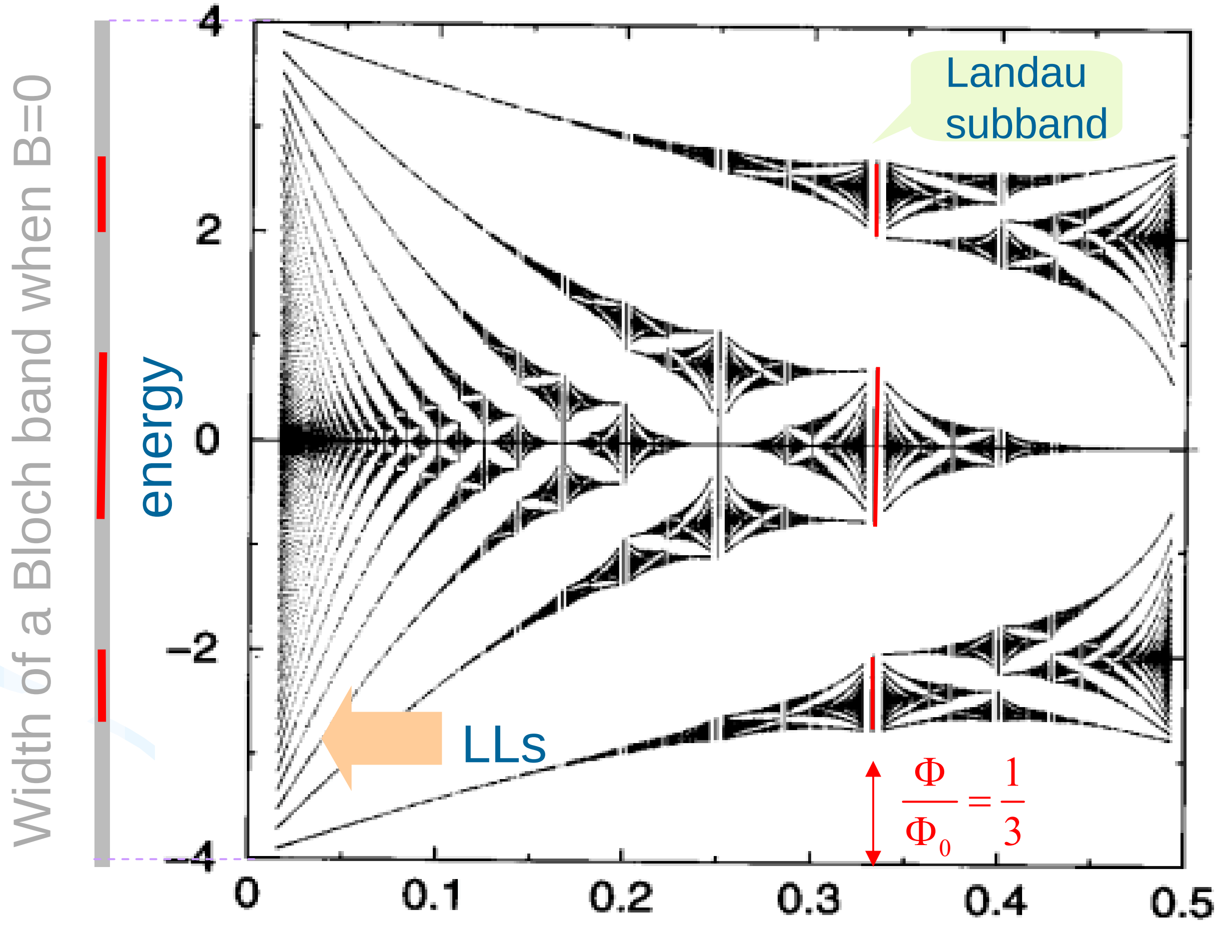
this *n* is the (first) Chern number that

characterizes the *topology* of a fiber bundle

(base space: BZ; fiber space: U(1) phase)

