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Carrier self-energy and photoluminescence spectrum in quantum Hall system

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Abstract

In this paper, we study electronic screening and its subsequent influence on the electron self-energy in quantum Hall system. We focus on two issues: how the width of quantum well and the strength of disorder affect the electron self-energy. It is found that a wider quantum well has a smaller self-energy correction, and a stronger disorder yields a smaller self-energy oscillation with respect to the magnetic field. We have also compared the theoretical many-body energy spectra with actual photoluminescence spectra of a modulation-doped quantum well in a strong magnetic field.

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Calculation of electron self-energy in quantum Hall system is crucial in understanding phenomena such as exchange-enhanced Zeeman splitting [1], the blue shift of photoluminescence (PL) spectrum near integer filling factors [2,3], the energy spectrum in modulated systems [4], and hysteresis effect [5,6]. The simplest many-body correction is calculated using the Hartree–Fock approximation (HFA). This is not necessarily a good approximation but often is chosen for its simplicity. In some cases, screened HFA is used [1,4], but in the absence of justification this may not necessarily yield an improved result over the HFA. In fact, it is known that, in the field-free case [7], screened HFA yields a worse result. Also, in the calculation of the PL spectrum for a quantum Hall system, using the screened HFA would predict a red shift, instead of the observed blue shift, near integer filling factors.

In this paper, we calculate the electron self-energy beyond the screened HFA by including the Coulomb-hole term. We study in detail the change of self-energy as various physical parameters, such as the width of quantum well and the strength of disorder, are changed [8]. [1] Calculating the conduction electron energy together with the valence hole energy enables us to calculate the PL spectra since the latter are largely determined by the energy difference between electrons and holes. The outline of this calculation has been published in the study of PL in Refs. [2,3]. It is believed

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[1] This paper studies how the distance between the electron layer and the hole layer affects the PL spectrum.

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that their calculations are applicable to dirtier samples where disorder effect is dominant, and many-body interactions are added later on as perturbations.

On the contrary, for high mobility samples where many-body effects are dominant, various different spectral features might appear. For example, the *abrupt* red shift near filling factor $\nu = 1$ has been attributed to different recombination mechanisms at $\nu = 1^+$ and 1^- [9–11]. In the vicinity of $\nu = 1$ quantum Hall ferromagnet, it costs less energy to create a skyrmion than to create a quasi-particle. Therefore, skyrmion-related features of the PL spectrum may appear [12–14]. Some groups observed a low-energy satellite peak in PL as $\nu > 2$, which has been explained as a signature of Anderson-Fano-like resonance [9,11,15]. In other occasions, multiplet structure near the neutral exciton recombination line has been observed, which is due to either charged excitons or low-lying many-body excitations [16,17]. The motivation of this work is to explain the PL spectra reported in Ref. [18], in which none of these many-body features seem to be relevant. The observed blue shift near integer filling factors is best explained using Katayama's and Uenoyama's slightly different theories [2,3]. Katayama and Ando use the plasmon-pole approximation, while Uenoyama and Sham use the quasi-static approximation, to obtain the dielectric function. We will follow the latter approach, which is computationally less involved, but is expected to be less accurate at high magnetic fields [19].

In the quasi-static random phase approximation, the electron energy is composed of the bare particle energy, the screened exchange energy and the Coulomb-hole energy [20–23]: [2]

$$E_n = E_n^0 + \Sigma_n^{\mathrm{sx}} + \Sigma_n^{\mathrm{ch}}, \tag{1}$$

where $E_n^0 = (n + 1/2)\hbar\omega_c + g_e\mu_e B$, n is the Landau level (LL) index, g_e and μ_e are the effective g-factor and the magnetic moment of electrons, respectively. For the many-body corrections, we have [3]

$$\Sigma_n^{\mathrm{sx}} = -\sum_{n'} \int \frac{\mathrm{d}^2\mathbf{q}}{(2\pi)^2} \tilde{V}_{nn'}(q)\nu_{n'},$$

$$\Sigma_n^{\mathrm{ch}} = \frac{1}{2}\sum_{n'} \int \frac{\mathrm{d}^2\mathbf{q}}{(2\pi)^2}[\tilde{V}_{nn'}(q) - V_{nn'}(q)], \tag{2}$$