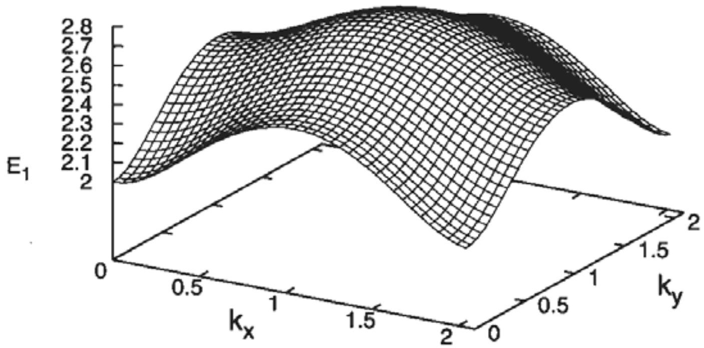
Berry curvature and Hofstadter spectrum

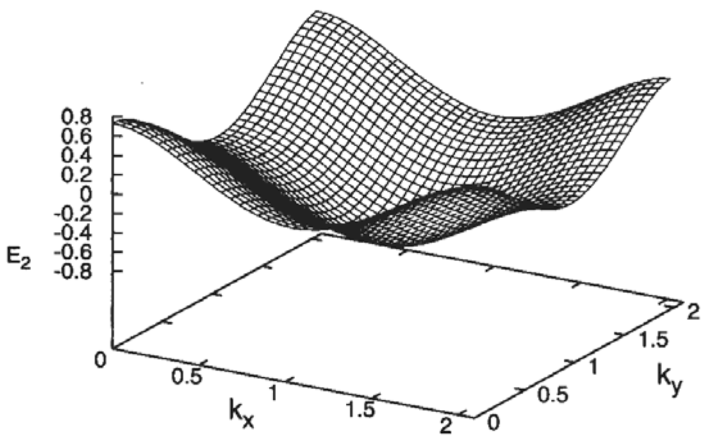
2DEG in a square lattice + a perpendicular B field
tight-binding model: (Hofstadter, PRB 1976)

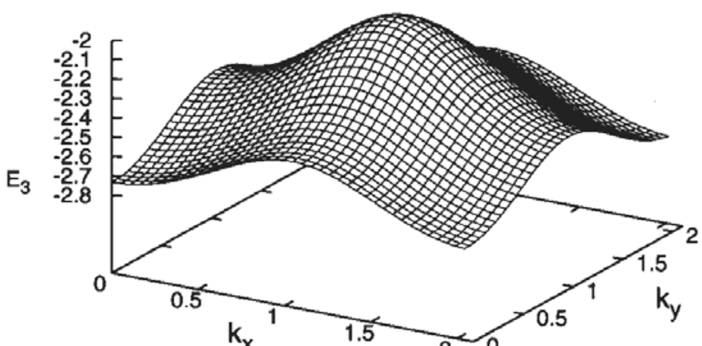


Magnetic flux (in Φ_0) / plaquette

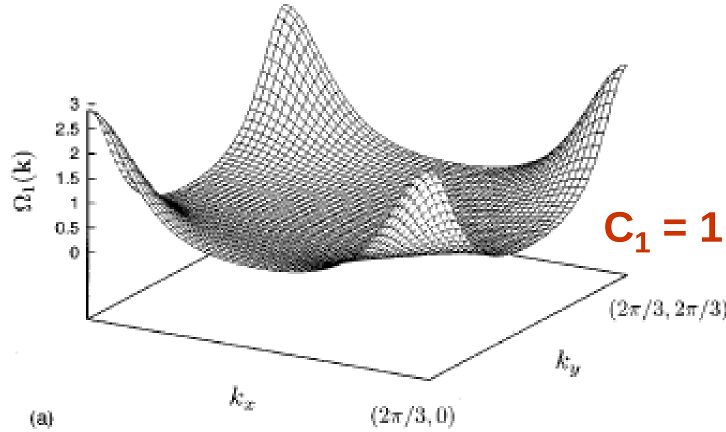
Bloch energy *E*(*k*)



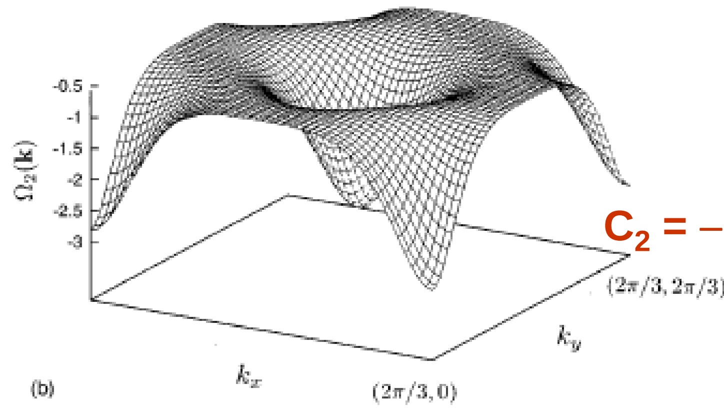




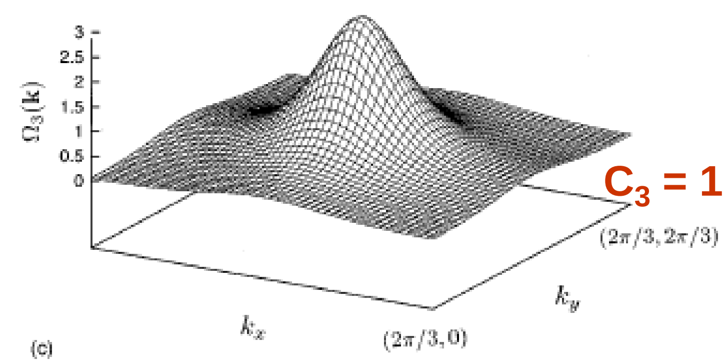
Berry curvature Ω(*k*)



$C_1 = 1$



$C_2 = -2$



$C_3 = 1$

Re-quantization of semiclassical theory

$$L_{eff} = \left(\hbar\vec{k} - e\vec{A}\right)\cdot\dot{\vec{r}} + \hbar\dot{\vec{k}}\cdot\vec{R} - E(\vec{r},\vec{k})$$

Bohr-Sommerfeld quantization

$$\frac{1}{2}\oint_{C_m}\left(\vec{k}\times d\vec{k}\right)\cdot d\hat{z} = 2\pi\left(m + \frac{1}{2} - \frac{\gamma(C_m)}{2\pi}\right)\frac{eB}{\hbar}$$

Berry phase $\gamma(C_m) = \oint_{C_m} \vec{R}\cdot d\vec{k}$

Would shift quantized cyclotron energies (LLs)

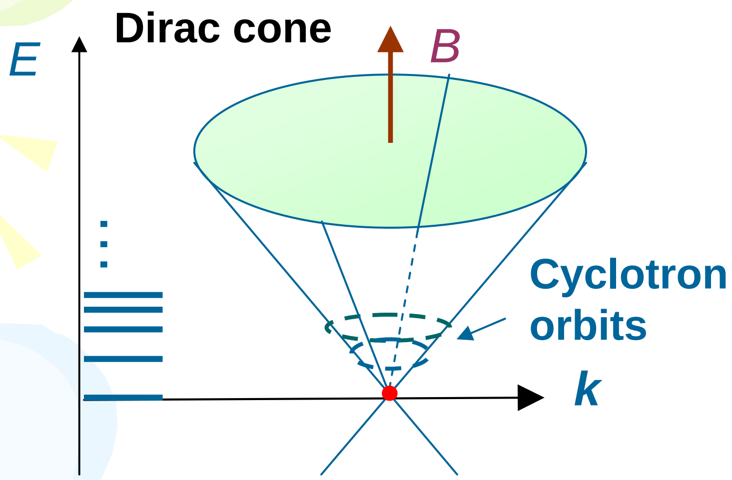
- **Bloch oscillation** in a DC electric field,

  re-quantization → **Wannier-Stark ladders**
- **cyclotron motion** in a magnetic field,

  re-quantization → **LLs, dHvA effect**
- …

Now with Berry phase effect!

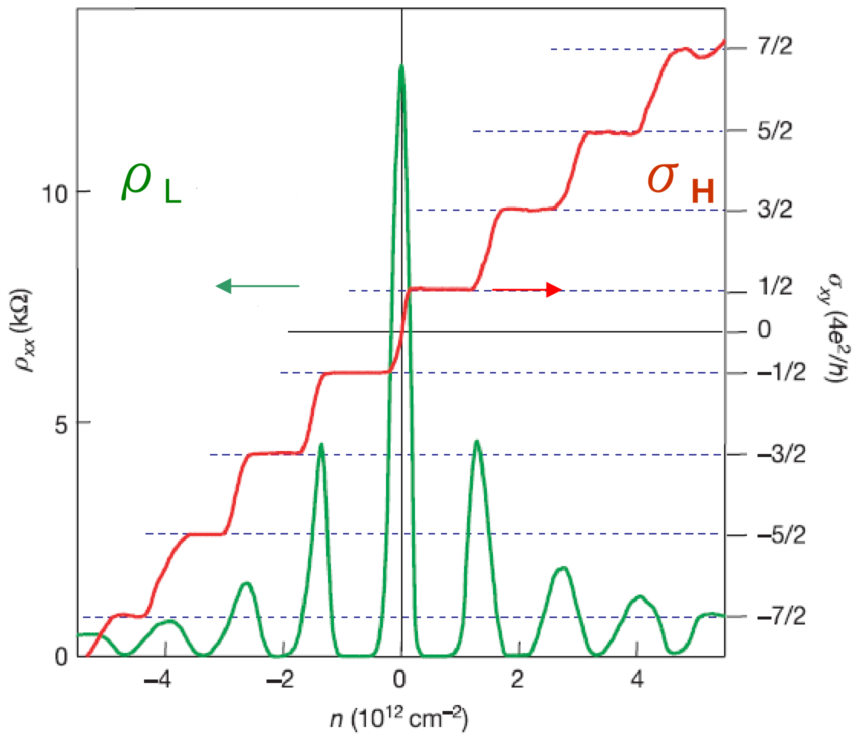
cyclotron orbits (LLs) in graphene ↔ QHE in graphene