where q is the magnitude of the momentum $\mathbf{q}$, $V_{nn'}(q) = F^{\perp}(q)F^{\parallel}_{nn'}(q)V(q)$, $F^{\perp}(q)$ is the form factor of the electron envelope function along the z-direction, $F^{\parallel}_{nn'}(q)$ is the form factor associated with the Landau orbitals in the x–y plane, and ν_n is the electron filling factor in the nth LL. The tilde indicates that the Coulomb potential $V(q) = 2\pi e^2/\varepsilon_0 q$ is screened, $\tilde{V}(q) = V(q)/\varepsilon(q)$, where $\varepsilon(q)$ is the static dielectric function to be explained in more details below. The form factors are determined by

$$F^{\perp}(q) = \int \mathrm{d}z \int \mathrm{d}z' |\phi(z)|^2 |\phi(z')|^2 \mathrm{e}^{-q|z-z'|},$$

$$F^{\parallel}_{nn'}(q) = |\langle n|\mathrm{e}^{\mathrm{i}\mathbf{q}\cdot\boldsymbol{\xi}}|n'\rangle|^2 = (n'!/n!)(q^2 l_B^2/2)^{n-n'}\mathrm{e}^{-q^2 l_B^2/2}[L_{n'}^{n-n'}(q^2 l_B^2/2)]^2 \quad (\text{for } n \geqslant n'), \tag{3}$$

where $\phi(z)$ is the electron envelope function along the z-direction, $\boldsymbol{\xi}$ is the cyclotron coordinate, $l_B = \sqrt{\hbar/eB}$ is the magnetic length, and L_α^β are the associated Laguerre polynomials. Notice that in the self-energy corrections above, the screened exchange part is a function of the carrier population. Therefore, for a system with dilute carriers, such an energy correction can be neglected because there is almost no quantum exchange effect. This is the case when these formulas are used to calculate the self-energy of dilute photo-excited valence holes in the PL experiment. Because of the low hole density, the self-energy for holes are dominated by the Coulomb-hole term only.

The dielectric function describes the effectiveness of screening and it is sensitive to the location of the Fermi level. The discrete nature of the LLs leads to prominent variation of this quantity as the magnetic field is changed. It can be separated to a dominant part related to intra-LL transitions and a minor part related to inter-LL transitions [3],

$$\varepsilon(q) = 1 + F^{\perp}(q)V(q)[D(E_{\mathrm{F}}) - \Pi_{\mathrm{inter}}(q)]. \tag{4}$$

The former is proportional to the disorder-broadened density of states (per unit area) $D(E)$, which is assumed to be a Gaussian (for each LL) with a width Γ.

[2] See Appendix B of Ref. [23] for a very nice derivation.

That is,

$$D(E) = \sum_n \frac{s}{2\pi l_B^2} \sqrt{\frac{2}{\pi}} \frac{e^{-2(E-E_n/\Gamma)^2}}{\Gamma}, \quad (5)$$

where s (=1 in our case) is the spin degeneracy. The width Γ depends on the magnetic field, $\Gamma = (\alpha/\sqrt{B})\hbar\omega_c$, in which B is in units of Tesla and α depends on sample quality and is of the order unity. The inter-LL part $\Pi_{\text{inter}}(q)$ is given as

$$\Pi_{\text{inter}}(q) = \frac{s}{2\pi l_B^2} \sum_{nn'} \frac{\nu_n - \nu_{n'}}{E_n^0 - E_{n'}^0} |\langle n|e^{i\mathbf{q}\cdot\boldsymbol{\xi}}|n'\rangle|^2. \quad (6)$$

It becomes more important in a weak magnetic field due to the smaller LL separations. In addition, its presence is also required for the dielectric function to satisfy the f-sum rule [3].

Combining this dielectric function with the formulas (1)–(3) above, we have calculated the electron self-energy for a n-type modulation-doped InGaAs/GaAs QW. The QW is approximated as a square well with a height V_0 and a width d. The electron envelope function is approximated as a Gaussian with a width σ, $|\phi(z)|^2 = \sqrt{\sigma/\pi}e^{-\sigma z^2}$. It is then convenient to obtain an analytic form for the form factor $F^{\perp}(q) = (2/\sqrt{\pi})e^{q^2/2\sigma}\text{erfc}(\sqrt{q^2/2\sigma})$. The width σ is determined by minimizing the electron energy in the Hatree approximation [24]. For a QW with a height $V_0 = 57$ meV and a width $d = 200$ Å [18], the optimized parameter for the Gaussian is $\sigma = 2.0 \times 10^{-3}/\text{Å}^2$. For the InGaAs/GaAs QW sample being used in the PL experiment [18], the electron effective mass is 0.0622 m_e, $g_e = -2$, $\varepsilon_0 = 12$, and the electron density after optical pumping is $2.7 \times 10^{11}/\text{cm}^2$. The parameter α in the LL broadening is chosen as 1/2, which gives $\Gamma = 4.68$ meV at $B = 10$ T. For comparison, the cyclotron energy is 18.6 meV at 10 T. It is known that the electron cyclotron energy $\hbar\omega_c$ is proportional to B and typical electron interaction energy $e^2/\varepsilon_0 l_B$ is proportional to $\sqrt{B}$. For reference, these two energies become equal at $B = 6.32$ T.