$$\Omega(\vec{k}) = \pi\delta(\vec{k}) \rightarrow \gamma_C = \pi$$

$$\begin{cases} E(k) = \hbar v_F k \\ \pi k^2 = 2\pi\left(m + \frac{1}{2} - \frac{\gamma_C}{2\pi}\right)\frac{eB}{\hbar} \end{cases}$$

$$\Rightarrow E_n = v_F\sqrt{2eB\hbar\left(n + \frac{1}{2} - \frac{\gamma_C}{2\pi}\right)}$$



Novoselov et al, Nature 2005

Berry phase in solid state physics

- ❖ Persistent spin current
- ❖ Quantum tunneling in a magnetic cluster
- ❖ Modern theory of electric polarization
- ❖ Semiclassical electron dynamics
  - ✓ Quantum Hall effect
  - ✓ Anomalous Hall effect
  - ✓ Spin Hall effect

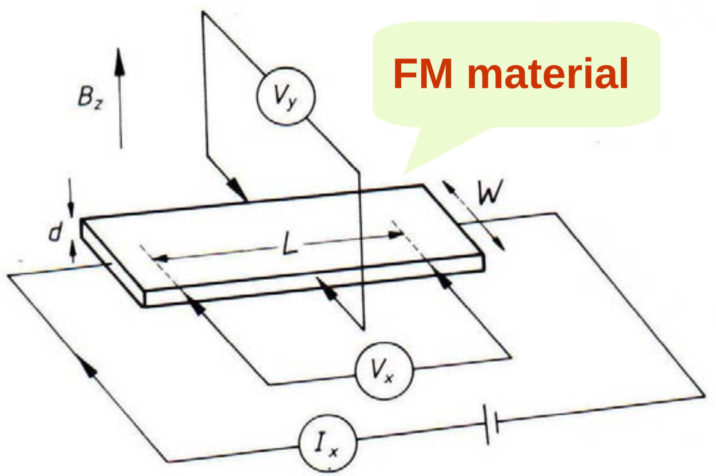
Mokrousov's talks this Friday

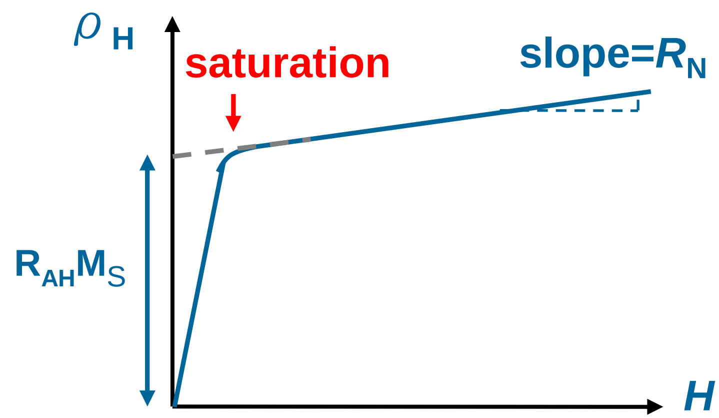
Buhmann's next Thu (on QSHE)

Poor men's, and women's, version of QHE, AHE, and SHE

Anomalous Hall effect (Edwin Hall, 1881):

Hall effect in ferromagnetic (FM) materials





The usual Lorentz force term

$$\rho_H = R_N H + \rho_{AH}(H),$$

Anomalous term

$$\rho_{AH}(H) \equiv R_{AH} M(H)$$

Ingredients required for a successful theory:

- magnetization (majority spin)
- spin-orbit coupling

(to couple the *majority-spin* direction to transverse orbital direction)

Intrinsic mechanism (ideal lattice without impurity)

PHYSICAL REVIEW VOLUME 95, NUMBER 5 SEPTEMBER 1, 1954

Hall Effect in Ferromagnetics*

ROBERT KARPLUS,† *Department of Physics, University of California, Berkeley, California*

AND

J. M. LUTTINGER, *Department of Physics, University of Michigan, Ann Arbor, Michigan*

(Received May 21, 1954)

Both the unusually large magnitude and strong temperature dependence of the extraordinary Hall effect in ferromagnetic materials can be understood as effects of the spin-orbit interaction of polarized conduction electrons. It is shown that the interband matrix elements of the applied electric potential energy combine with the spin-orbit perturbation to give a current perpendicular to both the field and the magnetization. Since the net effect of the spin-orbit interaction is proportional to the extent to which the electron spins are aligned, this current is proportional to the magnetization. The magnitude of the Hall constant is equal to the square of the ordinary resistivity multiplied by functions that are not very sensitive to temperature and impurity content. The experimental results behave in such a way also.

- Linear response
- Spin-orbit coupling
- magnetization

gives correct order of magnitude of ρ_H for Fe, also explains $\rho_{AH} \sim \rho_L^2$ that's observed in some data

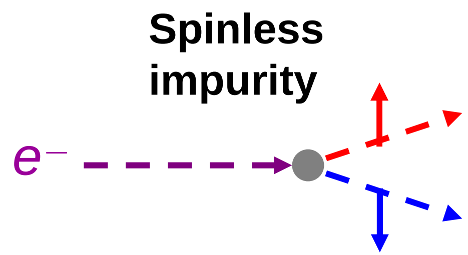
Smit, 1955: KL mechanism should be annihilated by (an extra effect from) impurities

Alternative scenario:

Extrinsic mechanisms (with impurities)

- **Skew scattering** (Smit, Physica 1955)

$\sim$ Mott scattering

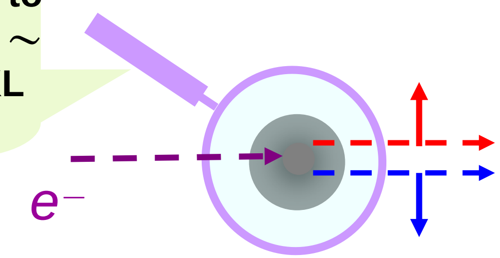


$$\rho_{AH} \sim \rho_L$$

- **Side jump** (Berger, PRB 1970)

anomalous velocity due to electric field of impurity $\sim$ anomalous velocity in KL

(Crépieux and Bruno, PRB 2001)



$$\rho_{AH} \sim \rho_L^2$$

$$\delta \approx 1A$$

2 (or 3) mechanisms: $\rho_{AH} = a(M)\rho_L + b(M)\rho_L^2$

In reality, it's not so clear-cut !

Review: Sinitsyn, J. Phys: Condens. Matter 20, 023201 (2008)

CM Hurd, *The Hall Effect in Metals and Alloys* (1972)

"The difference of opinion between Luttinger and Smit seems never to have been entirely resolved."

30 years later:

Crepieux and Bruno, PRB 2001

"It is now accepted that two mechanisms are responsible for the AHE: the skew scattering… and the side-jump…"

However,

Science 2001

Spin Chirality, Berry Phase, and Anomalous Hall Effect in a Frustrated Ferromagnet

Y. Taguchi,[1] Y. Oohara,[2] H. Yoshizawa,[2] N. Nagaosa,[1,3] Y. Tokura[1,3]

An electr complex acts as an effect wh surement that the ferromag the spin c tilting.

VOLUME 88, NUMBER 20 PHYSICAL REV

Anomalous Hall Effect in Fe

T. Jungwirth,[1,2] Qian Niu

[1]*Department of Physics, The Unive*

[2]*Institute of Physics ASCR, Cukrovarnic*

(Received 3 October 2001

We present a theory of the anomalous Hall effect in ferromagnetic (III, Mn)V semiconductors. Our theory relates the anomalous Hall conductance of a homogeneous ferromagnet to the Berry phase acquired by a quasiparticle wave function upon traversing closed paths on the spin-split Fermi surface. The quantitative agreement between our theory and experimental data in both (In, Mn)As and (Ga, Mn)As systems suggests that this disorder independent contribution to the anomalous Hall conductivity dominates in diluted magnetic semiconductors. The success of this model for (III, Mn)V materials is unprecedented in the longstanding effort to understand origins of the anomalous Hall effect in itinerant ferromagnets.

The Anomalous Hall Effect and Magnetic Monopoles in Momentum Space

Zhong Fang,[1,2*] Naoto Nagaosa,[1,3,4] Kei S. Takahashi,[5] Atsushi Asamitsu,[1,6] Roland Mathieu,[1] Takeshi Ogasawara,[3] Hiroyuki Yamada,[3] Masashi Kawasaki,[3,7] Yoshinori Tokura,[1,3,4] Kiyoyuki Terakura[8]

Science 2003

Efforts to find the magnetic monopole in real space have been made in cosmic rays and in particle accelerators, but there has not yet been any firm evidence for its existence because of its very heavy mass, $\sim 10^{16}$ giga–electron volts. We show that the magnetic monopole can appear in the crystal momentum space of solids in the accessible low-energy region ($\sim$0.1 to 1 electron volts) in the context of the anomalous Hall effect. We report experimental results together with first-principles calculations on the ferromagnetic crystal $SrRuO_3$ that provide evidence for the magnetic monopole in the crystal momentum space.