In Fig. 1, we plot the screened exchange energies and the Coulomb-hole energies for *electrons* in the lowest three LLs. The filling factors are 1–3 when the magnetic fields are 11.12, 5.56, and 3.73 T, respectively. When $B > 11.12$ T, the electrons are populated in the LLL so only Σ_0^{sx} and Σ_0^{ch} are of interest. As the magnetic field gets smaller, electrons are populated in more than one LL and several self-energies

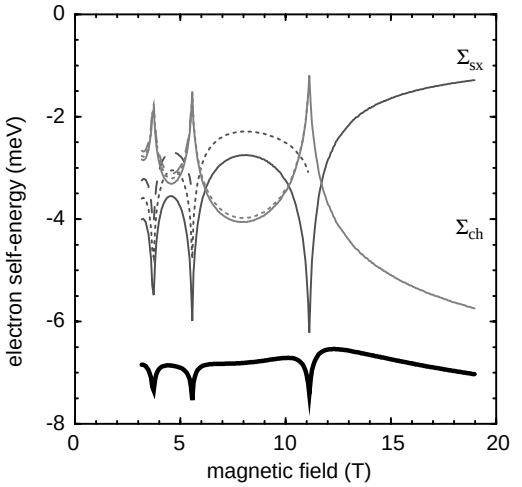

Fig. 1. Screened exchange energies and Coulomb-hole energies for the lowest three LLs are shown ($\alpha = 1/4$). The curves with downward cusps are Σ_n^{sx}; the curves with upward cusps are Σ_n^{ch}. Solid lines, dotted lines, and dashed lines are for $n = 0$, 1, and 2, respectively. The bold solid line below is the total self-energy for the LLL, which is the sum of the two solid lines above.

for different LLs are plotted. It can be seen that the Coulomb-hole energies for different LLs have roughly the same values, which makes the curves difficult to be distinguished in weak magnetic fields.

At integer filling factors, the electron gas is insulator-like and the screening is less effective. Therefore, there is a larger (in magnitude) negative screened exchange energy. This leads to the downward cusps in the figure. The Coulomb-hole energy is the energy reduction due to the charge deficiency around a charged carrier. At integer filling factors, the electrons are "locked" at their respective Landau orbitals, which means that it is more difficult to distort the wave function and create a Coulomb hole. Therefore, the negative Coulomb-hole energy is smaller (in magnitude) at integer filling factors. This gives the upward cusps in the figure. For electrons, the total self-energy, which is the sum of the two terms above, is less-sensitive to the filling factor because this two opposite trends from the screened exchange energy and the Coulomb-hole energy cancel each other [2,3]. (see the bold line in Fig. 1).

Naively, we expect the magnitudes of the cusps at integer filling factors to be proportional to $\sqrt{B}$, because the electron–electron interaction, which

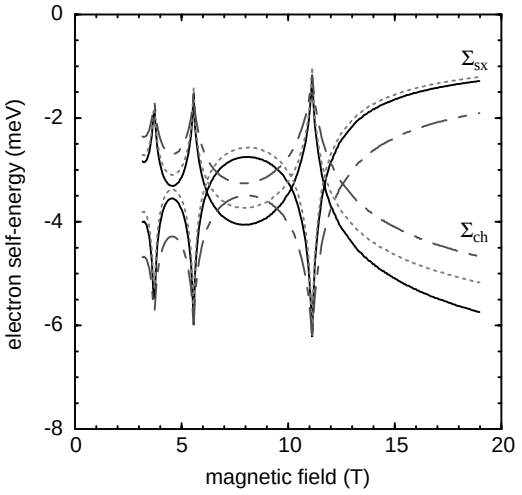

Fig. 2. Screened exchange energy and Coulomb-hole energy for electrons in the LLL. Solid lines are for a QW with width $d = 200$ Å; dotted lines are for a QW with a width $d = 300$ Å. The impurity broadening parameter α is 1/4 for both cases. Dot-dashed lines are for $d = 200$ Å and $\alpha = 1/2$.