And many more …

Karplus-Luttinger mechanism:

Mired in controversy from the start, it simmered for a long time as an unsolved problem, but has now re-emerged as a topic with modern appeal. – Ong @ Princeton

Old wine in new bottle

Karplus-Luttinger theory (1954)

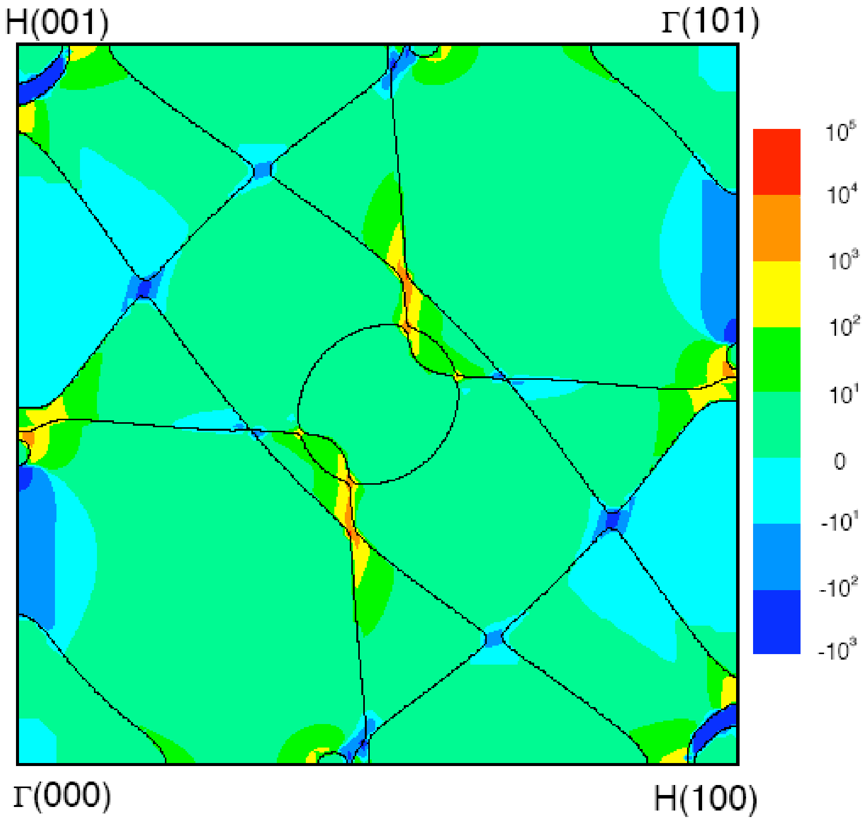
= **Berry curvature theory** (2001)

$$\sigma_{AH} = \frac{e^2}{\hbar}\int_{filled}\frac{d^3k}{(2\pi)^3}\vec{\Omega}(\vec{k}) \neq 0$$

$\rightarrow$ intrinsic AHE

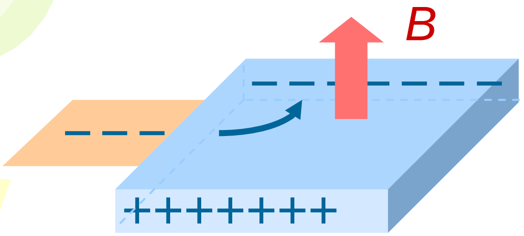
- same as Kubo-formula result
- *ab initio* calculation

Berry curvature of fcc Fe
(Yao et al, PRL 2004)

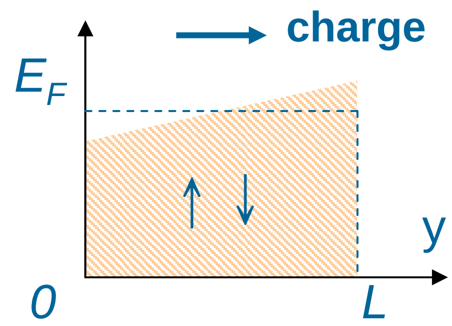


Ideal lattice without impurity

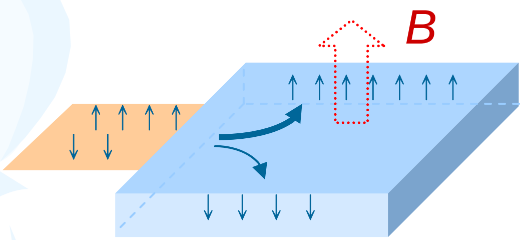
• **classical Hall effect**



✓ Lorentz force

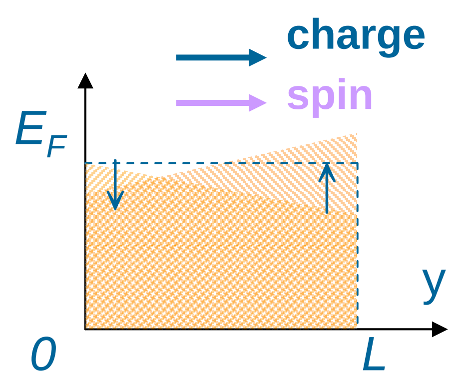


• **anomalous Hall effect**

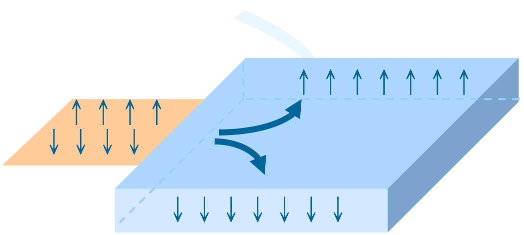


✓ Berry curvature

✓ Skew scattering

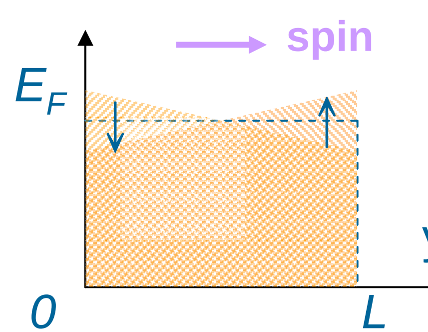


• **spin Hall effect**



✓ Berry curvature

✓ Skew scattering

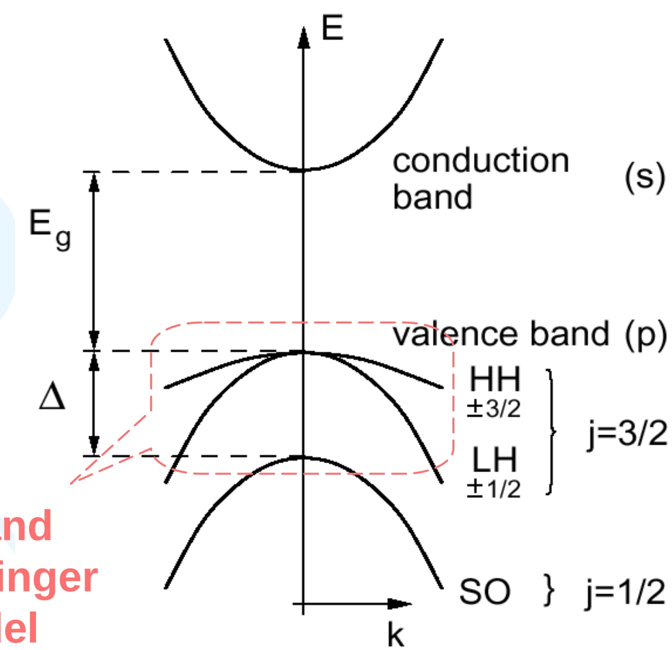


No magnetic field required !

Murakami, Nagaosa, and Zhang, Science 2003:

Intrinsic spin Hall effect in semiconductor

Band structure



4-band Luttinger model

The crystal has ***both*** space inversion symmetry and time reversal symmetry !

- Spin-degenerate Bloch state due to Kramer's degeneracy

→ Berry curvature becomes a 2x2 matrix (non-Abelian)