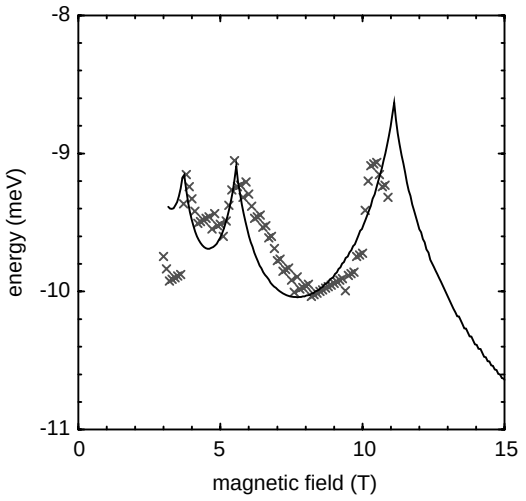

Fig. 3. Crosses are the PL energies for electrons transiting from the lowest electron LL to the highest valence hole LL that is permitted by the selection rule. The part linearly increasing with B has been removed, as explained in the text. The solid line is the theoretical result ($\alpha = 1/2$).

gives rise to the screened exchange term and the Coulomb-hole term, is proportional to $\sqrt{B}$. However, the effective interactions in different LLs can be very different because of the Landau-orbital form factors $F^{\perp}_{nn}$, which reduce rapidly as n gets larger. This is why the cusps diminish faster than $\sqrt{B}$ as B is decreased.

It is expected that, because the effective two-dimensional Coulomb interaction is reduced in a wider QW, the self-energy will be smaller. It is interesting to know quantitatively the connection between the width of the QW and the self-energy correction. In Fig. 2, we show the self-energy for the electrons in the LLL in a wider QW (dotted lines). It can be seen that the energies at the cusps remain largely unchanged, but the energies at half-integer fillings are indeed reduced by several meVs. We have also investigated the effect of impurity broadening on the self-energy. In general, thinner disorder-broadened LLs would give larger self-energy oscillations. In Fig. 2, we see that as the parameter α is varied (dot-dashed lines), the cusp energies also remain largely unchanged. But the energy shifts at half-integer fillings are significant. Unlike the changes due to finite QW width that have both Σ_0^{sx} and Σ_0^{ch} shift up. The shifts due to impurity broadening are opposite for Σ_0^{sx} and Σ_0^{ch}. Therefore, it would give a very minor change to the total self-energy. Even though we only show the result for the LLL in Fig. 2, the behavior for higher LLs are basically the same.

In Fig. 3, the theoretical PL energy is compared with the experimental data. The PL energy is the difference between the electron energy and the hole energy, $E^{PL}_{NN'} = E^{e}_{N} - E^{h}_{N'}$, where E^{e}_{N} is composed of three parts as shown in Eq. (1), E^{h}_{N} is composed of the bare particle energy, to be calculated from the Luttinger formalism [18], and the Coulomb-hole energy. The selection rule requires $\Delta N = 1$, where $N = n + m_j - 1/2$, in which n is the LL index, and m_j is the spin-component in the z-direction. We have calculated the Coulomb-hole energy for *holes* using the following parameters: hole effective mass $m_h = 0.094 m_e$, and the QW at the valence band has a depth $V_0 = 68$ meV and a width $d = 200$ Å. The solid line in Fig. 3 represents $\Sigma_N^{sx(e)} + \Sigma_N^{ch(e)} - \Sigma_{N'}^{ch(h)}$. The crosses are the PL peak energies being measured [18]. To emphasize the oscillating behavior, we have subtracted from the experimental data a part γB that is

linearly proportional to the magnetic field. The slope γ ($=0.9$ meV/T) is the sum of the slopes from the bare-electron energy and from the hole energy calculated using the eight-band Luttinger formalism. The data has also been vertically shifted to fit the curve. The only essential adjustable parameter in this calculation is the disorder-broadening parameter α, which is chosen to be $1/2$ for both the electrons and the holes. It can be seen that the magnitudes of the dips between the cusps agree roughly with the experimental data, supporting the aforementioned mechanism for the blue shift.

To summarize, we have investigated in details the influence of the QW width and the impurity broadening on the self-energy of the electrons in quantum Hall systems. It is found that a wider quantum well has a smaller self-energy correction, and a larger level-broadening yields a smaller self-energy variation with respect to the magnetic field. Comparison with experiment shows that the calculation can explain the observed data reasonably well. These quantitative results might be helpful for experimentalists to fine-tune the spectral properties of their samples.

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