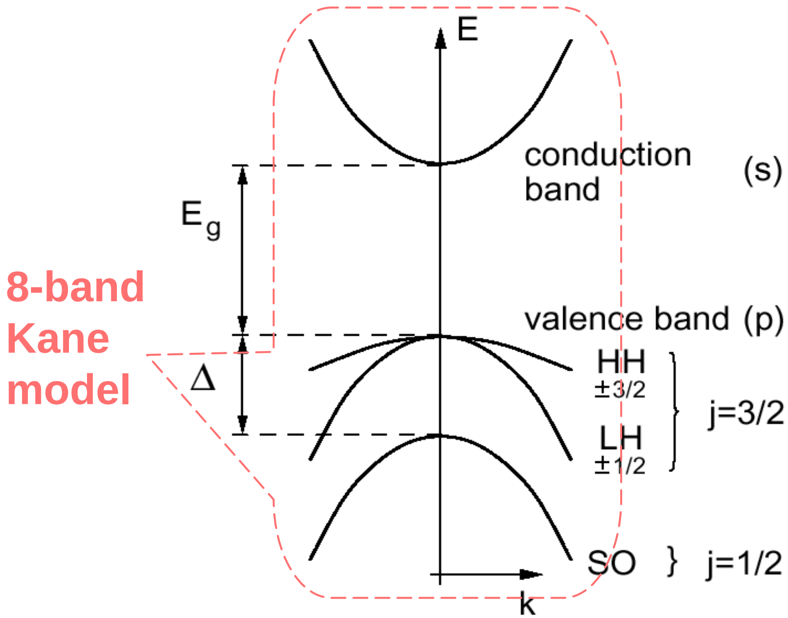
- **(from Luttinger model)** Berry curvature for HH/LH

$$\vec{\mathbf{\Omega}}_{HH}(\vec{k}) = -\frac{3}{2}\frac{\hat{k}}{k^2}\boldsymbol{\sigma}_z$$

$$\frac{d\vec{x}}{dt} = \frac{\partial E_n(\vec{k})}{\hbar \partial \vec{k}} - \frac{e}{\hbar}\vec{E} \times \left\langle \vec{\mathbf{\Omega}}_n \right\rangle$$

Spin-dependent **transverse velocity**
→ SHE for holes

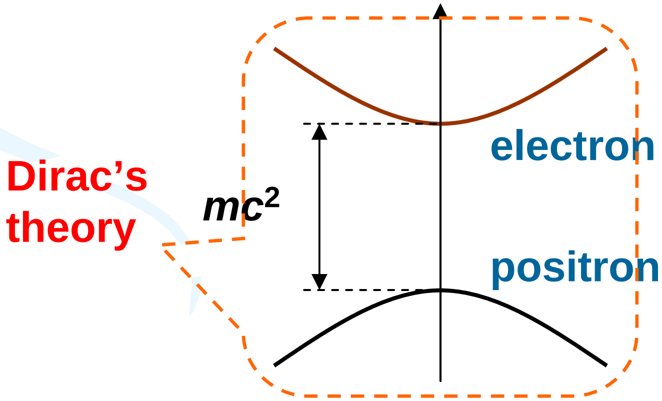
Only the HH/LH can have SHE? : Not really



- **Berry curvature for *conduction electron:***

$$\vec{\mathbf{\Omega}} = \alpha\,\boldsymbol{\sigma} + O(k^1)$$

spin-orbit coupling strength



- **Berry curvature for *free electron* (*!*)*:***

$$\vec{\mathbf{\Omega}} = -\frac{\lambda_C^2}{2}\boldsymbol{\sigma} + O(k^1)$$

$$\lambda_C = \hbar / mc$$
$$\approx 10^{-12}\,\mathrm{m}$$

Chang and Niu, J Phys, Cond Mat 2008

Observations of SHE (extrinsic)

Observation of the Spin Hall Effect in Semiconductors

Y. K. Kato, R. C. Myers, A. C. Gossard, D. D. Awschalom* Science 2004

Direct electronic measurement of the spin Hall effect

S. O. Valenzuela[1]† & M. Tinkham[1] Nature 2006

PRL 98, 156601 (2007) PHYSICAL REVIEW LETTERS week ending 13 APRIL 2007

Room-Temperature Reversible Spin Hall Effect

T. Kimura,[1,2] Y. Otani,[1,2] T. Sato,[1] S. Takahashi,[3,4] and S. Maekawa[3,4]

Giant spin Hall effect in perpendicularly spin-polarized FePt/Au devices

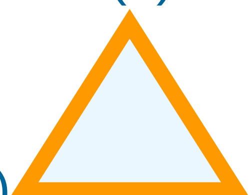
TAKESHI SEKI[1]*, YU HASEGAWA[1], SEIJI MITANI[1], SABURO TAKAHASHI[1,2], HIROSHI IMAMURA[2,3], SADAMICHI MAEKAWA[1,2], JUNSAKU NITTA[4] AND KOKI TAKANASHI[1] Nature Material 2008

Observation of Intrinsic SHE?

- Summary

Spin		Bloch state
• Persistent spin current • Quantum tunneling	• AHE • SHE	• Electric polarization • QHE

- Three fundamental quantities in *any* crystalline solid



Thank you!

Slides :
http://phy.ntnu.edu.tw/~changmc/Paper

Reviews:

- Chang and Niu, J Phys Cond Matt 20, 193202 (2008)
- Xiao, Chang, and Niu, to be published (RMP?)